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The indexation of financial assets : an economic analysis for monetary policy

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The Indexation of Financial Assets : An Economic Analysis for Monetary Policy

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JURY DU MEMOIRE :

MM. Jacques de GROOTE
Charles JAUMOTTE

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C O N T E N T S

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INTRODUCTION

During the last three decades, a number of countries have experimented with the index-linkage of fixed-interest financial assets in face of the continuous post-war inflation.
The most striking and well-known examples are Finland and Israel, but in other industrialized countries, the index-linkage has been applied to some financial assets as well : in France, private and nationalized companies issued index-linked bonds, in the Netherlands and in Denmark mortgage loans have been recently tied to an index as well.

Such an index-linkage of financial assets is, of course, in many respects a basically institutional arrangement. Hence a theoretical approach to such a measure should at best be elaborated in view of economic and, more precisely, monetary policy.

Unfortunately, decisions to link financial claims to an index in face of a continuous inflation have never been thought of as a means of economic policy in at least two countries where the index-linkages were applied on a large scale : in both Israel and Finland, the implementation of financial index-linkages was achieved under particular historical circumstances. As a result, one may not come relevant and general conclusions on the effects of an index-linkage from the only empirical observation of those experiments.

Hence, what is needed is an elaborate theoretical analysis of a general character so as to be in a position to evaluate ex ante the costs and benefits associated with index-tied financial instruments in a free-market economy that experiences a continuous inflation.
Account of particular institutional characteristics may then be taken by appropriate modifications of the assumptions made in such a theory.
We aim at providing the policy-maker with a strong theoretical tool to evaluate the index-linkage of financial assets as a solution to the financial problems that arise in inflationary economies and to state the theoretical conditions for an efficient working of the index-linkage.

The financial problems arising in the face of inflation, will not be analyzed in the framework of a particular type of inflationary process as defined by its causes, i.e., for instance, a demand or a cost-push inflation. This would involve a loss in generality and any conclusion on the effects of a financial index-linkage would be subject to the particularities of the inflationary process assumed.

We shall rather contemplate the financial effects of inflation most generally defined as a permanent increase in value of a broadly representative index, whatever the causes of this increase in value may be.
In such a theoretical approach to financial indexation as a monetary policy measure, answers should be given to a few questions that are directly related to the practical problems of its implementation.

These questions could be formulated as follows :

- Is there any good reason to expect index-tied financial assets to be favourably accepted by the general public, in other words will they effectively compete with other types of assets that the public may have decided to hold in face of a continuous inflation ?
- The future real value of all monetary assets becomes uncertain in times of inflation and the index-linkage applied to those assets is likely to remove this risk-effect of inflation.
  As it seems hard to conceive to link all monetary assets to an index, is there any good reason to link one monetary asset rather than another monetary asset to an index ?
- What are the economic variables to be considered when assessing the chances for the index-linkage to be successfull and to work in the way expected ?
- What are the responsabilities of the monetary authorities in connection with the problems that may arise, once financial index-linkages have been effectively implemented ?
  Is there any need of management of index-tied financial systems ?
- Does the indexation technique serve other specific purposes of monetary policy, apart from its being a solution to some financial distorsions brought about by inflation ?

The first question will be answered in our chapter I.
The problem of the acceptance of index-tied monetary assets is crucial. Indeed, if nothing points to the potential demand for this type of asset, it cannot be issued as a debt-instrument, unless issues are made at very unfavourable terms. This problem will be tackled in the framework of a

general model of demand for assets.
It will be shown that inflation causes a reallocation of wealth from monetary into real assets. Considering the properties of real assets from the risk point of view, this reallocation may be expected to decrease general welfare. Consequently, if an index-linkage applied to monetary assets preserves the more optimal allocation under general price-stability, there can be no doubt about the demand for index-tied assets in inflationary economies.

Our chapter II demonstrates that monetary assets may be ranked according to the amount of risk borne in inflation, bonds being the most risky monetary asset and money the less risky monetary asset.
This is because interest-bearing monetary assets may change in nominal market-value which money cannot. If there is some relation between inflation and those changes in market-value, i.e. in rates of interest, long-term monetary assets will be more risky than short-term monetary assets and money. This relation between inflation and rates of interest will be examined in a macro-economic model. Its implications will be analyzed at a macro-economic level. The main result will be a general tendency to shorten the average maturity of monetary debts and assets, which materializes, for instance, in a gradual disappearance of markets for bonds and fixed-interest long-term capital.

In our chapter III, we aim at demonstrating how the index-linkage solves the problem of inflation - induced changes in market-value of long-term monetary assets.
This may be done through a simultaneous analysis of the difference in response of market-values of nominal and index-tied asset-prices to changing rates of inflation. As the power of the argument is in the comparison, we first develop a theory of the relation between the rates of interest on nominal and index-tied assets, which will be integrated into the macro-model of chapter II.
Some elements of this partial equilibrium will be tested in an econometric approach to the demand for nominal and index-tied assets in Finland.
It will be shown as a by-product that, if there is no relation between

the market-value of monetary assets, i.e. their rates of interest, and inflation, in which case no ranking of monetary assets according to the amount of risk borne in inflation can be established, the index-linkage will not effectively work as a risk-reducer.

In our chapter IV, some attention will be paid to the expected perverse effects of the index-linkage inherent in the dynamics of adjustment. As we will show, these perverse effects, may be avoided through a more active open-market policy and active management of index-tied financial systems. Moreover the index-linkage may be useful from many other points of view. It will make debt-management more effective in inflation and reduce the risk of default in stabilization-policies.

Those four chapters deal with, we think, the main elements to be considered in an approach to the index-linkage of financial assets as a measure of monetary policy.

We tried to keep the whole analysis as close as possible to current economic theory and techniques.
This work may be seem as an attempt to demonstrate how these current economic theory and techniques may be used in an appraisal of the costs and benefits associated with such an institutional arrangement as the index-linking of financial assets.

Chapter I

AN INQUIRY INTO THE POTENTIAL DEMAND FOR INDEX-TIED ASSETS

Introduction

This chapter is an inquiry into the possible demand for index-tied financial assets, most generally defined as securities whose interest-payments and redemption-values are linked to a specific value-index, so as to preserve their purchasing-power as measured by this index.

The analysis is deliberately developed in the context of inflation, not of deflation. It is a preliminary approach to a global discussion of the expected benefits and costs associated with index-linked financial assets.
Indeed, a first thing to do is considering all the elements that point to a potential demand for this type of assets in inflationary economies.
If there is no demand for this kind of assets, it cannot be issued as a debt-instrument, unless the market-rates of interest are very high.
Consequently, no debtor would raise loanable funds through the issue of index-tied claims and any further discussion on the merits of index-tied debts would be aimless.

The attractiveness of and hence the potential demand for index-tied assets depends on their ability to compete with other types of assets in times of inflation.

Consequently, the potential demand for index-tied assets must be evaluated in the framework of a system of demand-equations for assets.
The demand for each asset is assumed to depend on its own rate of return, on the rates of return on all the alternative assets and on a number of parameters that refer to the risk-properties of those assets.

In the framework of such a demand system, we shall try to evaluate the effects that inflation will have on the demand for broad groups of assets, once inflation comes to be expected.
Inflation will be defined as a permanent increase in value of a representative index that reflects the nominal prices of goods and services currently

consumed. Those goods and services are exogeneous to the model. The increase in the index must be regarded as an exogeneous variation in the parameters of the model that are "deflated" by the index.
The broad groups of assets which we shall consider are defined according to the impact of inflation on the real value of the incomes yielded by those assets.
The group of real assets contains all those assets yielding incomes that are measured by flows of goods and services produced.
The group of monetary assets contains all those assets yielding incomes that are fixed in terms of money.

The effets that expected inflation will have on the demand for monetary and real assets will be evaluated under the assumption that so far no index-tied asset is available.

After having shown that expected inflation increases the demand for real assets as a group at the expense of the demand for monetary assets as a group, we shall consider those real assets that are currently regarded as effective inflation-hedges.

Contemplating the empirical properties of those inflation-hedges, we may evaluate the empirical conditions of a portfolio-reallocation in times of inflation and compare the optimal portfolio in times of inflation with the optimal portfolio in times of general price-stability.

If the availability of index-tied assets - i.e. assets linked to the representative index refered to above - would have allowed a better allocation of portfolios in times of inflation, there can be no doubt about the potential demand for index-tied assets.

From a methodological point of view, the effets of inflation on the demand for monetary and real assets will be deduced as follows :

- The set of demand - equations refered to above will be deduced from a maximized utility function subject to a wealth constraint.
  The demand for each asset will prove to be a function of all the rates of return and the variance-covariance parameters.
  A comparative static analysis may be applied for a change in all those parameters.

So far the procedure is entirely analogous to Sh. Royama's model(1).

- As we focus the analysis on the demand for groups of assets rather than on the demand for a single asset, we shall resort to the second-order conditions for a maximum and to Hickx'inequalities between rates of substitution in order to get meaningful results for groups of assets. This application of Royama's model to a study of inflationary effects remains ours.

- We assumed that the effects of expected inflation could be validly approximated by small variations in the parameters that enter the demand-functions for assets, i.e. by a comparative statics analysis.
This implies that a state of inflation should be considered as very close to a state of general price-stability which might be true for very low rates of inflation only.

- The utility function chosen is quadratic. With this type of function optimum portfolios are calculated as functions of expected rates of return and of variance-covariance parameters that refer to the risk-properties of those assets. This technique was in fact innovated by Harry Markowitz(2) who proved that, if investors calculate the risk on assets by the variance of the rates of return, then utility functions will be quadratic.
J.-K. Arrow, however, has demonstrated that the investor's behaviour may not be validly expressed by a quadratic utility function, as such a function impliex a risk-aversion that increases with the investors' wealth. In other words, the richer the investor the greater is aversion to risk. Arrow rejects such a behaviour, because it seems to contradict empirical observation(3).
Hence we shall assume that the quadratic utility-function is a good approximation of the investor's true utility-function.

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(1) Sh. ROYAMA, "Substitution and Complementarity in the Choice of Risky Assets", in D. Hester and James Tobin: "Risk-aversion and portfolio-choice", N.Y. 1967.
(2) MARKOWITZ Harry: "Portfolio Selection.Efficient diversification of Investments", N.Y. 1959.
(3) ARROW John-K.: "Aspects of the Theory of Risk-hearning", Yyö Jahnsson Lectures, Helsinki 1965.

1.1. A GENERALIZED THEORY OF LIQUIDITY-PREFERENCE AND THE RANGE OF ASSETS CONSIDERED

The keynesian theory of liquidity-preference attempts to explain the transactional terms at which creditors are ready to transfer purchasing-power to debtors.

In equilibrium, the more investment is marginably productive, the more creditors are willing to finance it, as rates of interest are high.
The individuals'gain through interest-earnings coincides with the marginal productivity of the community's capital stock. Moreover, in the long-run, the growing capital stock tends to increase the marginal productivity of a given stock of labour and hence the real wage rate. So far the social benefits from investment go hand in hand with the individuals'gain from interest-earning assets.

Although such a theory gives insight in the process of capital accumulation, it is necessarily simplified, as it restrains too heavily the ultimate wealth-owners'choice between various types of assets.
One must realistically enlarge the keynesian range of assets that are all monetary, as far as ultimate wealth-holders are concerned, so as to include "real" assets that the public may consider as possible alternatives to monetary assets.
In the context of inflation, defined as a steady decline in the purchasing-power of money, this generalization of liquidity-preference becomes a crucial point. Indeed, as the real value of all assets labeled in monetary units is affected by the inflation so defined, inflation is likely to affect the demand for those assets, as soon as wealth-owners become inflation-conscious. They may consider, for instance, that in the face of inflation holding "real" assets safeguards their wealth in terms of purchasing-power more efficiently than holding monetary assets.
Would this mean that in times of inflation wealth-owners should convert all their monetary assets into real assets ?

Quite clearly we need a general theory of demand for assets to specify all the relevant parameters that individuals may be assumed to consider when deciding to hold their wealth in different forms.

Meaningful results may then be obtained if it can be shown how inflation affects those relevant parameters, when it comes to be expected.

Troughout the next theoretical developments, we will assume the absence of any index-linked assets; il we may confront later on the meaningful results to the so far plausible characteristics of index-tied financial assets, we may find out what elements point to the potential demand for index-tied assets in inflationary economies.

1.2. A MODEL OF DEMAND FOR ASSETS

1.2.1. Some behavioural assumptions

a) When allocating his wealth, the investor is assumed to be sensitive to the rate of return expected on his portfolio and to its risk.
Harry Markowitz (op. cit.) has shown that, if the investor measures the risk of his portfolio by the variance of the rate of return, his utility function must be quadratic.
The use of such a function is mathematically convenient and the quadratic function is a good approximation to other concave functions.
The investor's behaviour is assumed to be validly approximated by Tobin's (1) quadratic utility function :

$$u = (1+b)_y + b\, y^2$$ where u = ordinal utility function
y = total real wealth.

The function is assumed to be always increasing with real wealth. Hence we have :

$$\frac{\delta u}{\delta y} = (1+b) + 2\, b\, y > 0$$

$$y > -\frac{1+b}{2\, b} \qquad \text{for } b > 0$$

$$y < -\frac{1+b}{2\, b} \qquad \text{for } b < 0$$

b) Aggregation over the assets

The model of demand will be derived from a maximized utility function subject to a wealth constraint. This procedure may possibly not be used in the case of assets because of the extreme volatility of asset-market.

(1) TOBIN, James, "La préférence pour la liquidité en tant que comportement face au risque", in Richard S. THORN, "Théorie monétaire", pp. 153, 182, Dunod, 1971.

An approach by partial derivatives might then remain without any significance. This is why we assume partial aggregation over largely substituable assets, say, for instance, monetary claims with different maturities.
The prices and rates of return of the aggregates are then likely to be more sticky and not affected by rather erratic fluctuations.
We assume the investor allocates his wealth between "n" of such assets.

c) The investor's parametric environment.

Each asset "i" has a rate of return "g_i", which is a stochastic variable following a distribution that we need not specify further than by its mean and variance.
This rate of return is equal to current income earned on the asset, or the services provided, plus any possible appreciation or depreciation in the asset's price minus possible user-costs such as physical depreciation.

Mathamatically, we have

$$E(g_i) = \bar{g}_i \quad \text{for the mean}$$

$$E(g_i - \bar{g}_i)^2 = \sigma_{ii} \quad \text{for the variance}$$

$$E\left[(g_i - \bar{g}_i)(g_j - \bar{g}_j)\right] = \sigma_{ij} \quad \text{for the covariance between } g_i \text{ and } g_j.$$

The matrix $[\sigma_{ij}]$ of all the variance-covariance parameters must be positive semi-definite.
This is because risk cannot be negative, and if x_i is the amount of asset i held, total risk on the portfolio will be :

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sigma_{ij} x_i x_j. \text{ And } \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} x_i x_j \geq 0 \text{ hence } [\sigma_{ij}] \text{ is positive-semi definite.}$$

d) The time-horizon and maximizing behaviour.

The investor is assumed to maximize expected utility at the end of a single period. In the absence of debt issues against himself this investor's maximizing behaviour is subject to the constraint of his initial wealth.

1.2.2. Demand equations for assets

The main lines of the mathematical analysis below are analogous to Sh. Royama's approach. The application of the model as an approach to inflation remains ours.

By virtue of our hypothesis a) and d), the demand equations for assets may be yielded by maximizing the Lagrangean L :

$$L = E\left[u\right] + \lambda \left[\sum_{i=1}^{n} x_i - y_o \right]$$ where x_i = amount of asset "i"

y_o = amount of initial wealth.

or $$L = E\left[(1 + \quad)y + b\, y^2\right] + \lambda \left[\sum_{i=1}^{n} x_i - y_o \right]$$

where y is real wealth at the end of the period.

$$y = \sum_{i=1}^{n} x_i(1 + g_i)$$ where y is stochastic as the g_i's are stochastic.

hence $$L = E\left[(1+b)\sum_{i=1}^{n} x_i(i+g_i) + b\left(\sum_{i=1}^{n} x_i(1+g_i)\right)^2\right] + \lambda\left[\sum_{i=1}^{n} x_i - y_o\right]$$

$$L = (1+b) \sum_{i=1}^{n} x_i(i+\bar{g}_i) + b \sum_{i=1}^{n} \sum_{j=1}^{n} \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j)\right] x_i x_j + \lambda\left[\sum_{i=1}^{n} x_i - y_o\right]$$

One may usefully notice that

a) $L = (1+b)\, E(y) + b\left[E(y)\right]^2 + b.V$

where $E(y) = \sum_{i=1}^{n} x_i(i+\bar{g}_i)$ is the expected real wealth at the end of the period

and $V = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij}\, x_i\, x_j$ is the variance of the total return on the portfolio, i.e. its risk.

It follows that for risk-lovers $b > 0$

and for risk-averters $b < 0$.

b) A fundamental difference with neoclassical demand theory is that the "objective"parameters-yields and covariances - enter the utility function.

Maximizing L with respect to x_i, i=1...n, and λ we get :

$$\begin{cases} \dfrac{\delta L}{\delta x_i} = (1+\bar{g}_i)(1+b) + b \displaystyle\sum_{j=1}^{n} \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j)\right] x_j - \lambda = 0 \quad \text{for } i=1\ldots n \\ \dfrac{\delta L}{\delta \lambda} = \sum x_i - y_o = 0. \end{cases}$$

Those are the first order conditions.
The second order conditions will be specified in a comparative statics analysis below.

The first-order conditions yield a system of (n+1) simultaneous equations in (n+1) parameters $(1+\bar{g}_i)$ i=1...n and y_o.

For a given variance-covariance matrix, we may then express the vector $[\underline{x}, \lambda]$ as a function of the vector $[\underline{1+\bar{g}}, y_o]$.

This yields our demand equations :

$$x_i = x_i\left[(1+\bar{g}_1, \ldots (1+\bar{g}_n) ; y_o ; \sigma_{ke}\right]$$

for k,e = 1... n

i = 1... n.

and $$\lambda = \lambda\left[(1+\bar{g}_1), \ldots, (1+\bar{g}_n) ; y_o ; \sigma_{ke}\right].$$

Analyzing the effects of inflation on wealth-allocation behaviour amounts to considering its effects on the parameters entering the demand functions for assets.

The yield effects of inflation may be obtained by changes in the real rates of return. The risk-effects of inflation may be deduced from changes in the elements of the variance-covariance matrix. This is all a matter of comparative statics.

1.2.3. The yield-effect induced by inflation

1.2.3.1. Inflation and proportionnal changes in real rates of return

We define anticipated inflation as an expected increase in value of the representative index used by the investor when converting his nominal wealth into real wealth.

The index must obviously exclude the prices of any of the "n" assets in the investor's portfolio. It will rather reflect the value of money in terms of current consumption goods.

So defined the inflation affects the real rates of return on all those assets the income **cn** which is fixed in terms of money.

It does not affect the rates of return on all those assets yielding incomes measured in terms of real goods and services provided with prices increasing at the rate of our representative index.

Hence changes in the rate of inflation induce changes in the real rates of return on a large number of assets. Quite obviously we need an analytical tool that allows to deal with the demand for a group of assets.

The Hickxian theory of demand(1) states that for proportionnal changes in their prices a bundle of goods behaves as a single good. This statement is known as the Leontieff-Hickx Theorem.

Does the inflation bring about a proportionnal change in the real rates of return on a given bundle of assets ?

This will be true, if the real rate of return does not take into account possible price variations of assets and if it is defined as the income yielded by an asset, expressed as a percentage of its original price.
In this case, the real rate of return on typically monetary assets changes with the rate of inflation. The real rate of return on assets yielding incomes in the form of real goods and services will not be such a function of the rate of inflation.

If price variations were to be considered changes in real rates of return would not be proportionnal. One possible price-variation, for instance, could be that one induced by increasing rates of interest when capital markets tend to discount the expected inflation. Under such conditions, the longer the maturity of interest-hearing debts the greater the capital losses. This complicated matter has a more macro-economic character and will be treated in our next chapter.

We may not prove the Leontieff-Hickx Theorem in the case of asset-choices, unless we have calculated the rates of substitution among assets and the relations linking those rates. The rates of substitution are generated by a comparative statics analysis of a change in a rate of return, the relations by a development of the second order conditions.

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(1) Sir J. HICKX, Valeur et Capital, Paris, Dunod 1968, p. 45.

1.2.3.2. A Comparative statics analysis of a change in a real rate of return

Our first order conditions for an equilibrium were :

$$\begin{cases} (1+b)(1+\bar{g}_i) + b \sum_{j=1}^{n} \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right] x_j - \lambda = 0 \quad \text{for } i=1\ldots n \\ \sum_{i=1}^{n} x_i - y_o = 0 \end{cases}$$

How does the system respond to a variation in $\bar{g}_e$?

Differentiating for d $\bar{g}_e$, we have :

$$\begin{cases} b \sum_{j=1}^{n} \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right] \frac{\delta x_j}{\delta \bar{g}_e} + b(1+\bar{g}_i)x_e - \frac{\delta \lambda}{\delta \bar{g}_e} = 0 \quad \text{for } \forall\, i \neq e \\ (1+b) + b \sum_{j=1}^{n} \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right] \frac{\delta x_j}{\delta \bar{g}_e} + b(1+\bar{g}_e) + b \sum_{j=1}^{n} x_j(1+\bar{g}_j) - \frac{\delta e}{\delta \bar{g}_e} = 0 \\ \qquad \text{for } i \neq e \\ \sum_{i=1}^{n} \frac{\delta x_i}{\delta \bar{g}_e} = 0 \end{cases}$$

In matrix form, we have :

$$\begin{bmatrix} m_{11} & \ldots & m_{1n} & -1 \\ \vdots & & & \\ m_{e1} & \ldots & m_{en} & -1 \\ m_{n1} & \ldots & m_{nn} & -1 \\ -1 & \ldots & -1 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1 / \delta \bar{g}_e \\ \vdots \\ \delta x_e / \delta \bar{g}_e \\ \delta x_n / \delta \bar{g}_e \\ \delta \lambda / \delta \bar{g}_e \end{bmatrix} + \begin{bmatrix} (1+\bar{g}_1)x_e \\ \\ (1+\bar{g}_e)x_e + \sum_{j=1}^{n} x_j(1+\bar{g}_j) + \frac{1+b}{b} \\ (1+\bar{g}_n)x_e \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

where $m_{ij} = b \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right]$

Writing the successive matrixes as [M], [D], [G] and [O] we may write the system more conveniently as :

$$[M]\,[D] + [G] = [O]$$

and

$$[D] = -\,[M]^{-1}\,[G]$$

$$\text{where } [M]^{-1} = \begin{bmatrix} m_{11} & \cdot\cdot\cdot\cdot\cdot & m_{1n} & -1 \\ m_{e1} & & m_{en} & -1 \\ m_{n1} & & m_{nn} & -1 \\ 1 & & -1 & 0 \end{bmatrix}^{-1} = \frac{1}{v} \begin{bmatrix} v_{11} & v_{1n} & v_{1n+1} \\ v_{e1} & v_{en} & v_{en+1} \\ v_{n1} & v_{nn} & v_{nn+1} \\ v_{n+11} & v_{n+1n} & v_{n+1\ n+1} \end{bmatrix}$$

where v_{ij} is the cofactor of m_{ij} and v the determinant of [M]

For such a notation, the elements of [D] may be written as :

$$\frac{\delta x_i}{\delta \bar{g}_e} = - \left[\frac{1+b}{b} + \sum_{j=1}^{n} x_j \, (1+\bar{g}_j)\right] \frac{v_{ie}}{v} - x_e \sum_{h=1}^{n} (1+\bar{g}_h) \frac{v_{ik}}{v}$$

Sh. **Royama** (op. cit. p. 31) has shown that the second term at the right-hand side of the equation is a wealth-effect that is analogous to the income-effect in usual demand theory.

Hence the first term is the effect of a change in $\bar{g}_e$ on x_i provided that the investor is compensated for the change in $\bar{g}_e$ so as to leave him on the same level of **utility**.
This is essentially the interpretation of the substitution term in usual demand theory.

Writing the substitution term as S_{ie}, we have

$$S_{ie} = - \left[\frac{1+b}{b} + \sum_{j=1}^{n} x_j (1+\bar{g}_j)\right] \frac{v_{ie}}{v}$$

$$= - \left[\frac{1+b}{b} + E(y)\right] \frac{v_{ie}}{v}$$

The term in brackets is positive because $y > - \frac{1+b}{2\,b}$ for $b < o$.

1.2.3.3. A development of the second-order conditions

Following Samuelson (1), we may write the second-order conditions for our problem as :

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(1) SAMUELSON, P., Fondements de l'Analyse Economique, Gauthier-Villars, p. 322.

$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\delta L}{\delta x_i \, \delta x_j} h_i \, h_j < 0 \\ \sum_{i=1}^{n} (+1) \, h_i = 0 \quad \text{derived from} \quad \frac{\delta \left[\Sigma x_i - y_o \right]}{\delta x_i} \end{cases}$$

This becomes for Tobin's utility function :

$$\begin{cases} \sum_{i=1}^{n} \sum_{j=1}^{n} b \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right] h_i \, h_j < 0 \\ \sum_{i=1}^{n} h_i = 0 \end{cases}$$

The matrix $\left[\sigma_{ij} \right]$ is positive semi-definite.

We assume $\left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right]$ to be positive-definite.

Hence $b \left[\sigma_{ij} + (1+\bar{g}_i)(1+\bar{g}_j) \right]$ is negative definite for $b < 0$

This means that a regular maximum may be reached in the only case where the investor has an aversion to risk. As all our conclusions on the effects of inflation will be derived from the second order conditions for a maximum, we must continue the analysis under this **altogether** realistic assumption.

Hickx(1) derived from the second order conditions a number of rules that apply to the rates of substitution and may be used in the context of our problem as we have similar conditions for a maximum.

- Hickx derived his "third rule", $\sum_{j=1}^{n} p_j \, S_{ij} = 0$, from an expansion of the bordered Hessian relevant to the behaviour of the consumer.

Our bordered Hessian M is :

$$M = \left[\begin{array}{c|c} m_{ij} & -1 \\ \hline -1 & 0 \end{array} \right]$$

. Expanding by the last row we have

$$(-1)v_{r1} + (-1)v_{r2} + \ldots + (-1)v_{rn} + 0.V_{rn+1} = 0.$$

$$\text{Hence}(-1) \sum_{j=1}^{n} v_{rj} = 0 \Rightarrow \sum_{j=1}^{n} S_{rj} = - \left[\frac{1+b}{b} + E(y) \right] \sum_{j=1}^{n} \frac{v_{rj}}{v} = 0.$$

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(1) Sir J.R. HICKX, Valeur et Capital, Paris, Dunod 1968, p. 289.

- Deriving his "fourth rule" he found $\sum_{r=1}^{m} \sum_{s=1}^{m} \frac{U_{rs}}{U} z_r z_s < 0 \quad 1 \leq m < n$

where : U_{rs} is the cofactor of u_{rs}

(z_r, z_s) may have any value.

By the way of analogy, we have $\sum_{r=1}^{m} \sum_{s=1}^{m} \frac{V_{rs}}{V} z_r z_s < 0$

for $1 \leq m < n$ and any value of (z_r, z_s)

So we have : $$\sum_{i=1}^{m} \sum_{j=1}^{m} S_{ij} z_i z_j = - \underbrace{\left[\frac{1+b}{b} + E(y)\right]}_{>0} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{V_{ij}}{V} z_i z_j \quad 0$$

for $1 \leq n < m$ and any value of $(z_i z_j)$

We shall need this property in the analysis of all the inflation-induced effects.

1.2.3.4. Applying the Leontieff-Hickx Theorem

We may now devide the set of "n" available assets into two subsets according to the impact of changes in the general price level, as measured by our representative index, on the real value of incomes earned on those assets.

The first subset $(x_1 \ldots x_m)$ contains those assets earning incomes that are fixed in terms of money. They will be generally defined as "monetary assets". Real incomes earned on chose assets decline at a rate equal to the rate of inflation. The second subset $(x_{m+1} \ldots x_n)$ contains those assets earning incomes in terms of goods and services provided and hence the inflation leaves their real value unchanged.

We may now consider the global substitution effect that inflation induces in the wealth-allocation process.

The marginal change in demand for asset "i" in response to a change in a real rate of return $\bar{g}_e$ is expressed as (cfr p.11)

$$\frac{\delta x_i}{\delta \bar{g}_e} = - \left[\frac{1+b}{b} + \sum_{j=1}^{n} x_j (1+\bar{g}_j)\right] \frac{V_{ie}}{V} - x_e \sum_{k=1}^{n} (1+\bar{g}_k) \frac{V_{ik}}{V}$$

Writing the substitution effect as S_{ie}, we have :

$$\frac{\delta x_i}{\delta \bar{g}_e} = S_{ie} - x_e \sum_{k=1}^{n} (1+\bar{g}_k) \frac{v_{ik}}{v}$$

The variation in the expected real gain on asset "i" in response to a proportionnal change in the real rate of return $\bar{g}_e$ may be expressed as :

$$\bar{g}_i \frac{\delta x_i}{\delta \bar{g}_e / \bar{g}_e} = \bar{g}_e \, \bar{g}_i \frac{\delta x_i}{\delta \bar{g}_e} = \bar{g}_e \, \bar{g}_i \, S_{ie} - \bar{g}_i \, \bar{g}_e . \, x_e . \sum_{k=1}^{n} (1+\bar{g}_k) \frac{v_{ik}}{v}$$

The variation in the expected real gain on asset "i" in response to proportionnal changes in "m" real rates of return $\bar{g}_e$ on "m" monetary assets may be expressed as :

$$\sum_{e=1}^{m} \bar{g}_e \, \bar{g}_i \frac{\delta x_i}{\delta \bar{g}_e} = \sum_{e=1}^{m} \bar{g}_e \, \bar{g}_i \, S_{ie} - \sum_{e=1}^{m} \bar{g}_e . x_e \, \bar{g}_i \sum_{k=1}^{n} (1+\bar{g}_k) \frac{v_{ik}}{v}$$

The variation in the expected real gain on the whole group of "m" monetary assets in response to proportionnal changes in the "m" real rates of return on "m" monetary assets may be expressed as :

$$\sum_{i=1}^{m} \sum_{e=1}^{m} \bar{g}_e \, \bar{g}_i \frac{\delta x_i}{\delta g_e} = \sum_{i=1}^{m} \sum_{e=1}^{m} \bar{g}_e \, \bar{g}_i \, S_{ie} - \sum_{i=1}^{m} \sum_{e=1}^{m} \bar{g}_e . x_e . \bar{g}_i \sum_{k=1}^{n} (1+\bar{g}_k) \frac{v_{ik}}{v}$$

The first term of the right-hand side of the equation is the global rate of substitution, the second term is the global-wealth effect.
By virtue of our second-order conditions, we know that

$$\sum_{i=1}^{m} \sum_{e=1}^{m} \bar{g}_e \, \bar{r}_i \, S_{ie} > 0 \qquad \text{cfr } (1.2.3.3)$$

The sign of the substitution effect is definitely positive.
So the change in demand for monetary assets depends as the sign and magnitude of the wealth-effect.

In the case of demand for goods, Hickx considers negative income-effects as very unlikely, whenever the size of a group is large. His argument may be analogously invocated in the case of asset-choices.

So far, we achieve the next meaningful result :

When the investor expects an increase in the rate of inflation, he enlarges the share of real assets in his total real wealth, as the inflation effects negatively real gains on monetary assets.

This is the yield-effect of inflation.

The inflation induces however another effect that occurs quite independently from the yield-effect. This is the risk-effect.

1.2.4. The risk-effect induced by inflation

Inflation does not only affect the real gains expected on monetary assets, it also makes their future value in terms of purchasing power uncertain. This is to be regarded as an additional consequence of inflation.

Indeed, even when rates of interest on interest-bearing monetary assets allow for the expected rate of inflation, it remains basically uncertain whether factual inflation will exceed, be equal, or fall short of this anticipated inflation.

There is obviously bo reason to expect any risk-effect on real asset holdings. Real assets are subject to risks that may be due to their specific character, but unrelated to the developments in the purchasing-power of money.

Insight in the risk-effects of inflation may be given through a comparative statics analysis of changes in the variance-covariance parameters entering the demand functions for assets.

1.2.4.1. The response of demand to change in risk

Deriving the equilibrium conditions for a change in σ_{ij}, we get :

$$\begin{bmatrix} m_{11} & \cdots & m_{1n} & -1 \\ m_{i1} & \cdots & m_{in} & -1 \\ m_{j1} & \cdots & m_{jn} & -1 \\ m_{n1} & \cdots & m_{nn} & -1 \\ -1 & \cdots & -1 & 0 \end{bmatrix} \begin{bmatrix} \delta x_1/\delta\sigma_{ij} \\ \\ \\ \delta x_n/\delta\sigma_{ij} \\ \delta\lambda/\delta\sigma_{ij} \end{bmatrix} + \begin{bmatrix} 0 \\ bx_j \\ bx_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In abreviated form $[M]\,[D] + [X] = 0$

$$[D] = -[M]^{-1}[X]$$

hence :

$$\frac{\delta x_k}{\delta\sigma_{ij}} = \frac{-b\left[x_i\, v_{jk} + x_j\, v_{ik}\right]}{v}$$

Adopting Royama's notation $T^k_{ij} = \frac{\delta x_k}{\delta \sigma_{ij}}$ we can easily derive the risk-effects of inflation from the second order conditions.

1.2.4.2. The response of demand for all monetary assets to a global change in risk on monetary assets

Considering the bundle $(x_1 \dots x_m)$ of monetary assets, we derive the sign of a change in demand for all monetary assets in response to a change in the risk-parameters of monetary assets σ_{ij} $i=1 \dots m$, $j=1 \dots m$.

The choice of the risk parameters that inflation is likely to affect results from the following considerations.

First of all, there is no reason to expect the yields on real assets to become more uncertain or risky in times of inflation.

Hence, real yields on real assets will tend to display in times of inflation the same variance - and covariance - parameters as under conditions of general price-stability.

Secondly, it seems to be arbitrary to assume the covariances between monetary and real assets to be affected by inflation in a positive or negative direction.

Thirdly, something definite may be said about the effects of inflation on those risk-parameters that refer to monetary assets only.

Quite clearly, inflation tends to increase the variances of real yields on monetary assets. Under conditions of general price-stability, the expected real yield g on a monetary asset may be written as :

$$g = i + \left(\frac{p_e}{p} - 1\right)$$

where i is the market-rate of interest

p_e is the expected market-value of the asset considered

p is its current price.

hence $\left(\frac{p_e}{p} - 1\right)$ is the expected capital-gain or loss.

Under conditions of general price-stability, the risk of a monetary asset may be expressed by the variance of its rate of return and we have :

$$\sigma^2_g = \sigma^2_{\left(\frac{p_e}{p} - 1\right)} \quad \text{as } i \text{ is fixed.}$$

In inflation, the investor calculates his real yield as follows :

$$g = i + \left(\frac{p_e}{p} - 1\right) - \Theta$$

where Θ is the expected rate of inflation.

The rate of inflation Θ expected by the investor and by which he converts a nominal rate of return into a real rate of return is in fact the mean-value of a distribution of possible rates of inflation.
In other words, if he expects a rate of inflation of 5 %, this must be regarded as the mean value of a distribution that extends over a range of, for instance, 1 % to 9 %.

What is σ_g^2 going to be, if the investor deflates his rates of return by the rate of anticipated inflation ?

In inflation, we have

$$g = i + \left(\frac{p_e}{p} - 1\right) - \Theta$$

and

$$\sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p} - 1\right)} + \sigma_\Theta^2 - 2\sigma_{\Theta,\left(\frac{p_e}{p} - 1\right)}$$

where σ_Θ^2 is the variance of a probability distribution of rates of inflation

$\sigma_{\Theta,\left(\frac{p_e}{p} - 1\right)}$ is the covariance between the rate of inflation expected by the investor and capital gains or losses.

As a result, expected inflation tends to increase the variance of real rates of return on monetary assets if :

$$\sigma_\Theta^2 - 2\sigma_{\Theta,\left(\frac{p_e}{p} - 1\right)} > 0$$

First of all, σ_Θ^2 is necessarily positive.
Secondly, what may the sign of $\sigma_{\Theta,\left(\frac{p_e}{p} - 1\right)}$ be expected to be ?

Changes in market-values of monetary assets depend on variations in the rate of interest.
Hence, if there is no relation between the rate of anticipated inflation and the nominal rate of interest, there will be no relation between the rate of anticipated inflation and changes in the assets'market-value and we have :

$$\sigma_{\Theta,\left(\frac{p_e}{p}-1\right)} = 0 \quad \text{and} \quad \sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p}-1\right)} + \sigma^2_{\Theta}$$

If there is a functional relation between Θ and the nominal rates of interest, this function will be most probably positively sloped, i.e. nominal rates of interest increase with the expected rate of inflation (1).
Consequently, the increase in the rate of anticipated inflation will correspond to capital-losses and decreases in the rate of anticipated inflation will imply capital-gains. Consequently, in mathematical terms, we have :

$$\sigma_{\Theta,\left(\frac{p_e}{p}-1\right)} < 0 \quad \text{and} \quad \sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p}-1\right)} + \sigma^2_{\Theta} \underbrace{- 2\sigma_{\Theta,\left(\frac{p_e}{p}-1\right)}}_{>0}$$

As under conditions of general price-stability the variance of real yields on monetary assets was equal to :

$$\sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p}-1\right)},$$

while under conditions of inflation it is equal to :

$$\sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p}-1\right)} + \sigma^2_{\Theta}$$ if nominal rates of interest do not increase with the anticipated rate of inflation

or $$\sigma_g^2 = \sigma^2_{\left(\frac{p_e}{p}-1\right)} + \sigma^2_{\Theta} \underbrace{2\sigma_{\Theta,\left(\frac{p_e}{p}-1\right)}}_{>0}$$ if nominal rates of interest increase with the rates of inflation

we have $\sigma_g^2 \Big|_{\text{price-stability}} < \sigma_g^2 \Big|_{\text{inflation}}$.

The covariances of real yields on monetary assets will increase as well under conditions of inflation. This is because monetary assets undergo inflationary erosion, as soon as inflation is defined as a decrease in real value of money.
All monetary assets share this risk to the same extent if nominal rates of interest do not tend to allow for the anticipated rate of inflation. In

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(1) This relation and its implications will be largely discussed in our chapter II.

this case, inflation depreciates equally the real value of money, savings-accounts, time-deposits, bonds, etc...
If nominal rates of interest increase with the rate of expected inflation, changes in the expected rates of inflation induced by changing factual rates of inflation will cause capital-losses and gains that tend to be larger with longer maturities of monetary assets. In this case, monetary assets do not equally share the risk induced by inflation.
It remains, however, that in both cases the inflation tends to increase the covariances of the real rates of return on monetary assets.

After having specified the relevant parameters that inflation tends to affect when it comes to be expected, we shall determine the effect that anticipated inflation has on the demand for all monetary assets taken as a group.

This meaningful result must be given by the sign of the expression :

$$\sum_{k=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\delta x_k}{\delta \sigma_{ij}}$$

Hence we have : $$\sum_{k=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\delta x_k}{\delta \sigma_{ij}} = \sum_{k=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} - b \frac{x_i v_{jk} + x_j v_{ik}}{v}$$

$$= - b \left\{ \sum_{i=1}^{m} x_i \left[\sum_{k=1}^{m} \sum_{j=1}^{m} \frac{v_{jk}}{v} \right] + \sum_{j=1}^{m} x_j \left[\sum_{i=1}^{m} \sum_{k=1}^{m} \frac{v_{ik}}{v} \right] \right.$$

where $-b > 0$; $\sum_{i=1}^{m} x_i = \sum_{j=1}^{m} x_j \equiv M \equiv$ total monetary assets

hence : $$= - b \ M \left\{ \left[\sum_{k=1}^{m} \sum_{j=1}^{m} \frac{v_{jk}}{v} \right] + \left[\sum_{i=1}^{m} \sum_{k=1}^{m} \frac{v_{ik}}{v} \right] \right\}$$

$$= -2 \ b \ M \left[\sum_{k=1}^{m} \sum_{j=1}^{m} \frac{v_{jk}}{v} \right]$$

By virtue of our second-order conditions, we know that :

$$\sum_{k=1}^{m} \sum_{j=1}^{m} \frac{v_{jk}}{v} < 0$$

As a **result**, we must have $\sum_{k=1}^{m} \sum_{i=1}^{m} \sum_{j=1}^{m} \frac{\delta x_k}{\delta \sigma_{ij}} < 0$

This means that anticipated inflation induces people to hold less monetary assets, as those assets tend to become more risky, when measured in real value. In other words, even when rates of interest compensate for the anticipated rate of inflation so as to suppress the yield-effect, the investor holds still a larger part of its assets in real form, as he does not regard such an appropriate increase in the rate of interest as protecting him against the risk affecting the real rate of return on his monetary assets.

1.2.5. The portfolio-effect of inflation

So far we have focused an attention on two independent causal channels through which inflation tends to affect the demand for monetary assets.

We shall call the combined effets of inflation "the portfolio-effect of inflation".

Although the portfolio effect of inflation on the demand for real and monetary assets is certain, we should now find out what the real conditions of such a portfolio reallocation are in the menue of real assets available today.

1.3. A FEW "INFLATION - HEDGES" CONSIDERED

1.3.1. Houses

Houses deserve especially our attention, as their purchase is in many cases the main target of wealth accumulation by private investors.

If there is no reason to expect the real rents to be affected by the rate of inflation, the market value of houses should increase at this rate for a given real discount factor.
But, while the general price-level of houses keeps pace with the inflation, the price of a single dwelling may be subject to large fluctuations, because the secundary market for dwellings is imperfect. The housing market deals

in highly differentiated goods and prices may vary with tastes, moderness and environmental elements.

Another point is that, when purchasing a house, the average investor must abandon any form of diversification, because the purchase of a house may absorb his total wealth.
If, however, wealth accumulation is wholly subject to the final purchase of a house, the investor may not be very sensitive to such an increase in total risk.

1.3.2. Shares

Shares are a financial and rather divisible form of real wealth. When measuring the costs and benefits associated with share-holdings, one must take into account the following issues :

- Indexes for share quotations and consumer prices over long periods tend to show close conformity; theoretical considerations too point to conclusions in this direction.
- The share market as a whole may be subject to cyclical variations that make the relation between the quotation index and the price index rather weak in the short run.
- Fluctuations in profit on individual shares may be large in both the short and the long run.
  Hence, yield stability can only be ensured by large portfolios covering a broad selection of types of shares.
- Share-holding implies on the investor's behalf some appropriate knowledge of market mechanisms which tends to set up an educational barrier to this type of inflation-hedge.
- The successfull expansion of mutual funds in the last decade tends to open the share-market to less professional and low-income investors.
- In very open economies, the share-market might not provide investors with opportunities to hedge against inflation, as soon as domestic inflation exceeds the world rate of inflation and worsens in this way international competitivity. Inflation-hedging properties of shares would then be restricted to a given range of variation in the domestic rate of inflation, unless the domestic rate of exchange is floating.

1.3.3. Productive capital goods

When inflation leaves the real value of flows produced by capital goods and the real discount factor to actualize them unchanged, productive capital goods keep their real value and provide a good opportunity to hedge against inflation.

This type of real asset, however, is par excellence an asset of which the rate of substitution to other assets is zero for almost all private investors except those invested with the specific ability to use them in a production process.

Moreover, they are subject to physical obsolescence and their prices may vary with the relative prices of the goods produced and hence with tastes and consumption patterns.

The marketability of this type of good is limited and secundary markets may be very imperfect.

1.3.4. Consumer durables

Consumer durables provide their owner with a number of services that he consumes. There might thus be some discussion on the question to know whether durables are really a form of wealth-accumulation or simply goods that it takes some time to consume.
This is to a large extent an empirical matter and very dependent on the investor's attitude towards durables as an alternative form of wealth-accumulation competing with other assets such as monetary assets and houses.

An empirical investigation conducted by Alan C. Hess(1) supports implicitly the view that durables are a good alternative form of wealth-accumulation. This author estimated a demand-function for durables, containing the rates of return on different possible alternatives: houses and monetary assets. The estimation yielded very significant results and does entirely corroborate our theoretical yield-effect of inflation, as the demand for durables proved to increase with the anticipated rate of inflation.

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(1) HESS, Alan C., "Houshold demand for durable goods, the influences of rates of return and wealths", The Review of Economics and Statistics, Feb. 1973.

1.3.5. Foreign currencies

Foreign currencies are sometimes regarded as good inflation-hedges. The matt r, however, is fairly com licated and no straight forward answer can be given. The underlying assumption is that the development of the purchasing-power of the national monetary unit is the basic component of the evolution of the rate of exchange with other currencies. The rate of exchange, however, is fixed by many other components as well.
Moreover, foreign currencies cannot be regarded as very effective inflation-hedges in a fixed exchange-rate system, where devaluations occur unexpectedly and with no precise timing.
All things considered, foreign currencies may not be regarded as good inflation-hedges.

1.3.6. Hedging against inflation and risk

So far we have considered a few real assets that are currently regarded as possible inflation-hedges.
If anything definite may be said about such a heterogeneous group of assets, it is that they are likely to be more risky than monetary assets. The main argument for this conclusion is that secundary markets for real assets, if any, are imperfect as they deal in highly differentiated goods. Consequently, one of such assets might not be sold at the average market-price, a some of its characteri tics may bring about a variation in its price relatively to the average price of the assets traded on the market.

2.4. INFLATION AND POTENTIAL DEMAND FOR INDEX-TIED ASSETS

If we want to find out what points to the potential demand for index-tied monetary assets we should first evaluate the changes in the investor's welfare, after he has operated a marginal reallocation of his wealth from monetary into real assets.

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(1) HESS, Alan C., The Review of Economics and Statistics, Feb. 1973 : "Household demand for durable goods, the influences of rates of return and wealth".

Immediate common sense dictates that the risk-effect of inflation tends to affect the investor's utility negatively.
In fact, when reallocating his wealth into larger shares of real assets, the investor tries to minimize the risk-effect of inflation by holding marginally less monetary assets. He would, of course, avoid any risk effect on monetary assets by holding his wealth only in real forms, but then he would certainly not minimize the total risk on his portfolio and probably not maximize the expected rate of return.
So we may assert that the optimal combination of risk and return in times of inflation is obviously worse than the optimal combination under conditions of general price stability.

Consequently there exists a potential demand for any device that would allow the investor to maintain the combination of risk and return that he considers as optimal under general price stability.
Consequently, such a device should protect the investor against all the risks that inflation entails on monetary asset-holdings.
An index-linkage of financial assets may thus prove to be advantageous to and hence demanded by the investor, if it entirely restores the conditions of portfolio-diversification under general-price stability.

As defined in our introduction, the index-linkage of financial assets safeguards the real value of interest-payments and redemption-value of financial assets. Does it necessarily follow from this that an index-tied monetary asset has the same risk-properties in times of inflation as a non-index-linked monetary asset under general price-stability ?
In other words, does the index-linkage restore the conditions of portfolio-diversification prevailing under general price-stability ?
This depends on a number of macro-economic conditions that will be analyzed in our chapter III.
Without proving it now, we may assert that the index-linkage removes all the risks inherent to inflation whenover the nominal rates of interest on ordinary monetary assets keep pace with the anticipated rate of inflation; index-linked monetary assets will prove to be less risky than ordinary monetary assets, if the nominal rate of interest on ordinary monetary assets increases by less than the rate of anticipated inflation.

Index-tied monetary assets will prove to be more risky than ordinary monetary assets,if there is no positive relation between the rate of anticipated inflation and the nominal rate of interest at all.

Consequently, we may assert that, if certain macro-economic conditions are achieved, the index-linkage of monetary assets will effectively allow the investor to diversify his portfolio between monetary and real assets in a more optimal manner and there is no doubt that index-tied assets would very well compete with other inflation-hedges.

1.5. DEMAND FOR REAL ASSETS AND RESOURCE-ALLOCATION

The usefulness of an index-linkage of financial assets may be appraised from another point of view that is, however, directly related to the investors' demand for real assets in times of inflation.

Indeed,in the absence of any index-linkage, with the exception of shares, all real assets demanded by net creditors such as dwellings and consumer durables are not financial counterparts of purchasing power transfered from creditors to debtors for investment-purposes.
The ownership of a house, for instance, may yield a high rate of return to its owner, because he regards a house as a good form of wealth-holding in times of inflation. Hence the rate of return on such an asset may be rather high in terms of individual ownership.
The social benefits from holding real assets such as dwellings and consumer durables are likely to be much smaller.
Indeed, if net creditors as a whole increase their demand for real assets in times of inflation, the prices of the existing stock of real assets will increase. As a result, the sectors concerned with the production of those real assets will run larger profits as they sell at higher prices. By competing on the market for productive factors they will bid productive resources away from other sectors.
Hence inflation tends to operate an arbitrary reallocation of resources resulting from the net creditors' need to hedge against inflation.
If net creditors maintain their current level of consumption out of a given

real income and allocate larger shares of their current savings into real assets, the production of those real assets may only be achieved through a reallocation of resources from other investment sectors to those sectors producing the real assets demanded by ultimate wealth-holders.
Consequently, the real assets produced - dwellings, consumer durables, etc - may be desirable from the point of view of net creditors demanding inflation-hedges, but they will be less desirable from the point of view of their social productivity,as they are not net additions to productive capital.
In other words, inflation tends to set up or to enlarge the discrepancy between individual and social return to investment.
Consequently, if the index-linkage of monetary assets takes away the risks of real value affecting monetary assets, it will induce the net creditors not to make those sterile investments. They will rather hold their wealth in index-linked financial counterparts of resources which the capital-market allocates to those sectors of economic activity with the highest marginal productivity. Hence, the index-linkage may be validly considered as an instrument of economic growth in times of inflation and as such socially desirable.

The strength of such an argument depends, of course, on the range of real assets institutionally available. More precisely, the existence of an extensive market for common stock may channel much of the accumulated savings towards highly productive activities.
In this case, the reallocation will be a function of the public's attitude towards the risks of share-holginds and of the distribution of wealth and income. This requires much empirical research for each particular case under review.

1.6. CONCLUSION

In the framework of a generalized theory of liquidity preference, we have shown that inflation tends to increase the demand for real assets at the expense of the demand for monetary assets taken as a group.

Those assets that are currently regarded as good inflation-hedges are nevertheless very risky and rather illiquid because of the non-existence or imperfection of secundary markets for real assets.

Index-tied assets will be demanded, as soon as they will prove to have some advantages when compared to other inflation-hedges : this may be expected to be a decrease in risk and an increase in liquidity.

The social demand for index-tied assets is not simply equal to the sum of the individual demands by the investors.
From an epistemological point of view, we should state the conditions for an aggregation over all the individual utility-functions of wealth-holders. To the best of our knowledge, this has not been done so far.
Hence we will assume that all the results deduced from an individual utility-function may be validly extrapolated over the whole group of wealth-holders.

Moreover, in the long-run inflation tends to induce an arbitrary reallocation of resources that may be expected to impede an optimal economic growth. This reallocation of resources would not occur in a system where index-tied assets are considered as better inflation-hedges than physical forms of wealth.

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Chapter II

INFLATION AND THE MATURITY OF MONETARY DEBTS AND ASSETS

Introduction

So far we have stated the theoretical underpinnings for the demand of index-tied monetary assets in inflationary economies.
Defining inflation as a permanent increase in value of a broadly representative price-index, it followed as a truism that the expected real value of incomes yielded by monetary assets becomes uncertain, as the expected increase in value of the index is uncertain : one may expect the index to increase at a given rate, but such an expectation cannot be held with certainty. Quite clearly inflation makes all monetary assets more uncertain with respect to their future real value and at a first view, the linkage of monetary assets to a represnetative index is likely eliminate this uncertainty.

Since all monetary assets become more risky, it would seem desirable to link all monetary assets to the index, including money itself.
This is probably unrealistic and hard to think of in terms of economic policy. One may reasonably assume that the index-linkage needs not being generalized to such an extent. One should accordingly evaluate the risks entailed by inflation on various types of monetary assets.
In other words, does inflation affect all monetary assets like money, short-term debts and bonds to the same extent ?
If it does not, we should be able to establish a ranking of monetary assets according to the risk borne in times of inflation by monetary assets of different types. The most risky one should be envisaged for a possible index-linkage.

Hence we shall state the conditions under which such a ranking can be effectively achieved. We shall demonstrate that whenever the level of nominal rates of interest is positively related to the expected rate of inflation, fluctuations in this expected rate of inflation will imply variations in the expected real value of interest-earnings on monetary assets and variations in current market-value of monetary assets.

Now the longer the maturity of a monetary asset, the larger the change in market-value for a given change in the rate of interest.
Consequently, if nominal rates of interest are adjusted for the expected rate of inflation, which may vary, long-term monetary assets will tend to be more risky in times of inflation than short-term monetary assets and money.

Those results will be extrapolated so as to show their macro-economic effects. Paying some attention to the lending-borrowing process, we shall deduce from the attitudes of debtors and creditors towards the average maturity of their monetary debts and assets a general tendency to shorten this average maturity in times of inflation.
This general tendency will materialize in a gradual disappearance of markets for long-term fixed-income capital.

The shortening of the average maturity of debts and assets seriously impedes the activity of those non-monetary financial intermediaries that borrow short and lend long.
Monetary financial intermediaries may be expected to be more willing to hold long-term monetary assets, as long-term holdings will prove to stabilize the income earned from money-creation under inflation.

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2.1. A DISCRIMINATION OF INFLATIONARY RISK-EFFECTS ON VARIOUS TYPES OF MONETARY ASSETS

As demonstrated in our chapter I, all monetary assets become obviously more risky in times of inflation than under conditions of general price-stability. One ought to know, however, whether or not all monetary assets are exposed to greater risk to the same extent. Indeed, as we explained in the introduction, an important questioh remains to be answered : what should be tied to the index ?

If all monetary assets become equally more risky in times of inflation, there is no reason to link one particular monetary asset to the index rather than any other monetary asset.

We must accordingly find out what the ranking of monetary assets according to the amount of risk borne in times of inflation is likely to be.

We may validly start the analysis from a statement made by James Tobin (1) on this topic :

> "All categories of government debt, including demand debt, share their principal risk, namely uncertainty about the purchasing-power of the dollar. Presumably each investor assigns to cash and to other obligations fixed in units of currency a real rate of return based on his expectations of the change in the price-level. If his mean expectation is inflation at a rate of 3 per cent per annum, his expected real rate of return on cash is - 3 per cent per annum. Likewise he will subtract 3 per cent from the expected money return on any other obligation of fixed money-value in order to arrive at its expected real rate of return. But no one is sure aboute the price-level. It may rise more than the expected three per cent, or it may rise less, even fall. This is a risk of holding any asset of this kind. It is shared equally by all government debt instruments. If inflation takes away 50 per cent of the real value of cash during the next decade, it will also take away 50 per cent of a 10-year bond... An investor cannot defend himself against risks of this kind by spreading investments among different kinds of government obligations."

Tobin's argument on the risky nature of monetary assets in inflation may be formalized as follows :

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(1) TOBIN, James, "An Essay on the Principles of Debt Management", in Fiscal and Debt Management Policies, prepared for the Commission on Money and Credit, Englewood Cliffs, N.J. Prentice-Hall, 1963.

If "a" and "b" are both monetary assets and g_a and g_b are the real rates of return on those assets, we may write

$$g_a = i_a + \left(\frac{p_{ae}}{p_a} - 1 \right) - \Theta$$

$$g_b = i_b + \left(\frac{p_{be}}{p_b} - 1 \right) - \Theta$$

where 1) i_a and i_b are market-rates of interest on "a" and "b"

2) $\left(\frac{p_{ae}}{p} - 1 \right)$ is the expected change in market-value of monetary asset "a" during the holding period

3) $\left(\frac{p_{be}}{p_b} - 1 \right)$ idem for "b"

4) Θ is the mean value of expected inflation.

Following Tobin's statement, the absolute risks on "a" and "b" are expressed as :

$$\sigma^2_{g_a} = \sigma^2_{\left(\frac{p_{ae}}{p_a} - 1 \right)} + \sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{ae}}{p_a} - 1 \right)}$$

$$\sigma^2_{g_b} = \sigma^2_{\left(\frac{p_{be}}{p_b} - 1 \right)} + \sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{be}}{p_b} - 1 \right)}$$

Consequently, monetary assets "a" and "b" become equally more risky in times of inflation, provided that :

$$\sigma_{\Theta,\left(\frac{p_{ae}}{p_a} - 1 \right)} = \sigma_{\Theta,\left(\frac{p_{be}}{p_b} - 1 \right)}$$

i.e. the covariances between the expected rates of inflation and the market-value of monetary assets are equal for assets "a" and "b".

This is an assumption that Tobin fails to make when pretending :
"An investor cannot defend himself against risks of thos kind by spreading investments among different kinds of government obligations."
In other words, assuming that all monetary assets become equally more risky amounts to saying that the covariances of changes in capital value and the expected rate of inflation are equal for all monetary assets.
This does not hold when comparing any interest-bearing monetary asset with money itself. Money cannot change in current market-value, as its relative price in terms of itself is equal to one.

All other interest-bearing monetary assets may change in current-market value and this market-value may be in some relation to the expected rate of inflation. Hence, it is wrong to state that cash and bonds are equally affected by the rate of inflation. In fact, if inflation comes to be expected, it could affect cash and bonds quite differently, owing to the possibility for bonds to change in market-value, which cash does not.
Moreover, possible variations in market-value of monetary assets will tend to increase with the maturity of monetary assets.

As a result, if "a" is a short-term interest-bearing debt and "b" a bond, we will have :

$$\sigma^2_{g_a} = \sigma^2\left(\frac{p_{ae}}{p_a} - 1\right) + \sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{ae}}{p_a} - 1\right)}$$

$$\sigma^2_{g_b} = \sigma^2\left(\frac{p_{be}}{p_b} - 1\right) + \sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{be}}{p_b} - 1\right)}$$

where $\sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{ae}}{p_a} - 1\right)}$ is the increase in risk on asset "a" as due to inflation.

$\sigma^2_{\Theta} - 2\,\sigma_{\Theta,\left(\frac{p_{be}}{p_b} - 1\right)}$ is the increase in risk on asset "b" as due to inflation.

The covariance between the rate of expected inflation and changes in market-value in the case of bonds will be much larger than the covariance between the rates of expected inflation and changes in market-value in the case of short-term debts.

In other words, Tobin's statement holds in the only case where there is no relation between the current-market value of monetary assets and the anticipated rate of inflation.

Now there is much empirical evidence that nominal rates of interest are positively related to the expected rate of inflation. Consequently, there is a definite relation between the expected rate of inflation and market-prices of monetary assets. Hence there must be a ranking of various types of monetary assets according to the amount of risk borne in times of inflation. The necessary condition for such a ranking being the relation between nominal rates of interest and the rate of expected inflation, we shall first develop a model where this relation may be most easily grasped.

2.2. A MODEL OF ADJUSTMENT OF RATES OF NOMINAL INTEREST TO ANTICIPATED INFLATION

2.2.1. Assumptions

a) The model considered is purely keynesian. Keynesian theory generates equilibrium prices for monetary assets from equilibrating conditions that prevail on the commodity and the money-bond markets.
If it may be shown how inflation affects those markets, we will have a good theory of asset prices as determined by inflation.

b) Our model is static. We do not mean that inflation is a static process, but the very expectation of it is a behavioural parameter that may affect short-run equilibriums.

c) Wealth is held in three forms: shares, money and interest-bearing monetary assets with various maturities.
Shares are considered as rights to the property of real productive capital. But, focusing the attention on rates of interest of monetary assets, we assume all investment financing to be achieved through monetary debt-issues.

d) The economy is assumed to be at full-employment. Productive ressources have been allocated between investment and consumption sectors according to the community's willingness to consume and to save out of a given real aggregate income.
Inflation is not supposed to alter this willingness in any way.

e) We assume the absence of government interference in the form of fiscal or monetary policy.
As for monetary policy, we assume the absence of it in the sense that the monetary sector has a passive attitude towards inflation and wants neither to stop nor to accelerate the process. Once it occurs inflation is adequately financed so as not to interfer with the components of real income and employment.

f) The inflation is defined as a general rise in prices reflected in the increase of a broadly defined index as the G.N.P. deflator.
No reference is made to particular types of inflation as defined by their causes, this would introduce a loss of generality.
If we had done so we would have had a large number of degrees of freedom in our model. It would than suffice to specify any set of assumptions to reach any result.

h) The economy is supposed to be a closed economy.

i) People try to anticipate the inflation. No difference is provisionnally made between long-run and short-run expectations. In other words, if people come to expect a rate of inflation of 5 %, they anticipate this rate over each short period and over the whole period considered in the long-run.

2.2.2. Inflation and the marginal efficiency of capital

By virtue of our hypothesis i) people anticipate the future rate of inflation. This anticipated inflation affects the long-term expectations of the entrepreneurs concerning the future nominal returns on the existing capital-stock.

In a strictly closed ecpnomy, where the value of real flows produced by capital goods are unrelated to the inflation, an anticipated rate of decrease in the real value of money increases proportionnally the expected net nominal income forthcoming out of an existing capital stock.

Keynes'marginal efficiency of capital is defined as a discount-factor that equalizes the value of future net income streams yielded by productive capital goods to the current replacement cost of those capital-goods.

By current replacement-cost is meant the current price at which a capital good is sold. It depends entirely on the efficiency of production, the cost of labour, capital and entrepreneurship in the investment-sector.

There is, of course, no direct relation between current nominal prices of investment-goods and future inflation, even when it comes to be expected.

There is, however, as we have shown, a proportionnal relation between the present value of net nominal profit streamsexpected from capital goods and the expected **inflation**.

Hence the discount-factor equalizing the value of expected net profit-streams to the current replacement-cost must be a function of the rate of inflation.

Mathematically, the marginal efficiency of capital "m" is defined as :

$$k\, p_I = \int_o^T p(t).q(t)\; e^{-mt}.dt$$

where k : physical amount of capital.

p_I : current investment price or replacement cost per unit.

$p(t)$: net nominal income per unit produced at time t.

$q(t)$: number of units produced at time t.

T : date of complete obsolescence of k

We may take values of k so that $q(t) = 1$

We assume $q(t) = q(o) = 1$ for $t \in [o \dots T]$

The "real" net income per unit produced is assumed to be constant for $t \in [o \dots T]$. We may assume $p(o) = 1$.

For such a notation, as soon as a given rate of inflation " " comes to be expected, we have :

$$k\, p_I = \int_o^T e^{\Theta t}\, e^{-m't}\, dt = \int_o^T e^{-(m' - \Theta)t}\, dt$$

where m' is the marg. eff. of cap. when a rate Θ comes to be expected.

Current replacement cost remaining unchanged by future inflation we have :

$$k\, p_I = \int_o^T p(t).q(t)\; e^{-mt} dt = \int_o^T e^{-mt} dt = \int_o^T e^{-(m' - \Theta)t}\, dt$$

which implies : $m = m' - \Theta$
hence : $m' = m + \Theta$

This means that the marginal efficiency of capital, when inflation comes to be expected, is equal to the marg.eff.of.cap. under conditions of general price stability plus the rate of anticipated inflation.

Since the entrepreneurs tend to enlarge their capital-stock to the point where the m.e.c. is equal to the current rate of interest, inflation will induce them to invest.

If their excess-demand for investment goods may not be matched by an increased supply of invesment-goods, the rate of interest will have to adjust for the rate of anticipated inflation and at equilibrium we must have:

$$m' = m + \theta = i \quad \text{and} \quad m = i - \theta$$

This implies that investment becomes a function of the real rate of interest. The whole equilibrating process through the rate of interest as proposed by Keynes may then be rewritten in terms of a real rate of interest.

Hence there will always be a real rate of interest that maintains total real demand at its full-employment level and allows for a given rate of expected inflation.

In our next section, we shall analyze all the elements of this adjustment process of a nominal rate of interest to anticipated inflation.

2.2.3. <u>The mechanisms of adjustment</u>

In this section, we shall proceed by a comparative statics analysis and consider what happens when the public comes to change its expectations of future inflation.

This may be done by considering the response of the economic variables for a change in inflationary expectations from 0 % to θ %.

Finding an equilibrium point for a rate of anticipated inflation "θ", we may analogically apply the analysis to any change in inflationary expectations, increases and decreases, from any given level of expectations.

In the following sections, we shall apply our conclusions to an attempt to answer two fundamental questions :

1) Is there any justification for an index-linkage of monetary assets in an economy that tends to anticipate the rate of inflation and consequently, to avoid the redistributive effects of inflation that the index-linkage is supposed to preclude.

2) What is the influence of inflation on the financial lending and horrowing process from the point of view of the term-structure of monetary debts ?

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The previous section has shown why investment becomes a function of the real rather than of the nominal rate of interest, as soon as entrepreneurs have come to expect inflation.
Consequently, a rise in the rate of expected inflation from 0 to Θ induces a decrease in the real rate of interest of Θ % and an excess-demand for investment-goods.

Through the keynesian multiplier process the increase in total real demand will be equal to the excess-demand for investment goods multiplied by the inverse of the propensity to save.

As by the vitrue of our assumption a) the economy is at full-employment this total excess-demand cannot be matched by an increase in real supply, as all productive ressources are fully employed. Consequently the excess-demand induced by anticipated inflation will cause the general price-level to rise.

The rises in the general price-level result from two major effects.
As inflation affects the demand for capital-goods, the excess demand will first reduce the stocks available and then result into pure increases in the average price of investment goods relatively to the average price of consumption goods. However, as the higher real income earned in the investment sector is distributed among the productive ressources and partly saved, partly consumed, it tends to increase the relative price of consumer-goods also. This is the multiplier process.
Another factor of rising price-levels is that, as the investment sector becomes more profitable because of its higher relative prices, it tends to bid productive resources away from the consumption-sector. But, as by virtue of our hypothesis d), the propensity to consume remains unaltered by expected inflation, prices rise in the consumption sector for both the resources employed and the commodities produced.

As prices rise, the nominal balances needed by the public for its transaction purposes rise proportionnally to the prices.
This reduces the amount of idle money and hence increases the nominal rate of interest. As long as the nominal rate of interest has not allowed for the expected rate of inflation excess-demand continues to exist and consequently prices to rise.

The extent to which prices will have to rise depends entirely on the conditions prevailing on the money-market.

If the money-market is rather tight, the inflation-induced increase in real demand will by its very effect on the transaction-balances induce large increases in the rate of interest.
Small increases in the general price-level may be sufficient.

If the moneymarket supplies large amounts of idle money, large increases in the general price-level are needed to expand the nominal value of transaction balances and to decrease correspondingly the balances of idle money, so as to increase the nominal rate of interest.

Our analysis may be easily formulated in mathematical terms :

Writing the symbols : $\frac{y}{p}$ = real income; $\left(\frac{y}{p}\right)_0$ real income at full employment

A = real autonomous expenditures

i = nominal rate of interest

= expected rate of inflation

g = interest elasticity of the investment curve; g 0

s = marginal propensity to save.

We have :

$$\frac{y}{p} = \frac{1}{s.p}\left[A.p + g.i\right] = \left(\frac{y}{p}\right)_0 \qquad \text{when no inflation is expected}$$

and

$$\frac{y}{p} = \frac{1}{s.p}\left[A.p + g(i - \Theta)\right] \qquad \text{as inflation comes to be expected at a rate } \Theta.$$

$$\frac{y}{p} = \frac{1}{s.p}\left[A.p + g.i\right] - \frac{g.\Theta}{s.p}$$

$$\frac{y}{p} = \left(\frac{y}{p}\right)_0 - \underbrace{\frac{g.\Theta}{s.p}}_{\text{excess-real demand induced by } \Theta.}$$

For equilibrium in the real demand sector, we must have :

$$d\left(\frac{y}{p}\right) = \frac{1}{s.p}.g\ di - \frac{g}{s.p}.\Theta = 0 \quad \text{at full-employment}$$

$$\Longrightarrow \boxed{di = \Theta}$$

This implies for the monetary sector :

$$\left(\frac{m}{p}\right) = L\left(i, \frac{y}{p}\right) \quad \text{with} \quad L'_i < 0$$

$$d\left(\frac{m}{p}\right) = \frac{dm}{p} - \left(\frac{m}{p}\right).\frac{dp}{p} = L'_i\ di + L'_{\left(\frac{y}{p}\right)} . d\left(\frac{y}{p}\right).$$

But as we know that $d\left(\frac{y}{p}\right) = 0$ and $di = \Theta$, we must have :

$$d\left(\frac{m}{p}\right) = \frac{dm}{p} - \frac{m}{p}.\frac{dp}{p} = L'_i.\Theta.$$

As we have assumed the absence of monetary policy, we must have $\frac{dm}{p} = 0$

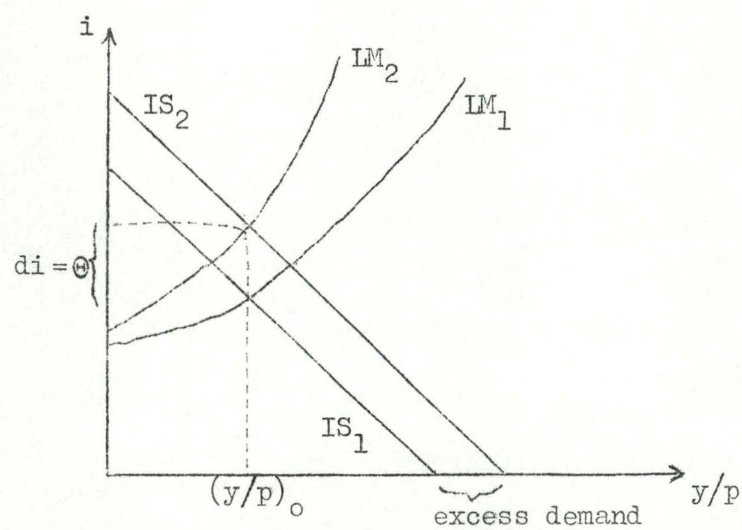
and : $$d\left(\frac{m}{p}\right) = -\frac{m}{p}\frac{dp}{p} = L'_i \quad .$$

hence : $$\frac{dp}{p} = -\frac{L'_i.\Theta}{\left(\frac{m}{p}\right)}$$

consequently : $\frac{dp}{p} = 0$ for $L'_i = 0$ tight money-market.

$\frac{dp}{p} = \infty$ for $L'_i = -\infty$ very easy money-market.

Graphically, we have :



The whole disequilibrium analyzed here is due to an over-optimistic behaviour of the entrepreneurs,who enlarge the volume of their current investment to meet the exceptionnal inflation-profits opportunities. It is a matter of wealth redistribution from creditor to debtor applied to private investment. The effective volume of private investment has not changed,as the public pretended to carry on its current level of consumption and as resources could not be bid away from the consumption sector. Full-employment has been maintained.
Only both the price-level and nominal rates of interest are much higher.

The analysis may logically be applied to any change in inflationary expectations. Increases in inflationary expectations imply increases in the nominal rates of interest, decreases in the anticipated rates imply decreases in the nominal rate of interest for any initial level of inflationnary expectations.

Thus, in the framework of our assumptions,expected inflation becomes an additionnal source of variations in the nominal rates of interest.
In mathematical terms, we know that :

$$i = r + \Theta \qquad \text{when } r = \text{real rate of interest}$$

$$\text{var}(i) = \text{var}(r) + \text{var}(\Theta) + 2\,\text{cov}(r, \Theta).$$

As we have shown that under our assumptions $\text{cov}(r, \Theta) = 0$, i.e. the rate of expected inflation does not affect the real rate of interest, wa have :

$$\text{var}(i) = \text{var}(r) + \text{var}(\Theta),$$

which clearly shows that the variance of expected inflation is a net additionnal source of fluctuation in the nominal rate of interest.

2.3. THE EFFECTS OF INFLATION ON THE TERM STRUCTURE OF MONETARY DEBTS AND ASSETS

So far we have focused the attention on the relation between anticipated inflation and a single rate of interest assumed to be a long term rate of interest.
We have shown why anticipated inflation becomes a net additionnal source of fluctuation in the nominal rate of interest.

Well-developed capital markets, however, offer a wide range of monetary assets with maturities of different length. Once we know the relevant features that creditors and debtors consider, when holding monetary assets or issuing monetary debts with different maturities, and how inflation acts upon those elements of diversification, we will be able to deduce the influence that inflation is likely to have on the term structure of the community's monetary portfolio, when it comes to be expected.

2.3.1. The debtor's concern with the term structure of his monetary debt

We shall not depart from our hypothesis that all debtors seek to raise funds to finance investment in productive capital. As they are engaged in the production process we may assume that, once they have invested the funds raised into real capital, they do not consider any possible speculation on the prices of their physical assets by selling them on secundary markets before the date of physical obsolescence. In other words, productive capital goods will be considered as illiquid.

This implies that the debtor's financing period will be long and, when issuing debts against himself, the debtor seeks to minimize the present value of his interest-payments over the entire lifetime of his productive assets.

The debtor may issue either short term-debts and refinance his investment whenever they come due, or issue long-term debts with a maturity corresponding to the length of his physical investment.
When choosing the first extreme solution, the anticipated present value of his interest payments is subject to uncertainty and depends on what the debtor expects short term rates are going to be.
When adopting the second extreme solution, the debtor knows with certainty the present value of his interest-charges over the whole period.

Actually the debtor will choose neither the first nor the second strategy, but he will finance his real investment partly with short-term debts, partly with long-term debts.
If he regards long-term rates of interest as being temporarily high, he will be induced to sell short term,debts as he expects to finance his

assets more cheaply at future long-term rates of interest. By doing so, he seeks to minimize his total capitalized debt-charges, but faces the uncertainty of future rates of interest. If he has an aversion to this uncertainty, he will confine such a strategy to part of his debt only, balancing the expected gain on interest charges against the risk involved.
If he regards long-term rates of interest as being temporarily low, he will finance a larger part of his assets with long-term debts, a minor part of his total debt remaining short, so as to meet the possibility of still lower long-term rates of interest in the future.

Hence, at any moment in time the debtor will diversify his monetary liabilities among different terms, so as to minimize the present value of his debt charges for a given level of risk.

This is nothing particular to inflation, but by virtue of what we know about the analytical link between anticipated inflation and the nominal rate of interest, we may deduce the effect of inflation on this diversification process.

Indeed, while expected nominal debt-charges are certain on long-term debts, expected real debt-charges on long-term debts become uncertain in times of inflation, as debtors cannot be sure that the anticipated inflation for which they allow in their interest payments will effectively occur.
If factual inflation tends to exceed the rate of inflation anticipated at the time he went into a long term debt, real debt charges will prove to be lower and long-term rates of interest higher.
If, factual inflation falls short off the anticipated rate of inflation real debt charges will prove to be larger than expected and long-term rates of interest lower.
As on the other hand, nominal rates of interest on short-term debts allow for the expected rate of inflation and as future short term rates of interest will allow for the rate of inflation anticipated by the time of their issue, the present value of real debt charges on short term financing keeps its relative risk prevailing under conditions of general price stability.
Indeed, any divergence of factual inflation from its anticipated level affects the real value of those debt-charges over very short periods and, if

people do permanently correct their anticipations of inflation with regard to what they know inflation was in the past, nominal rates on future short-term debts will reflect future anticipated rates of inflation.

Thus, as a result of inflation, long-term financing becomes relatively more risky in real terms, while short-term financing becomes hardly more risky than in times of general price stability.
Consequently, inflation will tend to shorten the average maturity of the debtor's monetary liabilities.
This does not mean, of course, that investment will be wholly financed by short-term debts. When shortening the average maturity of his debt, the debtor will simply balance the decrease in risk by short-term financing under inflation against the increase in risk inherent to the refinancing process. If the debtor was to structurate his debt in times of inflation in the same manner as in times of general price stability, he would require a decrease in the nominal rate of interest on long-term debts as a risk-premium.

2.3.2. The creditor's concern with the term structure of his monetary assets

When considering the creditor's attitude towards the term structure of his monetary assets, one has to consider an additionnal feature, which is the holding period of assets.
Indeed, creditors may regard accumulated wealth as a source of non-labour income or as a financial support for speculation on the prices of assets. In the first case, assets will be held until their date of redemption and probably later on refinanced, in the second case, monetary assets may be sold on secundary markets before the date of their redemption whenever their market-prices exceed the prices at which they were bought.

Those creditors, whose holding period is long, diversify their assets among different terms so as to maximize the present value of future income streams earned on asset-holdings. Their behaviour is quite analogous to that of debtors financing real investment over a long period.

When holding long term monetary assets, those creditors enjoy the cer-

tainty of nominal income over a large part of their holding period.
When holding short-term monetary assets, they face some uncertainty on their nominzl income earned over the whole period,as they cannot know what future nominal rates of interest are going to be at the date of redemption of their short-term monetary assets.
The part of short-term monetary assets in the total monetary wealth will be a positive function of expectations of later increases in the long-term rate of interest. The part of long-term monetary assets will be a function of expectations of either further stability in the long-term rate of interest or possible decreases in the long-term rate of interest.
At any moment, the term structure of monetary assets will be so defined as to yield a maximum level of expected real income for a given level of risk.

The effect of inflation on the creditor's willingness to hold monetary assets with various maturities may now be immediately deduced.
While, in times of general price stability, the real income earned on long term monetary assets was certain over a long period, it ceases being certain in times of inflation.
As,on the other hand current and,future short-term rates of interest allow for the rate of inflation anticipated by the time of their issue, inflation does not hardly affec the risk on real income earned on short-term monetary assets prevailing under conditions of general price stability.

Consequently, real income on long-term assets becoming more uncertain in times of inflation and real income on short-term holdings being hardly more risky than in times of general price stability, the creditor with a long holding period will shorten the average maturity of his monetary assets. If he was to hold the average maturity of his assets unchanged, he would require an increase in the nominal rate of interest by way of a risk premium.

We may now consider the creditor's attitude towards the term structure of his monetary assets, when his holding period is short.
In this case, the ordering of monetary assets with different maturities from the point of view of risk is inversed, when compared to the ordering prevailing for a long holding period.

There is no price-risk involved in holding short-term debts the maturity of which coincides approximately with the length of the holding period, at the end of the period those short-term edbts are redeemed at their par value. If, by contrast, the creditor holds long-term monetary debts over short periods, he faces the uncertainty of market-prices at which he may sell his long-term debts.

Indeed, for a given rise or decrease in the nominal rate of interest, the longer the maturity of his assets, the larger the capital-losses or gains on his assets.

Following Hickx(1), long-term rates of interest tend to be higher than short-term rates, because debtors are ready to pay a higher rate of interest for the convenience of a long-term disposal of loanable funds, compensating creditors for the risk involved in this type of investment.

Consequently, creditors may choose among more risky, but more profitable long-term monetary assets, and less risky, but also less profitable short-term monetary assets.

Hence the optimal term structure of monetary assets will yield a maximum return, calculated as current interest-earnings plus possible price-variations on assets, for a given level of risk.

The effect of inflation on the term structure of monetary assets may be deduced as follows.

Since inflation is in our model an additional source of variation in the nominal rates of interest and since, for equal changes in the nominal rates of interest, fluctuations in the market-value of monetary assets tend to be proportional to the length of their maturity, as measured from the time they were bought, inflation increases considerably more the risk on long-term monetary assets than on short-term monetary assets.

As a result, creditors with a short holding period will also shorten the average maturity of their monetary assets by reallocating their wealth from long-term holdings to short-term monetary holdings.

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(1) HICKX, op. cit., p. 138.

If creditors were not shorten the average maturity of their monetary assets, they would require an increase in the rate of interest on their long-term assets by way of a risk premium.

This analysis of the creditor's behaviour shows as a by-product that increasing the nominal rates of interest so as to allow for anticipated inflation is a very ineffective way of protecting monetary assets against inflation. Indeed, if the holding period is long, inflationary expectations are nothing but a guess about the future.
If the holding period is short, even a rate of inflation that proves to be perfectly anticipated ex post over the entire holding-period does not really protect the creditor against inflation-induced variations in the market-value of long-term monetary assets.
Indeed, it would suffice that debtors have come to anticipate a different rate of inflation to prevail in the future immediately after the holding period to bring about changes in supply on the bond market and hence changes in market-prices.

Finally, we may conclude that, whatever a creditor's holding period may be, he will shorten the average maturity of his monetary wealth in times of inflation, as a continuous decline in the purchasing-power of money increases risk on long-term monetary holdings relatively more than on short-term monetary holdings.

This conclusion, however, is subject to an additional remark.

As the economy moves from general price stability to inflation and as inflation comes to be anticipated and increases the nominal rates of interest by the appropriate amount, creditors undergo an immediate loss in capital-value of their monetary assets. The more distant the date of redemption, the larger those capital-losses will be.
Shortening the average maturity of a monetary portfolio by selling long-term assets at their current market-price and buying short-term monetary assets would be a very costly switch and creditors are forced to hold those long-term assets. This is the well-known lock-in effect.

Whether the switch will then be operated or not depends entirely on the evaluation of risk. Operating the switch amounts to incurring immediately all the certain capital-losses and foregoing both a possible increase in asset-prices that might occur through future decreases in inflationary expectations, and a possible further decrease in asset prices that might occur through increases in future inflationnary expectations.

This very risk being larger on long-term monetary assets than on short-term monetary assets by virtue of our analysis, there will be a shift from outstanding long-term monetary assets to short-term monetary assets, although this shift will be marginal.

Hence we may now restate our conclusion in a more sophisticated manner:

> "When the economy moves from general price stability or less inflationary conditions to more inflationnary conditions there will be a shift from previously issued long-term monetary assets to short-term monetary assets. This shift remains marginal to the extent of the lock-in effect. This decrease in the creditor's willingness to accept newly issued long-term monetary assets will be much larger, as there is no such lock-in effect involved".

2.3.3. Inflation and long-term financing

So far we have contemplated the effects of inflation on the debtors' willingness to issue debts with various maturities and on the creditors' willingness to hold them, when anticipated inflation comes to be reflected by the appropriate amount in nominal rates of interest.

We have shown that under these conditions risk-averting debtors will shorten the average maturity of their debts and risk-averting creditors will shorten the average maturity of their monetary assets.
In other words, if they were not to shorten the average maturity of their respective debts and assets, debtors would require an additional risk-premium through a decrease in the long-term rate of interest to pay and creditors would require an additional risk-premium by an increase in the long-term rate of interest to receive.

Both requirements are of course uncompatible and this is why under inflationary conditions the economy will finance real investment with relatively long holding periods by short-term financing and refinancing programs.

This is obviously to be considered as something negative. When financing real investment, the community as a whole should go into a long-term commitment. Of course, a single individual does not go into a long-term commitment when buying long-term monetary assets, as he may sell them before the date of redemption on secundary markets, but the community as a whole does so, as debtors are to redeem their debts only when they come due.

2.3.4. Inflation and financial intermediation

The focus of the analysis has been so far on the effects of inflation on the lending borrowing-process, considering the attitude of primary debtors and primary creditors towards the maturity structure of their monetary debts and monetary assets. Loanable funds, however, are channeled for a major part through financial intermediaries selling their own debts to primary creditors and holding debts issues by primaru debtors.

Financial intermediaries are traditionally classified according to the type of debt they issue. Monetary financial intermediaries issue a non interest-bearing debt that may be used as a medium of exchange and is not subject to variations in a nominal market-price. More precisely the price of this debt in terms of money, i.e. of itself, is always equal to one. Non monetary intermediaries issue interest-bearing debts that may not be readily used as a medium of exchange. Those debts may be extensively classified according to their liquidity, marketability and risk.
As we have emphasized the theoretical link between anticipated inflation and nominal rates of interest, the traditional distinction between non-monetary and monetary intermediaries will prove to be crucial when considering the impact of inflation on their intermediation activities.

2.3.4.1. Inflation and non monetary financial intermediaries

Non monetary intermediaries, as defined above, offer a broad range of debts on which they pay interest as the public is to be compensated for the loss of liquidity and,for some types of debt,for the risk of market-price fluctuations involved in asset holdings that may not be readily used as a medium of exchange.

Their very existence may said to be justified by the need to adjust the primary debtors' demand for long-term capital financing real investment to the primary creditors' demand for less risky interest-earning assets. In other words, they borrow short and lend long.
A major component in the interest rate differential between the rate they pay on their liabilities and the rate they earn on their assets is a compensation for the risk involved in this type of intermediation.

Now,as macro-economic forces tend to increase nominal rates of interest by anticipated rates of inflation and since this very adjustment implies a much larger increase in risk on long-term monetary assets than on short-term monetary assets, non-monetary intermediaries experience a considerable increase in risk in their lending-borrowing activities under conditions of inflation.
This may be most easily grasped by the following example.
Consider a non-monetary intermediary financing his assets on the market for time deposits and holding claims with a much longer maturity.
As a simplification,we assume time-deposits to be issued and assets to be acquired at the same date. At this very date nominal rates of interest on both times deposits and assets allow for the rate of anticipated inflation. If the rate of anticipated inflation has come to change when time deposits must be redeemed and assets must be refinanced with new time deposits, the nominal rate of interest on time deposits will change,while the nominal earnings on asset-holdings are stuck to the previously fixed level.
As a result, the profitability of non-monetary financial intermediation is subject to additionnal variation, i.e. additional risk, in times of inflation.

Another type of risk that inflation tends to increase is connected with the costs associated with possible deposit withdrauwals.
If by the time that a non-monetary intermediary has to meet net deposit withdrauwals nominal rates of interest have changed in response to a change

in the anticipated rate of inflation, some assets must be sold at market-prices that may diverge from their initial market-value.
If nominal rates of interest have increased, the intermediary undergoes a capital-loss, if nominal rates of interest have decreased a capital-gain. Consequently, inflation tends to enlarge the fluctuations in costs associated with deposit withdrawals and hence in profitability of non-monetary financial intermediation.

Finally, inflation tends to affect a third type of risk,which is the risk of default. Indeed, nominal rates of interest on financial intermediaries'assets tend to keep pace with the rate of inflation as it is anticipated. As it may very well be that factual inflation does not coincide with anticipated inflation, the real interest charges to be paid by primary debtors are uncertain. If factual inflation exceeds anticipated inflation, real interest-charges decrease and fall below the expected level. As a result, risk of default diminishes.
If factual inflation falls short off anticipated inflation,real interest charges will surpass the expected level and, as a result, risk of default increases.

We may obviously conclude that by its very effects on interest rates, anticipated inflation tends to hamper the activity of non-monetary intermediaries borrowing short and lending long.
The difference in the average maturity of their liabilities and of their assets measures somehow the degree of risk entailed.
If non-monetary intermediaries do permanently operate at optimum levels of risk and return, the increase in risk induced by inflation must be somehow minimized or compensated for.
Risk may be diminished by either shortening the average maturity of their assets or lengthening the average maturity of their liabilities.
Lengthening the average maturity of their liabilities is impossible, by virtue of what we know about the primary creditors'preference for a shortening of the average maturity of his assets in times of inflation.
The non-monetary intermediaries will neither be compensated for the increase in risk through higher rates of interest on their assets, as the primary

debtors' require risk premia in the form decreases in the long-term nominal rates of interest.
Consequently, there is only one degree of freedom left and non-monetary intermediaries will shorten the average maturity of their assets so as to minimize the negative risk-effects of inflation.
This shortening of the average maturity of their assets corresponds precisely to the primary debtors'preference for a shorter maturity of liabilities in times of inflation.

2.3.4.2. Inflation and monetary financial intermediaries

The monetary channel of financial intermediation is quite differently affected by inflation, as monetary intermediaries earn some interest on their assets, but do not pay any interest (or hardly) on the debt they issue. This debt has the advantage of being currently used as a medium of exchange and of being riskless in nominal value, while interest-bearing assets may change in nominal prices, when rates of interest vary.

Moreover, one might almost disregard the debt character of bank-deposits as the public permanently needs them for transactional purposes and as a substitution of ultimate money to bank-deposits is hardly advantageous.

The Chicago School (1) has paid much attention to the revenue earned from money creation.
Friedman pointed out that money creation diverts real income from those holding liquid balances to those who create them.
As in times of inflation, people do permanently increase their nominal balances so as to keep a given level of real liquidity, there is a permanent process of real income redistribution from money-holders to money- issuers The level of real liquidity held by the public depends in turn on the magnitude of the rate of inflation. Money holders experience inflationary erosion as a tax on money balances and the higher the tax rate, i.e. the

(1) FRIEDMAN, Milton, "Government Revenue from Inflation", in Journal of Political Economy, Aug. 1971.

rate of inflation, the less they are ready to hold real liquidity.
More precisely, for a given anticipated rate of inflation, i.e. for a given tax rate, people hold real liquidity to the point where the last dollar held yields a marginal utility equal to the real rate of interest plus the rate of anticipated inflation. Then, using an analysis of the general equilibrium type, Friedman comes also to the conclusion that nominal rates of interest keep pace with anticipated inflation.
It is precisely because money issuers do permanently earn a fixed real rate of interest on assets, which they do permanently expand that inflation is profitable to money issuers.

We are not so much concerned here with a study of seignoriage associated with money creation, but we may use the tools of it to show why inflation affects monetary intermediation differently from non-monetary intermediation.

A fundamental assumption of the Chicago School's analysis of the revenue from money creation is that money issuers have an active attitude towards inflation, i.e. they determine an optimal growth of the nominal stock of money so as to maximize their revenue.
This is the opposite to our assumption e) which we will maintain.
Hence we try to see what happens to the monetary intermediary, when operating at a certain level of inflation, which is regarded as exogeneous, and what the monetary sector ought to consider, when holding assets with various maturities.

Following Ph. Cagan's analysis, the revenue from money creation is defined as :

1) The real income earned through interest receipts on outstanding assets.

2) Minus the real depreciation of those assets through inflationary erosion.

3) Plus any net addition to assets per time.

Hence real income from money creation may be written as :

$$N = \frac{D}{P} \cdot i - \frac{D}{P} \cdot \Theta + \frac{D}{P} \cdot \Theta$$

where : N = real income per period.

$\frac{D}{P}$ = real ontstanding debt and hence the real value of the monetary sector's assets.

i = the nominal rate of interest, which allows for the rate of inflation Θ, hence $i = r + \Theta$, where r = real rate of interest.

$- \frac{D}{P} \cdot \Theta$ = real depreciation of assets.

$+ \frac{D}{P} \cdot \Theta$ = real growth per time in outstanding money balances, as the public wants to keep a given level of real liquidity.

This last item may be considered as a net addition assets, since the monetary sector as a whole does not regard money as a debt, but earns a rate of interest on the assets bought with it.

Hence we have for N

$N = \frac{D}{P} i = \frac{D}{P}(r + \Theta)$ as the growth in the monetary sector's assets makes up for the real depreciation of its assets.

We shall try to point out what happens to the income earned by money issuers, according to the length in time of their credits, i.e. of their assets.

Assume an unexpected increase in the rate of inflation.
Nominal rates of interest will correspondingly increase and real liquidity demanded by money holders will decrease. If we may assume the money market as being tight this decrease in demand for real liquidity may be disregarded.

Real assets will now depreciate at a higher rate, but this is compensated by a higher rate of growth of assets.

The interest income from previously bought assets does not allow for this now higher rate of inflation. But newly bought assets bear a rate of interest that allows for the new rate of inflation.

Consequently, if the monetary sector buys systematically short-term assets, it will be able to meet very fast. the high nominal rates of interest, when rates of inflation are high.
If the monetary sector buys systematically long-term assets, its real income will increase very slowly, as a major part of its assets earns a real income corresponding to a lower rate of inflation.

Analogically real income from money creation decreases very fast, if the monetary sector holds short-term assets, when the rate of inflation and hence the nominal rate of interest has come to decrease; white it decreases only very slowly if the monetary sector holds long-term assets on which it earns a rate of interest corresponding to more inflationary conditions.

Consequently, we come to the conclusion that, when nominal rates of interest allow for the anticipated rate of inflation, which is not under control of the monetary sector, holding short-term assets amounts to facing possible increases in real income, when the rate of inflation increases and possible decreases in real income, when the rate of inflation decreases. Holding long-term assets does not protect against those fluctuations in real income from money creation, but tends to even them to a large extent.

We are fully aware that this is a very partial view on the motivation to hold assets with various maturities, but it shows clearly that monetary intermediaries are differently concerned with inflation when compared to non-monetary intermediaries, because they do not pay any interest on their debt. While non-monetary intermediaries lost net wealth when the rate of inflation came to rise unexpectedly, the monetary intermediaries do only forgo the opportunity of increasing their net wealth.

Consequently, while long-term monetary holdings increased the risk inherent to non-monetary intermediation, they decrease the fluctuations in income from monetary intermediation.

2.4. CONCLUSION

The purpose of this analysis was to demonstrate that monetary assets may be ranked according to the amount of risk borne in times of inflation, provided that nominal rates of interest tend to keep pace with the anticipated rate of inflation.

Under these conditions, variations in the expected rate of inflation lead to capital-losses or gains that are proportional to the maturity of monetary assets. As a result, bonds and long-term fixed income assets tend to be more risky in times of inflation than short-term monetary assets and money itself.

From the attitudes of both debtors and creditors towards the average maturity of their assets, we deduced a general tendency to shorten this average maturity in times of inflation.

The so modified preferences of primary creditors and debtors tend to worsen the working-conditions of non-monetary intermediaries borrowing short and lending long and they are expected to shorten the average maturity of their assets as well. Monetary intermediaries proved to be differently affected, as they do not pay any interest on their liabilities. They will rather stabilize their real income in times of inflation, when holding long-term monetary assets.

All things considered the inflation will lead to a gradual disappearance of bond-markets.
Consequently, if the index-linkage of monetary assets is to be envisaged, it must be on the grounds of a possible solution to this problem.
This view is clearly confirmed in a recent report of the O.E.C.D. Secretariat (1) :

> "... Such high money rates (compensating for the expected rate of inflation), moreover, could play a negative role on the investment activities of enterprises by inducing them to refrain from borrowing at long-term. The historical experience shows that a situation like that may lead to the collapse of the market for long-term funds. The utilization of index-bonds is a natural and interesting way out of such a dramatic situation, since these special securities will have great attractiveness long-term for small savers and will stimulate the placing of their savings in long-term financial instruments, thus contributing to the revival of the bond-market..."

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(1) O.E.C.D., Committee on Financial Markets : "Indexation of Fixed-Interest Securities", Paris, 15th June, 1973, pp. 11-12.

AN INVESTIGATION ON THE RISK-FREE NATURE OF INDEX-LINKED MONETARY ASSETS

Introduction

We started the analysis of the indexation of financial assets by an investigation into the potential demand for this type of asset in inflationary economies. We showed why monetary assets become all more risky in times of inflation. Although the index-linkage seemed to be the appropriate remedy to this risk-effect of inflation, we were still to find out what kind of monetary asset would be most affected by the risk of inflation.
We built a theory that made it possible to establish a ranking of monetary assets according to the amount of risk borne in times of inflation.
We found that long-term monetary assets are significantly more risky than short-term monetary assets. Hence we found what to link to the index, if this device proves to be an effective risk-reducer.

At this point of the analysis, we shall demonstrate that the index-linkage of monetary assets does indeed take away all the inflationary risks and particularly that one, which is related to change in market-value of monetary assets.

The effects of the index-linkage will be analyzed on a comparative basis : we shall consider an economy where both nominal and index-tied assets exist side by side.
In the framework of a macro-economic model, we may then consider how changes in the expected rate of inflation affect the rate of interest on nominal and index-tied bonds.

As all the strength of the argument is in the comparison, we shall first consider how the markets for index-tied and nominal assets are interrelated. We will show that the rates of interest on nominal assets will be equal to the rate of interest on real assets plus the expected rate of inflation.

This partial equilibrium result will be followed by an empirical investigation into the demand for nominal and index-tied assets in Finland.

It will be shown that the market forces, that tend to establish the suggested relation between the rates of interest, work indeed in the way expected.

The theoretical results of this partial equilibrium analysis of rates of interest on nominal and index-tied assets will be integrated into a macro-economic framework so as to analyze the effectiveness of the index-linkage as a risk reducer.

An empirical observation of the working of capital-markets in Israël will confirm our thesis that the index-linkage contributes to preserve the normal functioning of bond-markets.

From a methodoligical point of view, we shall proceed as follows :

- We first consider the analytical properties of a few indexes that have been effectively usedas well as a few additional clauses on the working of the index-compensation.
- A second part of this chapter deals with the relation between rates of interest on nominal and index-tied assets.
- An empirical investigation into the demand for nominal and index-tied assets in Finland will confirm the main theoretical underpinnings of the partial equilibrium analysis.
- Finally, we achieve the integration into the macro-economic model of the partial equilibrium analysis.

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3.1. THE INDEX-LINKAGE OF FINANCIAL ASSETS AS AN INSTITUTIONAL ARRANGEMENT

In this chapter, we try to analyze the index-linkage of financial assets as a possible solution to the problems arising under inflationary conditions.

Most of the recent experiments with index-linking in capital markets differ from the point of view of the indexes used and of some additionnal clauses on the working of index-compensation.
Another point is to know what should be linked to the index chosen : interest payments, the principal, or both.

3.1.1. The types of indexes used in recent experiments

We shall not consider all the historical and economic circumstances that have led to the use of one particular index rather than of any other type. We shall mention the main theoretical characteristics of the main indexes that have been historically used so as to remain at a fair level of generality.

a) The cost-of-linving index

The cost-of-living index (COL index) is most commonly thought of as being the adequate coefficient, because this index is assumed to be used by creditors,when converting their monetary wealth into real wealth.
The point is to know whether such an index may be validly agreed upon by both the debtor and the creditor.
If the COL index is broadly defined, the rate at which its numerical value increases per year is a good approximation of the rate of inflation.
As a result, the creditor will consider that his monetary assets linked to such an index keep their real value.
The debtor, however, is concerned not with the purchasing power of his debt but with his capacity to meet the interest-charges and particularly the index-charges.
Hence the debtor may agree upon such an index, if it is a good measure of the development of his nominal income. This is likely to be so whenever the index is very broadly defined. Indeed, if the debtor is engaged in some

productive activity, his goods or services produced will then be taken into account in the COL index. Consequently, the debtor would regard the COL index as an appropriate measure of the development of his nominal income. Capital goods, however, do not enter a COL index; but as the final products that they help to produce do enter the COL index, those debtors producing capital goods consider their nominal income as indirectly related to the COL index.
The government, as a debtor, may also agree on a COL index. Indeed, tax-receipts are likely to keep pace with the rate of inflation.

The COL index or wholesale price index have been used in all index experiments in Finland, with the exception of three bond issues linked to the exchange-rate with the pound sterling.
Index experiments in Israël involved the use of both the COL index and of the exchange-rate of the Israel-pound with the U.S. Dollar.

b) Price indexes related to particular products

Particularly in France (1), bonds were tied to the price or the output of particular products. This implied for the creditor to bear the risk of relative price changes which depend, of course, on current conditions of supply and demand on the market for the product considered.
Such an index-linkage over-protects the debtor and relieves him at least partially of a risk that should be inherent to his entrepreneurship.
In the absence of a more generally defined index, the public may very well be ready to buy those assets, but the particular index used makes those bonds very similar to shares, and consequently the creditor may have to diversify his portfolio among a large number of bonds with different indexes.

c) Indexes related to particular asset-prices

Financial assets have also been tied to indexes that refer to price developments of particular assets, which creditors may consider as possible forms of wealth holding.

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(1) B. KRAGH, "Index-clause in deferred payments", Econ.Bulletin for Latin America, vol. 2, n° 2, United Nations, Santiago de Chile.

In 1951, for instance, The Palestine Land Development Company issued 5 % detentures convertible into shares or plots of Land (1).
Issues of bonds convertible into shares or entitling their owners to purchase flats at prices fixed in advance were made as well (1).

Land and real estate ownerships are of course commonly considered as good inflation-hedges. As a result, linking bonds to such a type of index amounts to improving the terms at which issues can be made.
The issuer of this type of bonds must of course be engaged in a type of activity that guarantees a corresponding progression of his capacity to meet the compensation-charges to be paid.

d) Foreign exchange-rate-indexes

The Industrial Mortgage Bank of Finland has floated three bond issues, two in 1953 and one in 1955, linked to the exchange-rate of the pound sterling and providing 50 per cent compensation (1).
The use of such an index has been very exceptional in Finland, but Israel has gone much further in this respect.

Such an index is a good but indirect measure of inflation, if domestic inflation is a principal component of the fluctuations in the relative price of the national monetary unit expressed in terms of foreign monetary units.
One ought to notice, however, that if foreign countries do also experience some inflation and if inflation is also a major component in the fluctuations of their exchange rate, the creditor will only be protected to the extent that the domestic rate of inflation exceeds some "world rate of inflation".

Moreover, if foreign countries have a larger rate of inflation than the domestic rate of inflation, the exchange-rate index does not only fail to protect the creditor against domestic inflation, it also makes him worse off and he would better have his assets not linked to such an index at all.

If the index is nothing but the rate of exchange between the national monetary unit and another foreign monetary unit, this index will also

(1) KRAGH, B., op. cit.

depend on a number of contingencies affecting the value of this foreign-monetary unit. A devaluation of the foreign monetary unit, for instance, would be disastrous.

If the index is calculated over a large number of exchange-rates of the national monetary unit with respect to other monetary units, the index is likely to be more stable, as it depends less on the economic circumstances in one single country and more on the parameters of the domestic economy, among which the domestic rate of inflation.

Moreover, in a fixed exchange-rate system the compensation for inflationary erosion occurs only stepwise through successive devaluations of the national monetary unit. Those devaluations may be hard to foresee with respect to their magnitude and to their timing.

This discontinuity in compensation for inflation generatesmore uncertainty than one pretends to remove by this type of index-linkage.
Indeed, it is clear that borrowers, who had received loans after a devaluation and repaid it before the next one, do not pay any escalation-charges (and the lenders do not receive any) while borrowers whose loans were granted just before a devaluation would be changed with tremendous additional payments (1).
This amounts to saying that borrowers, whe had received their loans after a devaluation and repaid them before the next one make not real gains on their debts, while debtors who issued their loans just before the next devaluation make net real losses on their debts.
The latter category of debtors bear in fact the charges of compensation for the inflation that prevailed before the date of issue of their debts.

We may conclude that, even if the foreign exchange-rate index is considered as an effective protection against the risks of real value of loans, it entails another type of risk inherent to the uncertainty of the timing of successive devaluations.

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(1) Amotz MORAG : Value linking of Loans in Israel; in Fiscal and Monetary Problems in Developing States, Ed. D. Krivine, N.Y. 1967.

By the same token, this kind of index-linkage increases considerably the social costs of a devaluation policy, which creates strong internal disturbances on the national capital markets. It follows as a paradox that the government might less resort to a devaluation-policy in view of those social costs, but this would distroy the effectiveness of a foreign exchange-rate index as a protection against inflation.

3.1.2. The use of additional clauses on the working of index-compensation

The index-clause is sometimes accompanied by a few more arrangements on the working of the index-compensation. There may be a "floor" or a "ceiling" to the index-compensation.
Financial assets may be partially or totally affected by the index-linkage. Compensation may be achieved "stepwise" or continuously.

a) The floor-and ceiling-clauses

The floor is indicated by the value of the index at the date of issue. The floor-clause guarantees that the debtor will not be worse off than if he had purchased an ordinary bond (1).
As in times of inflation, the value of the index is certainly not expected to fall a floor-clause may be regarded as almost unnecessary.

For the same reason, a ceiling-clause becomes extremely important. A ceiling-clause means that no increases in the value of the index will be considered above a certain value of the index (2).
Indeed, once the ceiling has been almost reached, there is nothing to gain any more from the index-clause and selling an asset with such a clause may only be done at very unfavourable terms, as the asset must compete with newly issued bonds, of which the ceiling has not been reached yet.

(1) KRAGH, B., op. cit.

(2) KRAGH, B., op. cit.

b) Partial index-linkage-clauses

Bonds may also be issued with clauses refering to the proportion of the bond that is index-linked. The problem with partially index-tied bonds is that the share of the index-tied part in the total nominal value progresses with the inflation. This may be easily demonstrated through a numerical example (1).

Assume a bond issued at an initial nominal value of 200; 50 % of the bond is index-linked, 50 % is nominal. What happens when prices double in one period ?

We have :

Initial nominal value at t	Index-tied part	Nominal part
200	100 (50 %)	100 (50 %)
At time (t+1) we have		
300	200 (67 %)	100 (33 %)

If a bond with the same "partition"-clause is issued at (t+1) at an initial value of 300, this bond may only be issued at very unfavourable terms, unless the partition-clause guarantees that 67 % will be index-tied. Newly issued bonds, however, will have to compete with other previously issued bonds having many different dates of issue.
As a result, yield comparisons and hence price-setting becomes extremely difficult.

Moreover, after some time the "partition-clause" will have no sense any more, as the inflation will have reduced the nominal parts to very small amounts.

c) Escalator-clauses

Discontinuous compensation clauses, known as escalator-clauses guarantee the index-compensation only stepwise, i.e. for instance for every 5, 10 per cent. Such a clause seems to impede a reliable calculation of expected yields over a short period of time. It has the advantage, however, to simplify the accounting problems, which an indexation of financial assets necessarily implies.

d) A conclusion on the additional clauses

All things considered, the additional clauses on the working of index-compensation may be regarded as undesirable, as they make the investor's calculation of expected rates of return a most difficult task.

If, for any reason, any of the clauses considered is desirable, it should be introduced uniformly throughout the market so as to avoid artificial distorsions in the capital-markets and to allow them to function smoothly.

3.1.3. What should be linked to the index ?

Different elements of a monetary asset may be linked to the index : interest-payments and capital-value.
For long-term monetary assets this problem may appear to be of no importance. Indeed, if interest-payments on those assets are the only index-linked element, the current market-price of those assets will not be different from the market-price of a long-term asset the principal-value of which is linked to the index as well.
The present value of the reimboursement of long-term monetary debts is very low and will hardly enter the calculation of the market-price. Consequently it does not matter very much whether or not the principal-value is tied to the index, the present-value of index-linked interest-payements being much higher than the present-value of the reimbursement-value.
Once such a long-term monetary asset comes close to its date of redemption, the absence of a clause linking the principal value to the index makes it rather unprofitable to hold such an asset and rather costly to sell it, as the present value of the redemption-value is now much higher.

Consequently, we propose both interest-payments and capital-value to be linked to the index as far as long-term assets are concerned.

For the same analytical reasons, we propose the index-linkage of the principal value of short-term assets and accessorily of the interest-payments as well.

3.2. A PARTIAL EQUILIBRIUM ANALYSIS OF NOMINAL AND INDEX-LINKED BOND-MARKETS

Introduction

This analysis is a major topic in the existing literature on the indexation of financial assets.
The analysis is entirely due to the Swedish authors Tord Palander (1) and Guy Arvidsson (2).

The main purpose of this study is to show how the rates of interest and hence the market-prices of nominal and index-linked assets will be related to each other. The main result will be that the nominal rate of interest on nominal assets must be equal to the real rate of interest on index-linked assets plus the rate of expected inflation.

We may hardly innovate on this topic, as it deserved most of the attention of many authors dealing with the indexation of financial assets.

We shall try, hower, to present the analysis in a more formalized fashion.

To focus the analysis on the effects of the index-linkage, we shall make the following assumptions.

a) We study a market system where index-linked and nominal bonds are freely bought and sold.
The only difference between both types of bonds is a 100 % index-linkage of both the principal and the interest-payments of the index-linked type.
Hence the bonds will be considered as equal in face-value, maturity and marketability.
Whether they will be equivalent from the point of view of risk is a fundamental question to which the answer will be a conclusion of our chapter.
Hence we assume them provisionally to be equivalent from the risk point of view.

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(1) PALANDER, TORD : Värdebeständighet, Stockholm 1957.

(2) ARVIDSSON, Guy: "Reflections on index-loans", Skandinaviska Bankar Quarterly Review, january 1959.

b) We assume the absence of any differential tax-regime that would bring about any distorsion in the price-setting.
Hence, if nominal bonds are taxed, index-tied bonds should be taxed in the same way. This raises the question how index-compensations should be taxed. This seems to be a topic in itself and it does not imply any loss of generality to assume both nominal and index-tied bonds to be tax-exempt.

c) The analysis is partial in the sense that we consider the investor's choice between both types of bonds, once the investor has decided to keep a certain amount of his wealth in the form of interest-bearing assets rather than in the form of liquid balances.
Once we have know how index-linked and nominal markets operate we may restate this partial analysis in a macro-framework, including the problem of the demand for money,when alternative assets have been index-tied.

d) Our assumption a) ensures a high rate of substitution between nominal and index-linked bonds. As a result, the simultaneous approach of both the nominal and index-tied markets is justified.

The analysis will be conducted as follows :

- We shall first define verbally and mathematically the nominal and real rates of return on nominal and index-tied assets.
- We will show that there is a definitional relation between a nominal and a real rate of return on one single asset.
- Finally, we define the equilibrium relations that must exist between the rates of return on nominal and index-tied assets.
- The entire analysis will be graphically illustrated.

3.2.1. A calculation of rates of return on nominal and index-linked bonds

3.2.1.1. A few definitional elements

a) The flow of interest-payments

Nominal and index-linked bonds have the same face-value of, say, 100. The flow of interest-payments per period of time on nominal bonds is equal to :

$100 \times a_m$ where a_m is the rate of interest prevailing at the date of issue.

The flow of nominal interest-payments per period of time on index-linked bonds is equal to :

$100 \times a_r \cdot \frac{p(t)}{p(-\tau)}$ where $p(t)$ is the value of a broadly defined index at time t, and $p(-\tau)$ is the value of the index at time $t = -\tau$, the date of issue; a_r is the rate of interest on index-bonds at the date of issue.

b) Market-prices and redemption-values

If the investor buys nominal or index-linked bonds at time t = o, he must acquire them at market-price.

Hence we write $K_m(o)$ = market-value of nominal bonds at time t = o

$K_r(o)$ = market-value of index-linked bonds at time t = o

The redemption-value of the bonds will be written as :

$B_m(T)$ = redemption-value of nominal bonds when they come due at time T

$B_r(T)$ = redemption-values of index-bonds when they come due at time T.

c) A definition of a rate of return

The investor will buy nominal or index-linked bonds according to the magnitude of the expected rates of return on both investments. Hence we need a rigourous definition of a rate of return.

If the investor does not expect the rate of interest to change, he may define as a rate of return the discount-factor that equalizes all the present and future flows of interest-payments plus the final redemption-value at time t = T to the current market-value of the asset considered.

Such a rate of return may be defined in nominal terms or in real terms. The real rate of return discounts the real flows of interest-payments and redemption-value to the current market-price.
The nominal rate of return discounts the nominal flows of interest-payments and redemption-values to the current market-price.

Hence we may write the nominal rate of return on nominal bonds as i_m

" " real " " " i_r

" " nominal " " index-linked as r_m

" " real " " " r_r.

3.2.1.2. Computing the rates of return

When investing his wealth into index-linked or nominal bonds, the investor considers the index from the point of view of its possible variation in value, hence he must have in mind some idea about the future rate of inflation.

The initial value of the index that prevails at the time the asset is bought may be set equal to 1.

Consequently, if the investor expects a future rate of inflation Θ to prevail evenly from 0 to T, the expected value of the index at time t is equal to :

$$P(t) = P(o)\, e^{\Theta t} = e^{\Theta t} \quad \text{if} \quad P(o) \text{ is set equal to 1.}$$

We may now calculate different rates of return on nominal and index-linked bonds.

a) The nominal rate of return on nominal bonds is defined by the equation :

$$K_m(o) = \int_o^T a_m . e^{-i_m . t}\, dt + B_m(T)\, e^{-i_m T} \qquad (1)$$

or $$K_m(o) = \frac{a_m}{i_m} \left[1 - e^{-i_m T} \right] + B_m(T)\, e^{-i_m T} \qquad (2)$$

b) The real rate of return on nominal bonds must satisfy the equation

$$K_m(o) = \int_o^T a_m\, e^{-\Theta t}\, e^{-i_r t} .\, dt + B_m(T)\, e^{-\Theta T}\, e^{-i_r T} \qquad (3)$$

or $$K_m(o) = \frac{a_m}{i_r + \Theta} \left[1 - e^{-(i_r + \Theta)T} \right] + B_m(T)\, e^{-(i_r + \Theta)T} \qquad (4)$$

c) The nominal rate of return on index-linked bonds is defined by the equation

$$K_r(o) = \int_o^T a_r \cdot e^{\Theta t} e^{-r_m t} dt + B_r(t) e^{\Theta T} e^{-r_m T} \quad (5)$$

or

$$K_r(o) = \frac{a_r}{r_m - \Theta} \left[1 - e^{-(r_m - \Theta)T} \right] + B_r(T) e^{-(r_m - \Theta)T} \quad (6)$$

d) The real rate of return on index-linked bonds is defined by the equation

$$K_r(o) = \int_o^T a_r e^{-r_r t} dt + B_r(T) e^{-r_r T} \quad (7)$$

or

$$K_r(o) = \frac{a_r}{r_r} \left[1 - e^{-r_r T} \right] + B_r(T) e^{-r_r T} \quad (8)$$

3.2.1.3. The necessary relations between nominal and real rates of return on one type of bond

It follows as a direct corollary from our definition of real and nominal rates of return on one type of bond that the nominal and the real rate of return on nominal bonds are mathamatically related to each other. For the same reason, the nominal and the real rates of return on index-linked bonds must also be mathamatically related.

Indeed from the system

$$\begin{cases} K_m(o) = \dfrac{a_m}{i_m} \left[1 - e^{-i_m T} \right] + B_m(T) e^{-i_m T} \\ K_m(o) = \dfrac{a_m}{i_r + \Theta} \left[1 - e^{-(i_r + \Theta)T} \right] + B_m(T) e^{-(i_r + \Theta)T} \end{cases}$$

it follows as a truism that

$$\boxed{i_m = i_r + \Theta} \quad (9)$$

In other words, the money rate of return on nominal bonds is equal to the real rate of return on nominal bonds plus the rate of expected inflation. For the same reason, we may deduce a relation between the nominal rate of return and the real rate of return on index-linked bonds.

From the system :

$$\begin{cases} K_r(o) = \dfrac{a_r}{r_m - \Theta}\left[1 - e^{-(r_m-\Theta)T}\right] + B_r(T)\, e^{-(r_m-\Theta)T} \\ K_r(o) = \dfrac{a_r}{r_r}\left[1 - e^{-r_r T}\right] + B_r(T)\, e^{-r_r T} \end{cases}$$

it follows as a truism that

$$r_m - \Theta = r_r$$

or

$$\boxed{r_m = r_r + \Theta} \qquad (10)$$

In other words, the money rate of return on index-linked bonds is equal to the real rate of return on index-linked bonds plus the expected rate of inflation (which is the expected rate of increase in the value of the index).

3.2.2. The equilibrium relations between nominal and real rates of return on nominal and index-linked bonds

The relations (9) and (10) between nominal and real rates of return followed as a truism from our definitions of nominal and real rates of return.

The relations considered here are of a different nature and prevail only at the partial equilibrium point of both the nominal and index-linked markets.

If both markets are in equilibrium with regard to each other, the investors should be indifferent between nominal bond or index-linked bond-holdings. They will be indifferent to nominal or real bond-holdings, if the nominal rate of return expected on nominal bonds is equal to the nominal rate of return expected on real bonds.
If they calculate in real terms, they will be indifferent to nominal or real bond-holdings, if the real rate of return on real bonds is equal to the real rate of return on nominal bonds.

That such a relation between rates of return and hence between prices of nominal and index-linked bonds will exist at equilibrium may be proved ab absurdo.

If the nominal rate of return on nominal bonds exceeds the nominal rate of return on index-linked bonds, investors will sell their index-linked bonds and buy nominal bonds. They might even borrow in the index-linked market and lend in the nominal market. As a result, current market-prices of index-bonds will decrease and the nominal rate of return on index-linked bonds will necessarily increase. Conversely, current market-prices of nominal bonds will increase and nominal rates of return will consequently decrease. This mechanism operates up to the point where the nominal rates of return on nominal and real bonds are equal.

This equilibrium relation may be written as :

$$\boxed{i_m = r_m}$$

: the nominal rate of return on nominal bonds is equal to the nominal rate of return on index-linked bonds.

We know however that $\begin{cases} r_m = r_r + \Theta \\ i_m = i_r + \Theta \end{cases}$ which are derived from our definition of nominal and real rates of return

Hence we have one redundant relation $r_r = i_r$.

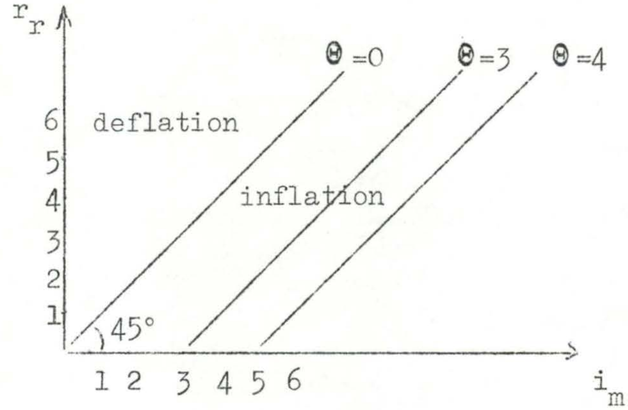
We may now find the mathematical relation between the prices of nominal and index-linked bonds.

Indeed, we have $i_m = r_m$ and $\begin{cases} r_m = r_r + \Theta \\ i_m = i_r + \Theta \end{cases}$

It follows immediately that in equilibrium : $\boxed{i_m = r_r + \Theta}$

As we know that for a given flow of nominal interest-payments and a given redemption value of nominal bonds, i_m determines the price of nominal bonds and that for the same reason r_r determines the price of index-linked bonds, the prices of nominal and index-linked bonds are interdependent.

This functional relation between the nominal rate of return on nominal bonds i_m and the nominal rate of return on real bonds $(r_r + \Theta)$ may be represented as below :



This relation between the market-rate of interest on nominal bonds and the market rate of interest on real bonds is subject to one important remark.

Indeed, comparing the nominal rate of return on nominal rate of return on nominal bonds with the nominal rate of return on index-linked bonds, or comparing the real rate of return on index-linked bonds with the real rate of return on nominal bonds is dependent on individual inflationary expectations and individual appreciation of risk.

We shall provisionally disregard the problems related to the dispersion of individual expectations,as we have assumed that all the lenders share the same anticipation of inflation. We must, however, take the risk-factor into account.

The nominal rate of return on nominal bonds, i.e. the market rate of interest on nominal bonds, is certain, the nominal rate of return on index-tied bonds, i.e. the market rate of interest on index-tied bonds plus the rate of inflation, is uncertain. The latter rate of return is uncertain, because it is expected to be of a given magnitude and the investors cannot be sure that factual inflation will effectively coincide with anticipated inflation.

Conversely, the real rate of return on index-tied bonds, i.e. the market rate of interest on index-tied bonds, is certain,while the real rate of return on nominal bonds, i.e. the market rate of interest on nominal bonds minus the rate of expected inflation, is uncertain. Indeed, investors may not be sure that factual inflation will coincide with anticipated inflation.

Now, if investors have a definite aversion to the risk on either the nominal or the real rate of return, this aversion will be reflected in the interest-rate differential on nominal and index-tied bonds.
If they prefer the certainty of the nominal rate of return on nominal bonds to the uncertainty of a nominal rate of return on index-tied bonds, they will buy nominal bonds, even if the interest-rate differential is exactly equal to the magnitude of the anticipated rate of inflation.
Hence nominal bond prices will increase and nominal rates of interest on nominal bonds will decrease. As a result, this will increase the interest-rate differential beyond the level where it was equal to the expected rate of inflation.

If investors prefer the certainty of the real rate of return on real bonds, i.e. of the market rate of interest on real bonds, to the uncertainty of the real rate of return on nominal bonds, i.e. the nominal rate of interest on nominal bonds as determined by the market, they will buy index-tied bonds, which will also enlarge the interest-rate differential beyond the level where it was exactly equal to the rate of anticipated inflation.

Consequently, we may maintain our conclusion that at equilibrium the nominal rate of interest on nominal bonds is equal to the real rate of interest on index bonds, provided that both types of risk are equally distributed among the market operators.

3.2.2. The equilibrium relation between subjective rates of return

So far we have laid stress on the relation that prevails in equilibrium between the rate of interest on nominal bonds and the rate of interest on index-tied bonds.

It may be, however, that investors speculate on future changes in the market-rates of interest which generate capital-gains or losses.

Consequently, the subjective expected rate of return may be smaller or larger than the current market-rate of interest.

If the investors do have such a speculative behaviour, the equilibrium relation between the subjective rates of return should be written as :

$$\left(i_m + \frac{d \log i_m}{dt} \right) = \left(r_r + \frac{d \log r_r}{dt} \right) + \Theta$$

where $\frac{d \log i_m}{dt}$ = expected capital gain or loss, expressed as a percentage of the initial market value of nominal bonds.

$\frac{d \log r_r}{dt}$ = expected capital gain or loss, expressed as a percentage of the initial market-value of real bonds.

If there is no reason to expect the nominal rates of interest to change differently from real rates of interest, we have again :

$$\frac{d \log i_m}{dt} = \frac{d \log r_r}{dt} \Rightarrow i_m = r_r + \Theta .$$

3.2.4. A graphical illustration of Palander's "Twin-markets system"

Much of the formal analysis may be easily grasped in a graphical illustration constructed by Robert V. Eagly (1).

The north-western quadrant is a two-dimensional space defined by the vectors i_m, the rate of interest on nominal bonds, and r_r the rate of interest on index-bonds.
The functions Θ relate the two rates of interest to each other for different rates of inflation and define in fact the equilibrium relations between the rates of interest.

The north-eastern quadrant is a two-dimensional space where the functions B_r^d are demand-functions for real bonds. The demand for real bonds is expressed as a positive function of the real rate of interest on real bonds.

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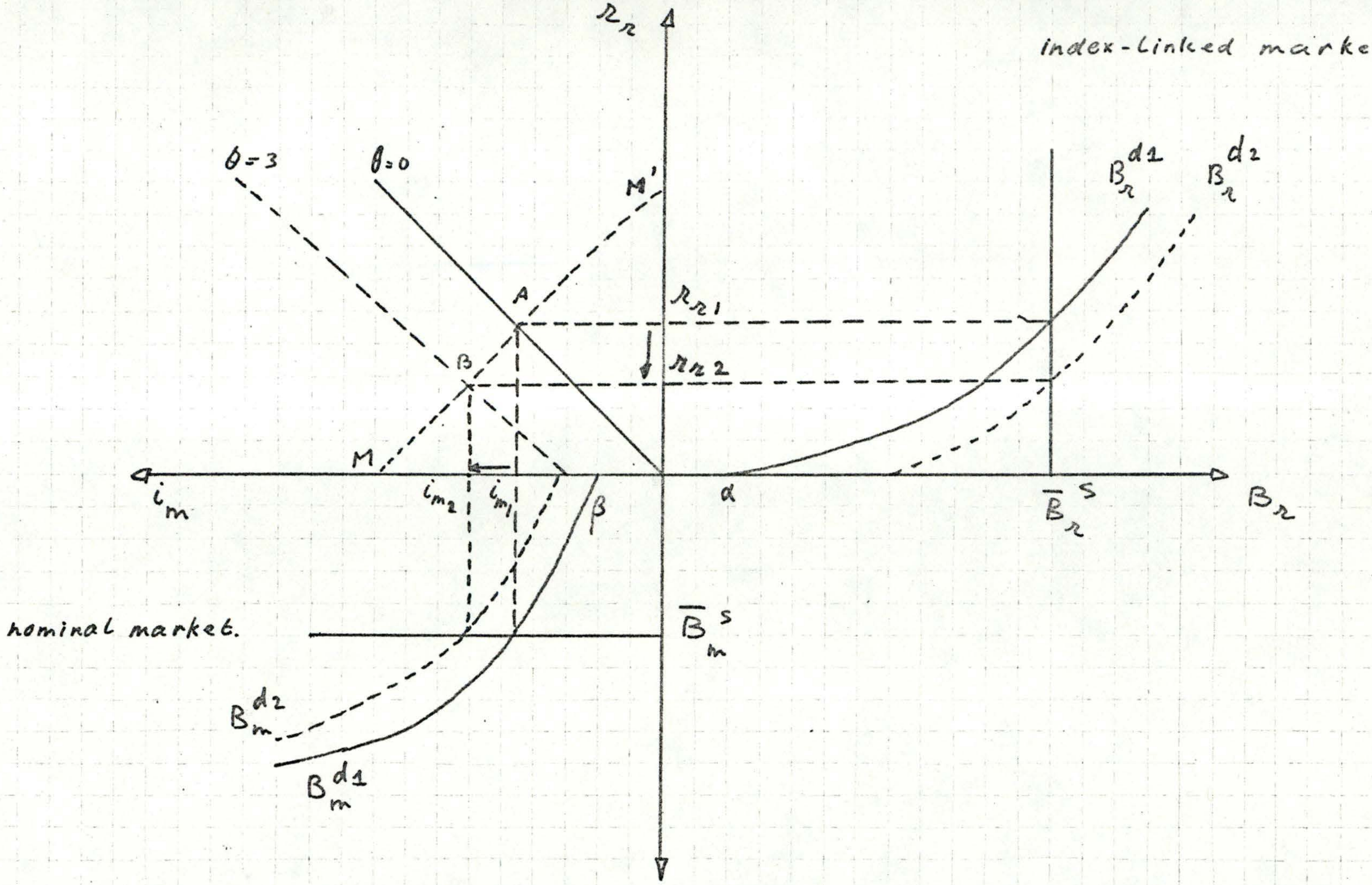
(1) Robert V. EAGLY, "On government issuance of an index-bond", in Finances Publiques, La Haye, n° 3, XXII, 1967, pp. 268-288.



index-linked market
nominal market.
θ=3
θ=0
M'
M
A
B
α
β

The intercept α means that even for a zero rate of interest on index-tied bonds people are still ready to hold some amount of their wealth in index-tied forms so as to safeguard their purchasing-power.

When holding index-tied bonds for a zero rate of interest on real bonds, people fail the risk of a capital-loss, if rates of interest increase. Consequently, we think that the intercept α in Eagly's system implies that people expect to gain more from an increase in the index than they might lose through an increase in the rate of interest on index-bonds.

As we lay stress on the demand for nominal and index-tied bonds, the stocks of bonds supplied are assumed to be given : $\overline{B}_r^s$ and $\overline{B}_m^s$

In the south-western quadrant, the function B^d expresses the demand for nominal bonds as a function of the nominal rate of interest on nominal bonds. The stock of nominal bonds supplied is given.

The intercept β means that no nominal bonds are demanded below a certain rate of interest, as nominal rates of interest are expected to be normally beyond this minimum rate of interest.

When no inflation is expected ($\Theta = 0$), the equilibrium relation between the rate of interest on nominal bonds and the rate of interest on real bonds is equality : $i_m = r_r$.
If demand and supply are given on the index-market, demand is antomatically determined on the nominal market.

Assume now that people come suddenly to expect a rate of inflation $\Theta = 3$. As the nominal rate of interest on nominal bonds is initially equal to the real rate of interest on real bonds, real bonds are expected to yield a higher nominal rate of interest than nominal bonds, or nominal bonds are expected to yield a lower real rate of interest than index-bonds.

Consequently, there will be an increase in demand for index-linked bonds and a decrease in demand for nominal bonds. This will result into an increase in index-linked bond prices and into a decrease of the rate of interest on index-linked bonds. Nominal bond prices will be lower and nominal rates of interest on nominal bonds higher.

The new equilibrium relation between the rate of interest will be given by the function $\Theta = 3$ and we have $i_m = r_r + 3$.

To what extent this new equilibrium relation has been established by a decrease in the real rate of interest rather than by an increase in the nominal rate of interest or vice versa depends entirely on the elasticities of both the demand curves and on the relative shifts in demand for nominal and index-linked bonds in response to changes in inflationary expectations.

An important result that we may deduce from Eagly's graphical construction is that, for given elasticities of the demand curves and for given propensities to shift the demand for nominal and index-tied assets in response to changing anticipations, there will be a locus of combinations of nominal rates of interest on nominal bonds and of real rates of interest on real bonds.

This locus of points is represented by the curve MM'. Point A is a combination of rates of interest for a zero rate of inflation, point B is a combination of rates of interest for $\Theta = 3$.

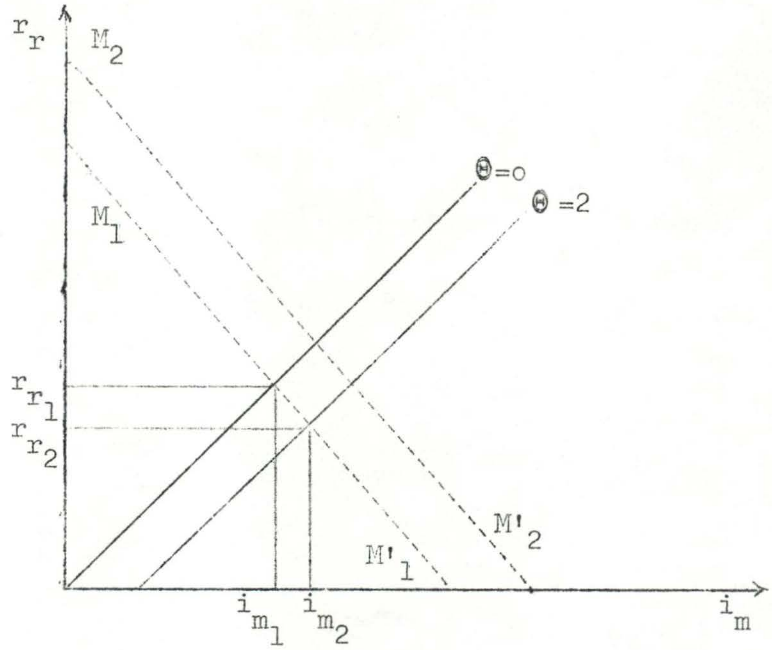
The slope of MM' will be a function of the elasticities of the demand-curves and of the propensities to shift the demand from one type of bond to another type of bond in response to changing rates of anticipated inflation.

If the demand-curves have similar interest-elasticities, but people tend to react to higher rates of inflation by increasing their demand for index-bonds rather than by decreasing their demand for nominal bonds, the MM' curve will be relatively steep.

If propensities to shift the demand from one type of bond to another type of bond are of about the same magnitude, but if the interest-elasticity of the demand-curve for nominal bonds is larger than the interest-elasticity of the demand curve for real bonds, the MM' curve will be relatively steep as well.

Such a locus of rates of interest for different rates of inflation depends on the relative stocks of nominal and index-tied bonds supplied.

Enlarging the supply of one type of bond will shift the locus to the right and probably modify its slope.
Enlarging proportionally the supply of nominal and real bonds will shift the locus to the right without affecting its slope.



M_1M_1' locus of rates of interest for a given supply of index-linked and nominal bonds.

M_1M_2' the same locus of points for a larger supply of nominal and index-linked bonds.

So far the analysis has been conducted under the assumption that inflationary expectations are identical for all the market operators, i.e. for all the lenders and borrowers.

If price expectations are dispersed, the interest rate differential which is established in the market will depend on the volume of indexed stock offered and the degree of dispersion of price expectations.
The differential will reflect the price-expectations of marginal bond holders only (1).

If only a small stock of index-tied bonds is placed on the market, it can be offered on relatively unfavourable terms so as to appeal only to those with the most inflationary price expectations.

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(1) Peter ROBSON, "Index-linked Bonds", Review of Economic Studies, Oct.1960.

Larger initial offerings would bring about higher rates of interest on index-linked bonds so as to induce holders of ordinary bonds with less inflationary expectations to take them up.
If the money rate of interest is assumed to be given, this would mean a gain in expected interest-earnings for the intramarginal holders of linked bonds.

This, however, is likely to be a minor issue, the main result of this analysis being that the "own" rate of interest on nominal bonds is equal to the "own" rate of interest on index-linked bonds plus the rate of expected inflation.

The macro-economic implications of this relation will be considered later on and we shall be in a position to use it as a proof for the risk-free nature of index-tied assets with respect to inflation.

3.3. EMPIRICAL EVIDENCE ON THE DEMAND FOR NOMINAL AND INDEX-LINKED TIME-DEPOSITS IN FINLAND *

Introduction

So far we have considered the theory of equilibrium of nominal and index-linked markets. The assets dealt in on those markets were assumed to be bonds. Empirical evidence for such a theory should logically be given by an investigation into the effective pricing-mechanism of such a market where both types of bonds are bought and sold side by side.

Now, the Finnish experience with the index-linkage of monetary assets is very broad and the index-linkages have spread over all types of monetary assets including bonds, but with the exception of money itself.
Shortly after world war II, government-issued bonds only were tied to the cost-of-living index, but as soon as the index-linkage turned out to be advantegeous to the creditors, the banking sector was to offer the same

* We are very much indebted to Mr Kari Puumanen of the Bank of Finland, for his permanent help and interest in this part of our research.

index-clause on the public's deposits in its competition for loanable funds.

Unfortunately one may not really speak of a bond-market in Finland. Rates of interest on bonds are under government control and do not depend on current supply and demand.
When applying to make a bond-issue, the issuer must provide information on interest and any other possible yield on the loan. When granting the permit, the Cabinet approves the conditions of the issue (1).
There is no secundary market for bonds and bonds are mainly held by their initial buyer throughout their lifetime. The limited size of the bond-market and its very institutional character prevented us from having any insight into the effects of the index-linkage on the pricing-mechanism of nominal and real bonds.

By contrast, the market for bank-deposits is extremely large and the main channel of financial intermediation in Finland. The predominance of this market is principally function of the public's attitudes : bank-deposits are regarded as an acceptable, secure and honourable type of asset, whereas investing in securities is regarded as something as gambling (2).

An important characteristic of the Finnish deposit-market is the small ratio of demand deposits to total bank deposits on the average less than 10 %. This is due to the fact that a faily large amount can be drawn on demand from the most popular term deposits, which formally require 6 months notice of withdrawal, without the interest being lost. Those deposits are called "ordinary-time-deposits". At the moment the upper limit for this type of drawing in 3000 FMK every month and the rate of interest is 4 % a year. As the demand deposits are not paid interest a considerable part of the liquid assets of households and small enterprises is placed in those term deposits (2). One may well consider those deposits as a kind of interest-bearing money.

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(1) Bank of Finland, Monetary Policy Dept., "Information on national bond-markets in Finland", sept. 12, 1972, p. 5.

(2) Bank of Finland, Financial markets in Finland, 1972, p. 3.

One important restriction of the previous partial equilibrium analysis remains to be discussed so as to enable us to draw relevant conclusions from the empirical picture.

Indeed, banks offered both nominal and index-tied time-deposits. Hence we may consider the effects of the index-linkage on the demand for both types of assets.

If, however, we insist upon the partial character of such an analysis, we we implicitly assume that all other assets may be disregarded from the point of view of the public's portfolio-choices : an increase in demand for index-linked time-deposits will be considered as a decrease in demand for nominal time-deposits and not as a decrease in demand for money or any other asset.

Such on assumption may be validly held under the conditions prevailing in Finland. As we have shown above, there is no effective bond-market in Finland. There is no secundary market for bonds, as bonds are held throughout their lifetime by their initial buyer. Consequently, bond-holdings are very illiquid.

There exists no Treasury bill-market nor any organized market for call-money or short-term money in general (1). Demand-deposits and currency, however, should be regarded as good substitutes for time-deposits.

Here again, Finnish institutional characteristics converge to justifying our partial approach. Indeed, demand deposits amount to no more than 10 % of total deposits, so if any substitution of time deposits for demand deposits, or vice versa, occurs it will be relatively small.

The core-factor, however, is that holders of demand-deposits in Finland do not hold time-deposits and vice versa. Demand deposits are mainly held by big firms and time-deposits by households. Households consider time-deposits as sufficiently liquid for their own transactional purposes, which firms do not. As a result, there exists no substitution between demand and time-deposits.

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(1) HANNI, Eila, "Inflation in Post-War Finland", Economic Studies XXXII, p. 177.

3.3.1. The data

a) The period considered

Banks decided in may 1955 that the public should be given the opportunity to hold part of its assets in index-tied forms. There was no question of automatically linking all deposits with the index. Index accounts of a new type were opened side by side with ordinary accounts. It was to the public to decide whether to invest in nominal or index-accounts.
As the index did not operate beyond a certain level that was not reached at the end of 1955, index-accounts turned out to be unprofitable and by the end of 1955 the commercial banks ceased accepting more index-accounts. The year 1956, however, proved to be a period of very strong inflation and in december 1956, the annual rate of inflation reached 14 %. Under those conditions of very strong inflation, the government index-bonds yielded very high rates of return when compared with the rates of return on nominal time-deposits. As a result, the banking-sector had to face a contracting deposit-market and to reconsider its attitude towards the index-linkage of at least part of its time-deposits. Index-accounts were accepted again from the beginning of 1957 until novembre 1968, when it was decided to discontinue all the index-linkages existing in the Finnish economy.

Our calculations have been made on quarterly data covering a period from january 1957 until december 1967.

b) Nominal and index-linked time-deposits in Finland

- Nominal time-deposits

Until 1.4.1959, banks offered one single type of time-deposit, called the "ordinary time-deposit".
This is the type of deposit in which households invest most of their liquid savings. Monthly withdrawals of up to 3000 FMK are allowed without loss of interest. Those deposits require 6 months notice of withdrawal.

After 1.4.1959, banks offered a second type of time-deposit, called the "high interest rate deposit", because they earn a higher rate of interest than ordinary time-deposits. Those deposits, however, are 12 month-deposits.

- Index-tied deposits

Index-tied deposits have been available in two forms. The A-account was a 100 % index-linked account, the B-account was a 50 % index-linked account.

Both types of deposits were 12 month - deposits. The rate of interest paid on those accounts was generally lower than the rate of interest paid on nominal deposits.

The capital-value only was tied to the cost-of-living index.

Index-tied assets could only be held with a minimum amount of 300 FMK.

Index-tied deposits have generally been subject to taxes, but tax-regimes changed many times. Nominal deposits have always been tax-exempted.

Index-tied deposits were clearly less liquid than the nominal "ordinary 6 months deposits".

c) The objective yield differential on nominal and index-tied deposits

By objective yield differential we mean the difference in return on nominal and index-linked deposits that results from differences in rates of interest and tax-regimes and not from the index-clause.

As we have mentioned sub b), rates of interest on nominal deposits have been generally higher than those on index-deposits. One particular feature of Finnish banking, however, is the absence of competition on the deposit-market through variations in the rates of interest. Consequently, those variations have been relatively small and so have been the variations in the interest-differential on index-linked and nominal accounts.

Changes in tax-regimes have contributed to variations in the objective yield differential as well. Il is worth noting that index-tied accounts were taxed as regards the nominal capital-value and the interest-payments. Hence the compensation for inflation was subject to taxation as well.

Changes in the interest-rate differential and in the tax-regimes have been taken into account simultaneously so as to obtain a curve for the objective yield differential on nominal and index-tied accounts.

3.3.2. A specification of the model

In our theoretical analysis, we considered the relation that prevails in equilibrium between the rates of interest on nominal and index-tied assets. The rates of interest were fixed by current supply and demand.

The logic of the estimated empirical relation is slightly different. We rather try to see how the share of index-deposits in total time-deposits changes in response to variations in the expected returns. If we are in a position to estimate such a function we may, of course, deduce a few results that are relevant for the theoretical analysis. More precisely, for given stocks of nominal and index-tied deposits supplied by banks, one may calculate the relation that must prevail in equilibrium between the rate of anticipated inflation and the interest-rate differential, if the public is to hold the stocks of both types of assets in the proportion supplied.
Such a relation shows, for instance, to what extent the banks must enlarge the interest-rate differential on nominal and index-tied deposits so as to keep index-tied and nominal deposits in a certain proportion in times of stronger inflation.

We are not interested in the absolute levels of index-tied or nominal deposits held, but rather in the share of index-tied or nominal deposits in the portfolio of total time deposits.
The variable so defined measures the public's willingness to hold any total stock of time-deposits supplied by banks in partly nominal, partly index-tied forms.

Following the theoretical analysis, the higher the rate of anticipated inflation the larger the share of index-tied deposits in the public's portfolio of time-deposits. Conversely, the higher the objective rate of return (rate of interest-tax rate) on nominal deposits relative to the rate of return on index-tied deposits, the larger the share of nominal deposits in the public's portfolio of time-deposits.

There is, of course, no quantitative information about the public's inflationary expectations. Consequently, we have to use a proxy for the anticipated rate of inflation.

One may reasonably adopt the Hickxian assumption that expectations of future price-increases are a function of price-variations observed in the past. Moreover, during the period under review the public expected twice the Finnmark to be devalued. This expectation of a possible devaluation led to the expectation of possible increases in the cost-of-living index and may have affected in this way the demand for index-tied deposits. We thus use a set of past rates of inflation and a devaluation-dummy as a proxy for the anticipated rate of inflation.

The relevant lag for past rates of inflation proved to be rather long. In fact, three different lags are to be taken into account, when regressing the share of index-tied deposits in total deposits against past rates of inflation.

Firstly, there is a lag between the effective rate of inflation and its effect on inflationary expectations : it may take some time after a change in the effective rate of inflation before the individuals expect the inflation to continue at its new rate. They will observe the new rate of inflation at least for some time in order to assess their subjective belief that it changes again.

Secondly, if individuals have come to expect a different rate of inflation, it may take some time again before they adjust the effective proportion of index-tied deposits in their portfolio to the proportion effectively desired.

Thirdly, there exists an institutional lag that is entirely due to the maturity of the different deposits. People must wait until the date of withdrawal before reallocating their portfolio.

As we know that index-tied deposits have a maturity of 12 months and nominal deposits a maturity of 6 months for "ordinary time-deposits" and of 12 months for "high interest-rate deposits", the institutional lag will tend to be longer, whenever the public wants to decrease the share of index-deposits in response to, for instance, decreases in inflationary expectations, whereas the lag will be shorter, whenever the public wants to increase the share of index-deposits in its deposit portfolio.

The objective rate differential was lagged as well. Since this rate differential is not anticipated or expected, but objectively determined, there is no lag due to an anticipating-process. The second and the third type of lag mentioned above remain relevant.

One may easily conclude from this that the total lag over the part rates of inflation ought to be larger than the total lag over past magnitudes of the objective rate differential.

A last important variable that enters the model is a tax-dummy having the value 0 from the first quarter of 1953 until the first quarter of 1964 and the value 1 for all the subsequent quarters.
This tax-dummy refers to a change in tax-regime of one important alternative form of wealth-holding which were dwellings.

> "In the early 1950 s a considerable incentive was established to invest in real estate of this kind by making the property and rental income on new dwellings tax-exempt for ten years. As a result of hedging against inflation, in particular, real estate prices in Helsinki, the center of rapid urbanization, have skyrocheted"(1)

Taking this asset into account does not amount to opening the model to a third asset. Acquiring dwellings is likely to follow an antonomeous trend, as the rate of substitution to such a short-term asset as time-deposits is likely to be close to zero. If the income - and property -tax regime of dwellings come to change drastically, it may have a strong impact on the level of the trend and this might affect the level of demand for inflation-proof deposits. Moreover, the effects are likely to be delayed, with little effect in the beginning of the new tax-regime and, after some time, with strong effects that progressively die out. Hence a lag distribution was adjusted on the change in tax regime as well.

In mathematical form, we were to estimate the following equation.

$$y_t = a_o + a_1 D + \sum_{i=o}^{11} w_i \; P_{t-i} + \sum_{i=o}^{5} v_i \; R_{t-i} + \sum_{i=o}^{15} s_i \; T_{t-i}$$

where y_t = share of index-tied deposits in the total stock of deposits (nominal + index - tied deposits)

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(1) DAHMEN, Erik, Suomen Aalondellinen kehitys ja talouspolitiika, p. 96, Bank of Finland Publications, c. 4, Helsinki, 1963.

a_o = constant

D = devaluation dummy

P_{t-i} = effective rate of inflation at time (t-i); percentage change on previous year averaged over three months

R_{t-i} = objective rate differential (interest differential + tax differential) at time (t-i)

T_{t-i} = value of tax-dummy at time (t-i)

3.3.3. The technique used

Before discussing the empirical results we shall briefly expose the main elements of the technique.

As we have shown above a realistic specification of a model of demand for index-tied and nominal deposits has to take into account a few lagged independent variables.
Moreover, the weights of past rates of inflation are likely to be differently distributed, when compared to the weights of past rate differentials or tax-effects.

Now the Almon-technique is flexible enough to allow the dependent variable to be regressed against different independent variables with independent lag distributions of different length and shape. Hence it corresponds very well to the problem tackled here.

The core of Almon's technique is to restrict the weights of lagged variables to lay on a polynomial of a given degree. On needs then to calculate a few points of the polynomial to get all the other values by simple interpolation.

Such a restriction solves the common problem of multicollinearity that arises many times, when the coefficients of lagged variables are estimated by simple least-squares.

In Almon's (1) terms, one estimates the function $y = \sum_{i=o}^{n-1} w(i)\, X_{t-i}$

(1) ALMON, Shirley, The distributed lag between capital appropriations and expenditures, Econometrica, vol. 33, n° 1 (jan. 1965).

where de $w(i)$'s are values at $x = o, \ldots, x = n-1$ of a polynomial $w(x)$ of degree $(q+1) < n$, where n is the length of the lag distribution.

If one knows (q+2) points on this polynomial, all the other points may be deduced by interpolation and one has :

$$w(i) = \sum_{j=o}^{q+1} \Phi_j(i)\, b_j \qquad i=o,\ldots, n-1$$

where b_j are the values of (q+2) points.

The interpolation polynomials are written as :

$$\Phi_o(x) = \frac{(x-x_1).(x-x_1)\ldots(x-x_{q+1})}{(x_o-x_1)(x_o-x_2)\ldots(x_o-x_{q+1})}$$

$$\Phi_1(x) = \frac{(x-x_o)(x-x_2)\ldots(x - x_{q+1})}{(x_1-x_o)(x_1-x_2)\ldots(x_1-x_{q+1})}$$

$$\vdots$$

$$\Phi_{q+1}(x) = \frac{(x - x_o)(x - x_1)\ldots(x - x_q)}{(x_{q+1}-x_o)(x_{q+1}-x_1)\ldots(x_{q+1}-x_q)}$$

Hence $w(x) = \sum_{j=o}^{q+1} \Phi_j(x)\, b_j$ is indeed a polynomial of degree q+1

If the weights are equal to zero before time o and after time n-1, one may take $x_o = -1$, $x_{q+1} = n$ and $b_o = b_{q+1} = o$

As a result, we have : $w(x) = \sum_{j=1}^{q} \Phi_j(x)\, b_j$

Consequently $y = \sum_{i=o}^{n-1} (i)\, X_{t-i}$ becomes $\sum_{i=o}^{n-1} \left[\sum_{j=1}^{q} \Phi_j(i)\, b_j \right] X_{t-i}$

and $$y = \sum_{j=1}^{q} b_j \sum_{i=o}^{n-1} \Phi_j(i)\, X_{t-i}$$

In other words, the Almon technique regresses y against "q" auxiliary variables of the form $\sum_{i=o}^{n-1} \Phi_j(i)\, X_{b-i}$ $\quad j=1\ldots q$.

The number of "q" variables is equal to the degree of the polynomial minus one.

3.3.4. The empirical results

The equation we propose to estimate is :

$$y = a_o + a_1 D + \sum_{i=o}^{11} w_i P_{t-i} + \sum_{i=o}^{5} v_i R_{t-i} + \sum_{i=o}^{15} S_i T_{t-i}$$

There are three lag distributions involved.

The lag distributions of past rates of inflation is supposed to lie on a 2d degree polynomial

" " " rate differentials " 1st degree polynomial

" " " values of the tax dummy is supposed to lie on 3d degree polynomial.

Hence we have 2 auxiliary variables for the rate of inflation, P_{A1} and P_{A2}

" 1 " variable for the rate differential, R_A

" 3 " variables for tax-dummies, T_{A1}, T_{A2} and T_{A3}.

The correlation-matrix for those auxiliary variables plus the devaluation-dummy and the constant term is

	P_{A1}	P_{A2}	R_A	T_{A1}	T_{A2}	T_{A3}	D	const.
P_{A1}	1.000	0.951	-0.312	0.059	0.089	0.110	0.446	0.000
P_{A2}	0.951	1.000	-0.333	-0.011	0.019	0.041	0.524	0.000
R_A	-0.312	-0.333	1.000	0.690	0.720	0.737	-0.310	0.000
T_{A1}	0.058	-0.011	0.690	1.000	0.995	0.986	-0.014	0.000
T_{A2}	0.089	0.019	0.720	0.995	1.000	0.998	-0.039	0.000
T_{A3}	0.110	0.041	0.737	0.986	0.998	1.000	-0.052	0.000
D	0.446	0.524	-0.310	-0.014	-0.039	-0.052	1.000	0.000
Const.	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

correlation with dependent variable :

0.734	0.703	0.064	0.627	0.619	0.613	0.514	0.000

One may usefully notice the absence of partial correlation between y and R_1. This corresponds entirely to Finnish banking conditions where variations in deposit rates of interest are very small end are by no means a weapon of competition.

The regression over the auxiliary variables yielded the following results.

	regression coefficients	standard errors	students t
P_{A1}	1.575	0.243	6.493
P_{A2}	1.267	0.221	5.730
R_A	-0.228	0.425	-0.536
T_{A1}	164.070	20.132	8.150
T_{A2}	-235.424	44.524	-5.288
T_{A3}	97.566	25.927	3.763
D	2.654	0.702	3.781
Const.	-5.899	1.285	-4.592

There is definite autocorrelation as $D - W = 1.149$
and $R^2 = 0.991$

The lag distributions are given by polynomial interpolation of the regression coefficients :

1) The weights W_i of lagged rates of inflation

	W_i	standard error	t-ratio
1	0.565	0.027	20.690
2	0.489	0.022	22.021
3	0.418	0.018	22.995
4	0.351	0.015	23.156
5	0.290	0.013	22.135
6	0.233	0.011	19.996
7	0.182	0.010	17.249
8	0.136	0.094	14.449
9	0.094	0.079	11.920
10	0.058	0.059	9.766
11	0.026	0.033	7.974
12	0.000	0.000	0.000

$$\sum_{i=1}^{12} w_i = 2.842$$

2) The weights, v_i, of lagged rate differentials

	v_i	standard error	t-ratio
1	-0.076	0.142	-0.536
2	-0.060	0.113	-0.536
3	-0.046	0.085	-0.536
4	-0.030	0.057	-0.536
5	-0.015	0.028	-0.536
6	0.000	0.000	0.000

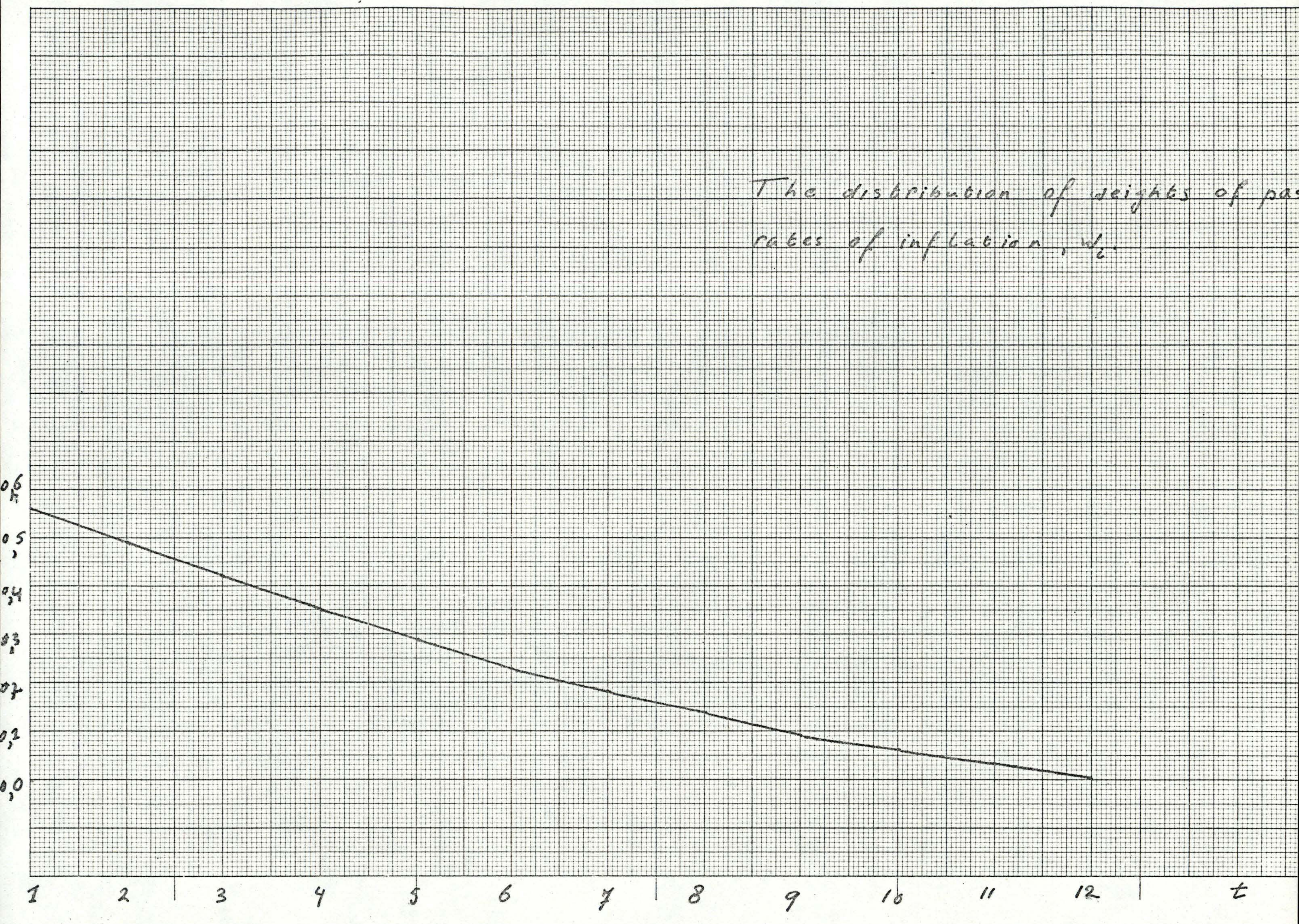
$$\sum_{i=1}^{6} v_i = -0.228$$

3) The weights, s_i, of lagged values of the tax-dummy

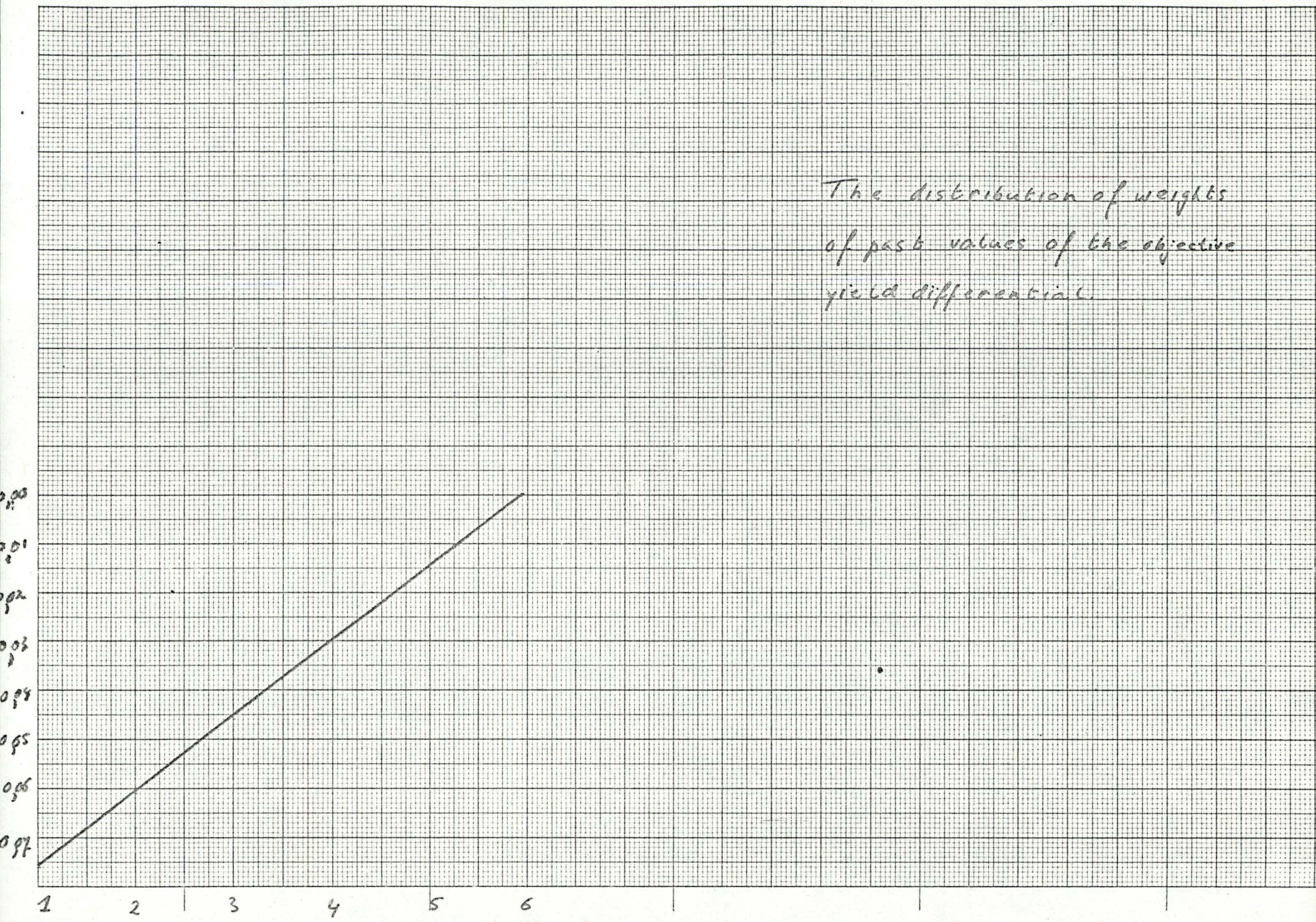
	s_i	standard error	t-ratio
1	0.658	0.607	1.084
2	0.521	0.354	1.472
3	0.574	0.211	2.718
4	0.775	0.179	4.328
5	1.085	0.194	5.607
6	1.462	0.198	7.372
7	1.866	0.185	10.085
8	2.256	0.165	13.711
9	2.592	0.156	16.590
10	2.832	0.173	16.394
11	2.937	0.205	14.351
12	2.865	0.232	12.333
13	2.576	0.239	10.786
14	2.029	0.210	9.648
15	1.184	0.135	8.799
16	0.000	0.000	0.000

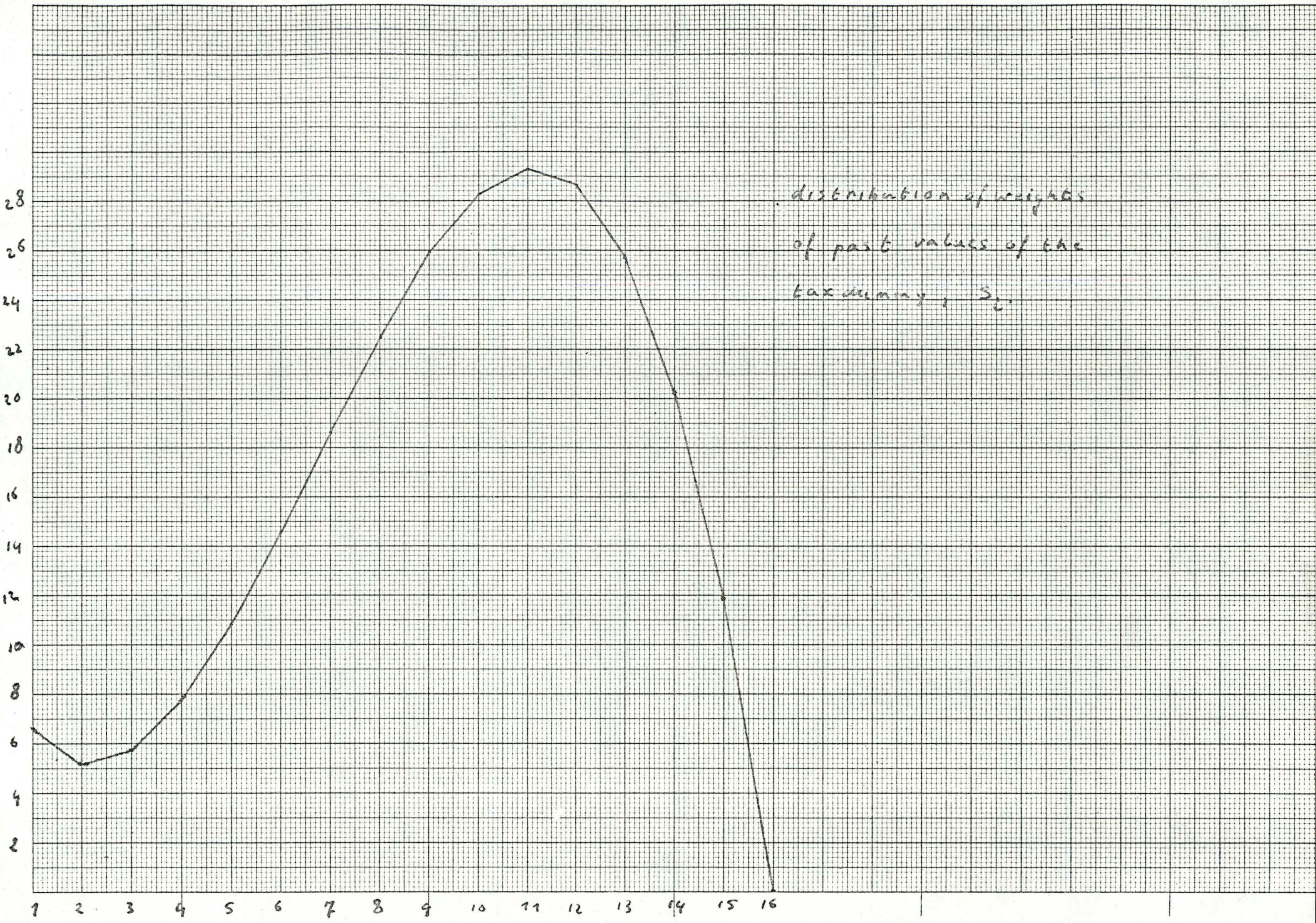
$$\sum_{i=1}^{16} s_i = 26.212$$

The distribution of the weights are represented on the next pages.



The distribution of weights of pas
rates of inflation, w_i
0,6
0,5
0,4
0,3
0,2
0,1
0,0
1
2
3
4
5
6
7
8
9
10
11
12
t

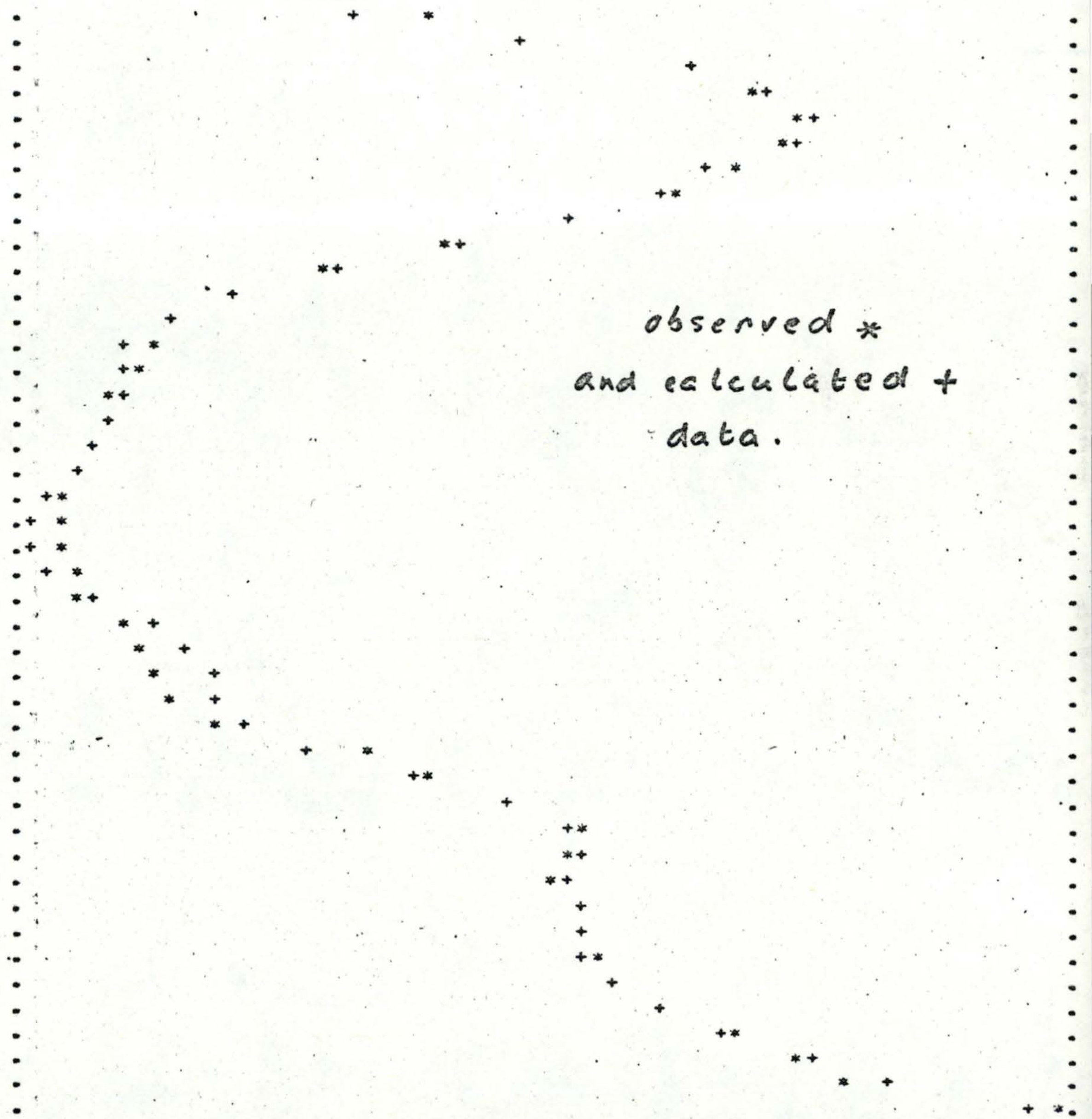

The distribution of weights of past values of the objective yield differential.
0,00
0,01
0,02
0,03
0,04
0,05
0,06
0,07
1
2
3
4
5
6


distribution of weights
of past values of the
tax dummy, S_i.
28
26
24
22
20
18
16
14
12
10
8
6
4
2
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

***** RESIDUAL ANALYSIS *****

OBSERVED	CALCULATED	RESIDUAL	% ERROR	OBS.
13.4200	10.8825	2.5375	18.9087	17
16.7200	17.0256	-.3056	-1.8279	18
22.8800	22.8933	-.0133	-.0580	19
25.0400	25.4912	-.4512	-1.8021	20
26.4800	26.9378	-.4578	-1.7288	21
26.0500	26.4289	-.3789	-1.4546	22
24.2300	23.3451	.8849	3.6522	23
22.3000	21.4511	.8489	3.8069	24
18.5600	18.2547	.3053	1.6450	25
13.9700	14.6092	-.6392	-4.5753	26
9.7500	10.1873	-.4373	-4.4854	27
6.3600	6.4848	-.1248	-1.9627	28
4.1600	4.3531	-.1931	-4.6429	29
3.6400	2.7706	.8694	23.8851	30
3.2800	2.4612	.8188	24.9628	31
2.2900	2.5397	-.2497	-10.9024	32
1.9100	2.0593	-.1493	-7.8154	33
1.4000	1.7865	-.3865	-27.6075	34
1.0300	.9976	.0324	3.1479	35
.6200	.1380	.4820	77.7463	36
.5100	-.4217	.9317	182.6886	37
.5800	-.6144	1.1944	205.9274	38
.8500	.1507	.6993	82.2746	39
1.0500	1.7284	-.6784	-64.6098	40
2.4200	3.4720	-1.0520	-43.4728	41
3.3700	4.8129	-1.4429	-42.8150	42
3.6900	5.7903	-2.1003	-56.9182	43
4.0800	6.1487	-2.0687	-50.7023	44
5.7200	6.9223	-1.2023	-21.0190	45
11.1700	9.0164	2.1536	19.2803	46
13.6800	12.8659	.8141	5.9510	47
16.3500	16.1337	.2163	1.3229	48
18.8300	18.3563	.4737	2.5157	49
18.1900	19.2391	-1.0491	-5.7675	50
18.1300	18.6823	-.5523	-3.0465	51
18.7900	18.7373	.0527	.2807	52
19.1500	18.8682	.2818	1.4716	53
19.5300	19.2268	.3032	1.5525	54
20.3200	20.2355	.0845	.4158	55
21.9600	21.6719	.2881	1.3120	56
24.6800	24.0368	.6432	2.6063	57
26.4200	26.9977	-.5777	-2.1868	58
27.9600	29.6666	-1.7068	-6.1045	59
35.8500	34.5488	1.3012	3.6296	60

RANGE = 36.4644



**=NOT USED IN REGRESSION

The sum of the weights is equal to the long-run elasticity of the independent variable.

The sum of the weights of past rates of inflation, $\sum_{i=1}^{12} w_i = 2.842$, measures the effect of a steady anticipated rate of inflation on the desired proportion of index-tied deposits in the total portfolio. The same may be said about the sum of the weights of the lagged rate differential.

Our equation may be written as :

$$y = -5.89 + 2.654\, D + \sum_{i=1}^{12} w_i P_{t-i} + \sum_{i=1}^{6} v_i R_{t-i} + \sum_{i=1}^{16} s_i T_{t-i}$$

where the w_i, v_i, s_i are given by the series above.

For a steady anticipated rate of inflation, rate differential and tax-regime, the equation may be written with it's long-run elasticities :

$$y = -5.89 + 2.654\, D + 2.842\, P - 0.228\, R + 26.212\, T$$

3.3.5. Conclusion

The empirical results support quite clearly the theoretical proposition that the demand for index-tied assets increases with the anticipated rate of inflation and decreases for larger values of the interest-rate differential. The signs observed correspond to the signs expected.

On may immediately deduce from this that there must be a locus of points that relates values of interest-rate differentials to rates of anticipated inflation in such a manner that the public's demand for a given proportion of index-tied assets in its total deposit-portfolio remains constant, and equal to, for instance, the proportion in which index-tied and nominal assets are supplied by banks.

Consequently, there is a strong analogy with Palander's "twin-markets" system where market-rates of interest on nominal and index-tied assets varied so as to establish a new rate-differential that made people ready to hold both types of assets in the proportions supplied.

The only difference in the context of our empirical test is that time-deposits are not marketable and do not display any price-variation so as to bear new market rates of interest. The estimated function shows however that banks may change the interest-rate differential on new deposits, if people expect a different rate of inflation. In terms of economic policy, the banks enjoy one degree of freedom, when establishing the new rate differential : they may change the rate of interest on nominal deposits, the rate of interest on index-tied deposits or both.

It remains, however, that the magnitudes of the coefficients relating anticipated inflation and the rate differential to the proportional demand of index-tied assets are of very different orders.
Indeed, writing the estimated function with its long-run coefficients, we have :

$$y = -5.89 + 2.654\ D + 2.842\ P - 0.228\ R + 26.212\ T$$

where P is a steady anticipated rate of inflation,
R the objective rate differential
D the devaluation dummy
T a tax dummy.

According to our theoretical analysis, which states that in equilibrium the rate differential must be equal to the rate of anticipated inflation, R = P, the coefficients of P and R in the estimated function should not significantly differ from each other, but they undoubtedly do.

Indeed, if we may provisionally disregard the other values so as to fours the attention on the relation between P and R, a slight increase in P must be followed by a large increase in the rate-differential, if y is to be kept constant, because the coefficient of R is very small compared to the coefficient of P. As a result, the theoretical equality does not seem to hold, as R must largely exceed P. This substantial difference in magnitude of the regression coefficients of P and R may be due to several causes.

From a theoretical point of view, the public's attitude towards risk could be predominant. Indeed, it may very well be that the public's concern in its choice between nominal and index-tied depositsis mainly to safeguard the real value of its savings.

This would mean that at least part of the public is ready to invest in index-tied deposits whatever the rate-differential may be, as people are willing to pay a high price for the certainty of the real value of their savings.
As we noticed in the theoretical analysis, this is a good reason for the equilibrium interest-rate differential not to be equal to the rate of anticipated inflation.

The reason, however, that is likely to be more relevant in the Finnish context is the absence of competition on the deposit-market through changes in rates of interest.
As we explained when discussing the data, a part of the objective rate differential results from changes in tax-regimes of different types of account. Such variations in the rate differential were not dictated by the needs of competition on the deposit market.
The t-ratios for the estimated weights of lagged values of the rate differential show that this variable is unsignificant, once its weights are restricted to lay on a first degree polynomial.
Polynomials with a higher degree were tried as well, but they yielded at least some weights with the wrong sign.
All things considered, the result for R reflects very well the conditions of Finnish banking.

One may well ask why banks were not led to competing more through variations in rates of interest on time deposits, as soon as they were to offer index-accounts. Indeed, at a first glance it seems to be highly desirable to have some control over the spreading of the index-linkage of the banks' liabilities and an active interest-policy seems to allow such a control.
If for instance, one particular financial institution has a far greater part of its deposits linked to the index than most of its competitors, it will have to charge its credit customers more, unless it can reduce the index-linked part of its liabilities to "normal" levels by larger interest-rate differentials on the public's deposits. Hence the index-linkage as such introduces an incentive to compete more through the rates of interest.

This stimulus to interest-rate competition was removed by the way compensation-charges were collected from the borrowers: all the financial institutions charged the borrowers with the same extra rate of interest in the framework of a pooling-system managed by the Bank of Finland.

Now, in the absence of a very significant relation between the proportion of index-tied assets demanded and the rate-differential, it may seem curious that people did not decide to hold almost all their deposits in index-tied forms as the rates of inflation have often exceeded the rate differential. This is most likely due to a liquidity effect in the choice between nominal and index-tied deposits.
Indeed, as explained in the data, part of nominal time deposits were very liquid and perfectly served the transactional purposes of households. Index-tied deposits were 12 month -deposits and far less liquid.

All things considered, the results point to the effective positive relation between the demand for index-tied deposits and inflation and to a negative relation between the demand for index-tied deposits and the objective differential in rates of return, measured as a combination of differences in rates of interest, tax-regimes and liquidity-proporties.

3.4. AN INVESTIGATION INTO THE RISK-FREE NATURE OF INDEX-TIED MONETARY ASSETS

Introduction

Our partial analysis showed how the princing-mechanism on markets for nominal and index-tied assets operates so as to establish an equilibrium relation between the rates of interest and consequently, between the market-prices of nominal and index-tied assets.
There must, however, be an equilibrium price of financial assets, since there is an equilibrium rate of interest defined at a macro-economic level. In our chapter II, we showed how expected inflation affects the nominal rate of interest as nominal bonds and consequently, how it affects interest-bearing monetary assets from the point of view of risk.

In this part of the analysis, we shall integrate the equilibrium relation between rates of interest on nominal and index-tied bonds into the same macro-economic framework.
We may then consider how expected inflation affects the equilibrium prices of nominal and index-tied assets.

In fact, we will show that, if the nominal rates of interest keep pace with the expected rate of inflation so as to keeps their real value, this will imply changing market-prices of nominal assets and unchanged market-prices of index-tied assets, as the expected rate of change in value of the index replaces for index-tied assets the changes in market-value that occur on nominal assets.

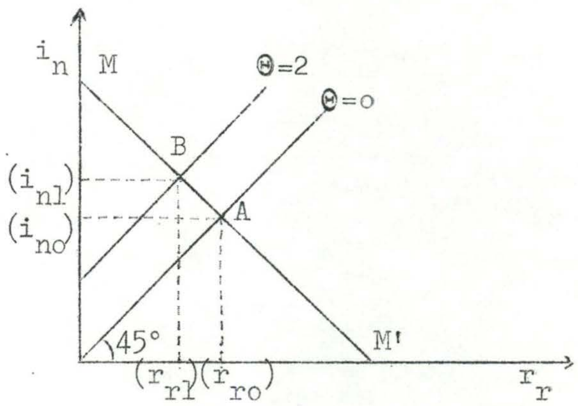
Methodologically, we proceed as follows :

- We first confront the partial equilibrium results to the demand for money.
- We resort to a comparative statics analysis in order to compare the response of market-prices of nominal and index-tied assets to changing rates of expected inflation.
- From this, we may deduce the conditions for the risk-free nature of index-tied assets and some attention will be paid to the contribution of the index-linkage to a revival of the bond-market in Israel.

3.4.1. The partial equilibrium analysis confronted with the demand for money

So far we have shown how market-rates of interest on nominal and index-tied assets are interrelated : in equilibrium the nominal rate of interest on nominal bonds will be equal to the rate of interest on index-tied bonds plus the anticipated rate of inflation.

We have shown as well that, for a given stock of nominal and index-tied assets supplied, there will be a locus of combinations of rates of interest on both types of assets, each combination refering to a specific value of the expected rate of inflation.
This was graphically illustrated as follows :



i_n : nominal rate of interest on nominal bonds.

r_r : real rate of interest on real bonds.

(r_{ro}, i_{no}) is a combination of rates of interest for a zero expected rate of inflation and a given stock of bonds supplied.

(r_{r1}, i_{n1}) is a combination of rates of interest for an expected rate of inflation of 2 % and a given stock of bonds supplied.

The results of this analysis may be integrated into a macro-economic model so as to get insight into the differences in response of market-prices of nominal and index-tied bonds to changing rates of anticipated inflation.

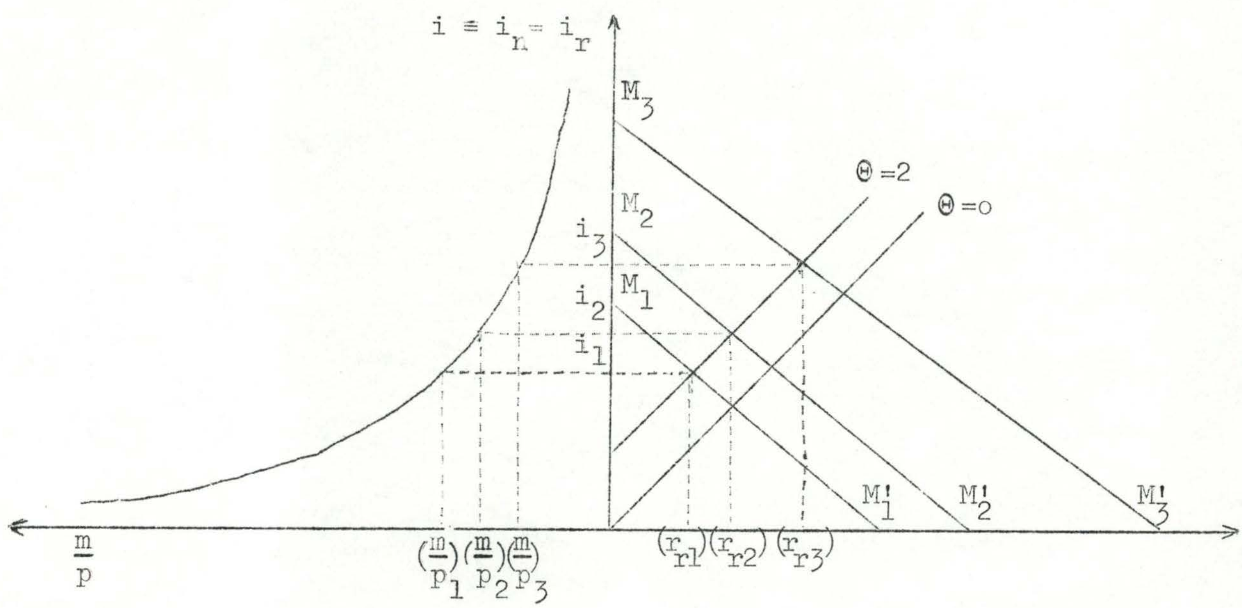
Before achieving this integration, we shall first confront those partial results with the problem of demand for money.
Indeed, by definition of the indexation technique money is not linked to the index, but it certainly is a close substitute to nominal and index-linked interest-bearing assets, which in our case are bonds. The level of money demanded will depend on the rates of interest on both types of assets, which in turn are definitely linked to each other.

We will be able to give an interpretation to the locus MM' in terms of demand of money.

Let us simply draw the keynesian liquidity-preference curve together with the equilibrium relations on Polander's twin-markets for nominal and index-tied assets.

The amount of money demanded will be a decreasing function of the nominal rate of interest.

We have :



The locus $M_1 M'_1$ corresponds to a given stock of nominal and index-linked bonds supplied. If the rate of expected inflation is equal to Θ = 2% the equilibrium combination on the nominal and index-linked market is equal to (i_{n1}, r_{r1}).

Those rates of interest are just high enough to induce people to hold the stocks of nominal and index-linked bonds supplied.

If the state of inflationary expectations does not change, and if the stocks of nominal and index-tied bonds supplied are increased proportionaly, the locus will be shifted to $M_2 M'_2$ as rates of interest on both types of bonds must increase to induce people to hold a larger share of their wealth in bonds. The equilibrium combination of rates of interest will be equal to (i_{n2}, r_{r2}). The locus $M_2 M'_2$ will correspond to a lower level of demand for money $(\frac{m}{p})_2$.

A shift of the locus to $M_3 M'_3$ through further increases in the supply of bonds will correspond to a level of demand for money $(\frac{m}{p})_3$.

This shows that all the locuses MM' correspond to given levels of demand for money. Moreover, the geometrical distances between two successive locuses tends to increase for a given decrease in the demand for money, owing to the gradual interest-inelasticity of money demand at levels of high rates of interest.

Consequently, the right-hand side of the figure may be used as a simultaneous illustration of the demand for money, money-bonds and index-linked bonds.

3.4.2. The response of market-prices of nominal and index-tied assets to changing rates of anticipated inflation

This section will be developed analogically to the macro-economic analysis of our chapter II.
The only difference in the assumptions will be that people, i.e. ultimate wealth-holders, may invest there wealth into a new type of asset: index-linked bonds. The alternative forms of wealth-holding will be money and nominal bonds. Once again, we assume the absence of equity-financing so as to focus the attention on the monetary assets only, of which one is linked to the index.

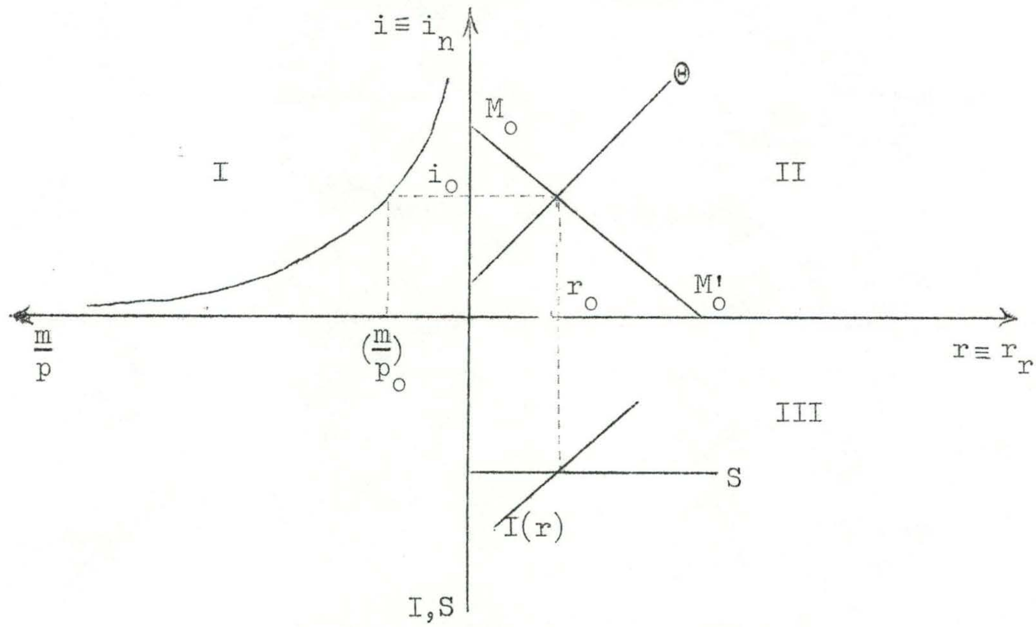
We shall proceed by a comparative statics analysis, where the changing parameter will be the expected rate of inflation.
The effect of the index-linkage may then be deduced from a difference in the change of market-value of nominal and index-tied bonds.

The equilibrium equations of this macro-model may be written as follows :

1) $I_o + g.r = S;\ g' < o$	ex ante investment is equal to ex ante saving, investment is a function of the real rate of interest, as in chapter II.
2) $(\frac{m}{p})_o = L(i)$ with $L_i' < o$	Money demanded is equal to money supplied. Money demanded is a decreasing function of the nominal rate of interest.
3) $r = i_n - \Theta = r_r$ or $i_n = r_r + \Theta$	The real rate of interest is equal to the nominal rate of interest on money bonds minus the expected rate of inflation.

Through arbitrage on the twin-markets for nominal and real bonds, the nominal rate of interest on money bonds minus the rate of expected inflation is equal to the real rate of interest on real bonds.

The model may be represented as follows:



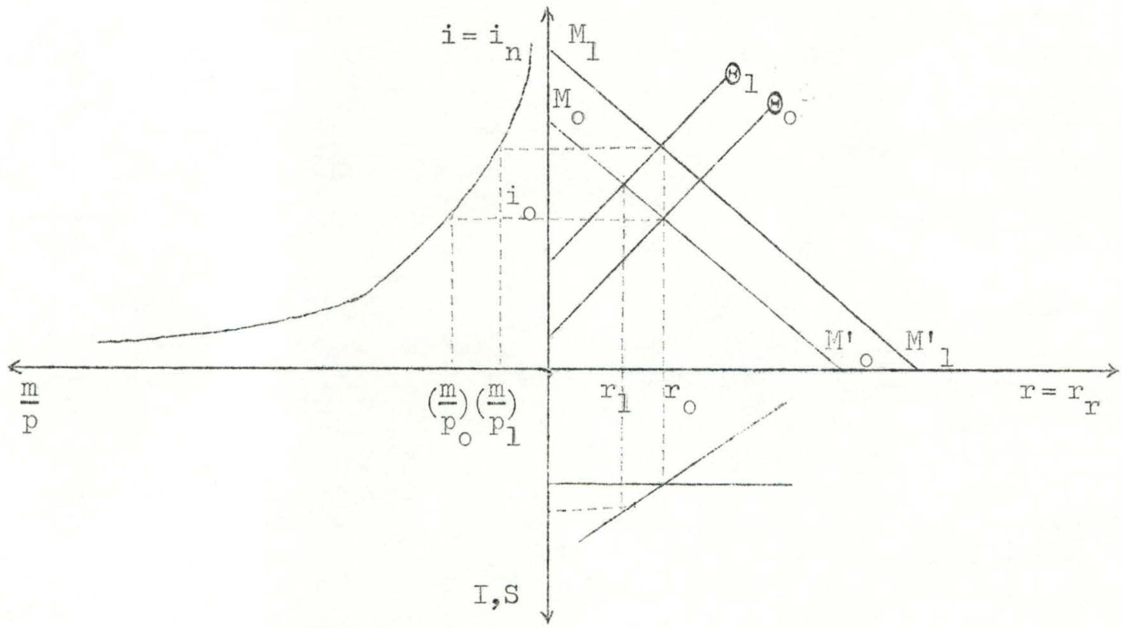
Part I of the figure represents a liquidity-preference curve.
Part II shows the locus of points $M_oM'_o$ that corresponds to a given level of bonds supplied and to a level of real liquidity demanded $(\frac{m}{p})_o$.
The function establishes the equilibrium relation between the rates of interest on nominal and index-tied bonds.
Part III shows the equilibrium relation I = S on the commodity market.
This equilibrium is reached for a real rate of interest $r_o = r_{ro}$ and hence for an "equilibrium price of index-tied assets", as this price is a function of r_r.

What happens if people come to expect a higher rate of inflation ?

The change in inflationary expectations, an increase for instance, will result into a shift in the Θ function.
Indeed, as people expect a higher rate of inflation, they will find it worthwhile to substitute index-tied monetary assets to nominal monetary assets. Consequently, prices of index-linked assets increase and the real

rate of interest on them falls. Conversely, prices of nominal assets decrease and nominal rates of intrrest on them increase.
If everybody changes his mind about future inflation at the same time, it may very well be that no transaction will effectively take place on the bond-market, but just because everybody expects the bond-prices and the rates of interest to change, they effectively do.

Graphically, we have a shift from Θ_o to Θ_1



For a given stock of bonds supplied, the shift from Θ_o to Θ, results into a decrease in the real rate of interest on index-bonds (an increase in their market-prices) and an increase in the nominal rates of interest on nominal bonds (a decrease in their market-price). As the new difference in rates of interest has been achieved through opposite variations in the rates of interest, the real level of rates of interest has decreased.

As a result, entrepreneurs will sell bonds at very favourable terms and acquire productive capital. This leads to excess-demand on the commodity and factor-markets and to rises in the price-level that decrease the value of real balances supplied. This process will continue up to the point where the real rate of interest has come back to its equilibrium value $r_o = r_{ro}$. The nominal rate of interest will have increased to the necessary amount so as to satisfy the equilibrium relation $i_n = r_o + \Theta$.

The point is that the real rate of interest comes always back to the level where it equilibrates the commodity-market, while nominal rates of interest on nominal bonds adjust for the rate of expected inflation so as to satisfy the equilibrium relation $i_n = r_o + \Theta$.

In other words, an index-tied bond bears a higher nominal rate of interest because the index is expected to increase at a higher rate, while ordinary bonds bear higher rates of interest after a necessary change in their market-value.

Mathematically, the comparative statics analysis amounts to differentiating the system :

$$\begin{cases} I_o + g.r = S \\ \frac{m}{p} = L(i) \\ r = i_n - \Theta = r_r;\ i = i_n \end{cases}$$ with respect to Θ, we have :

$$\begin{cases} g.d\,r = o \\ -\frac{m}{p}\frac{dp}{p} + \frac{dm}{p} = L_i'\,di \\ d\,r = d\,i_n - d\ \ = d\,r_r \end{cases}$$

Assuming the absence of monetary intervention, we have $\frac{dm}{p} = o$.

Hence :
$$\begin{cases} g\,d\,r = o \Longrightarrow d\,r = o \Longrightarrow d\,r_r = o. \\ -\frac{m}{p}\frac{dp}{p} = L_i'\,di = L_i'\,di_n \\ di_n = d\,\Theta \end{cases}$$

Consequently : $d\,r_r = o$ and $d\,i_n = d\,\Theta$.

$d\,r_r = o$ shows that the real rate of interest on index-linked bonds does not change and so their prices do not change (in response to a change in Θ !)

$d\,i_n = d\,\Theta$ shows that, as in our macro-model of chapter II, the nominal rate of interest on nominal bonds increases with the rate of expected inflation and so their prices must decrease, if the expected rate of inflation θ increases. The market-prices of nominal assets will increase, if the rate of expected inflation decreases.

3.4.3. The conditions for the risk-free nature of index-tied monetary assets and empirical evidence

Our comparative statics analysis shows quite clearly that under a number of theoretical assumptions used in a macro-economic framework, a change in the expected rate of inflation induces a corresponding variation in the nominal rate of interest on ordinary nominal bonds and hence a corresponding variation in market-value of outstanding nominal bonds.
The variation in the nominal rate of interest on index-tied bonds is entirely achieved through the expected increase in value of the index; the real rate of interest on index-tied bonds and hence their market-value, if they have been previously issued, remains unaffected by changing rates of expected inflation.

This result has been obtained, provided that some important assumptions were realized and, in a matter like this, the expected success of the index-linkage should be contemplated in the light of those hypotheses.

One fundamental assumption is that the issuers of bonds, who are supposed to be the investors in real productive capital consider the real value of a given rate of interest and not its nominal value in times of inflation.
Since planned investment is assumed to be a function of the real rate of interest and since the scope for ex ante investment is given by the amount of real aggregate planned saving, the real rate of interest must have a value where it equilibrates the keynesian equation $I = S$. This implies, of course, that nominal rates of interest keep always pace with the expected rate of inflation.
If bond-issuers and investors in real productive capital considered the nominal value of the rate of interest, without deflating it by the expected rate of inflation, the rate of interest equilibrating $I = S$ would be nominally fixed and hence variable in real terms. But, as there must be a twin-market equilibrium that yields a relation between the rates of interest

on the market-prices of both types of bonds so that the nominal rate of interest on nominal bonds is equal to the real rate of interest on real bonds plus the rate of anticipated inflation, fixing the nominal rate of interest amounts to permantly varying the real rate of interest on real bonds and hence to varying the market-prices of index-tied bonds, whenever the expected rate of inflation comes to change.

So we are led to the following important conclusion.
As we put it in the introduction to chapter I, if there is no relation between the expected rate of inflation and the nominal rate of interest on monetary assets, one cannot establish a ranking of monetary assets according to the amount of risk involved in times of inflation. In other words, as Tobin claimed, cash is as risky as bonds and, as a result, there is no reason to tie one particular monetary asset to the index rather than another. The analysis of our chapter III points to one additional feature which is that, should one particular monetary asset be tied to the index with respect to its principal and interest-payments under conditions where no relation exists between nominal rates of interest and expected rates of inflation, this index-tied asset would be risk-free for debtors not supposed to redeem them before maturity and for creditors holding them until they come due.
All creditors selling this index-tied monetary asset before maturity bear the risk of inflation-induced variations in market-value, as future market-values of this asset will depend on future expected rates of inflation.
As there is no creditor who does not envisage the possibility to sell his assets in case of, for instance, emergency, the index-linkage would entail an increase instead of a decrease in risk and any index-linkage would be impossible to implement.

This proves straightforward that an index-linkage of financial assets may only be implemented in a world where inflation is anticipated and commensurately reflected in the nominal rate of interest on nominal assets. This is precisely a world where, at a first glance, the index-linkage seems to be unnecessary.
Such a view, however, may only be held, if one disregards the risk-effect of inflation : future factual inflation may not coincide with its level

expected now, future expectations of inflation may be adjusted to the new levels of inflation, which changes nominal rates of interest and implies variations in market-value.

A second important assumption was that anticipated inflation acts upon the marginal efficiency of capital and hence on the demand curve for real investment, leaving ex ante saving unchanged.
In very general terms, this assumption disregards two main problems.

First of all, it may very well be that a particular inflationary process leads to redistributions of real income : for instance, wages may lag behind profits or profits behind wages.
Such a redistribution of income among broadly defined income-groups may very well affect aggregate ex ante saving, if those categories display different propensities to consume.
A redistribution of real income from low to high savings-groups increases ex ante saving, a redistribution of income in the other sense decreases ex ante saving. Consequently, the scope for ex ante investment will be a function of the rate of inflation and planned investment being a function of the real rate of interest, the equilibrium real rate of interest is itself a function of the rate of inflation.
As a result, an index-linkage of monetary assets does not necessarily decrease inflationary risk as their "equilibrium-price" will be subject to fluctuations with the rate of expected inflation.

A second problem is that consumption and saving out of a given real income may be a function of real net wealth. This problem has been treated by Robert Mundell (1) in rather simple terms. The main idea is that inflation is expected to decrease the real value of net monetary wealth which induces people to save more and consequently enlarges the scope for ex ante investment. The higher the rate of inflation, the stronger the wealth-effect and the larger the scope for ex ante investment. If the economy may be assumed to remain at full-employment, the real rate of interest must be a decreasing function of the rate of inflation.

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(1) MUNDELL, Robert, "Inflation and Real Interest", Journal of Political Economy, June 1963.

In this case as well, the index-linkage does not necessarily remove the risk-effects of inflation,as the market-prices of index-tied assets will fluctuate with the real rate of interest.
If, however, total net monetary wealth is tied to the index, inflation cannot have any wealth-effect and the index-linkage will prove to be an effective risk-reducer.

As a conclusion, we may say that the index-linkage of monetary assets will prove to reduce the risk-effects of inflation,whenever the real rate of interest will be unaffected by the rate of inflation, i.e. nominal rates of interest will tend to adjust to the anticipated rate of inflation. If, in equilibrium, the real rate of interest is a function of the rate of inflation, the index-linkage might reduce the risk-effect of inflation to the extent that real rates of interest fluctuate less than nominal rate of interest.

Under those assumptions, the index-linkage may certainly be applied to long term monetary assets like bonds.
Such a device does certainly contribute to the revival of a bond-market.

This is claerly confirmed by the experiments with index-linked bonds in Israel.

Michael Sarnat (1) emphasizes the importance of the index-linkage with respect to the securities-market in the following terms :

> "After a period of rather painfull adjustment, the further development of the securities-market was influenced by two major events. The first, which can be dated from 1955, was the large-scale introduction of a new investment medium, value-linked bonds. This had a significant impact on the structure of the new issue and secundary markets for securities, on the pattern of institutional and private investment, and on government and corporate financing".

That the index-linkage should be applied to bonds essentially follows from the author's coment on the working of the market for government and corporate bonds :

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(1) SARNAT, Michael, "The development of the securities market in Israel", Tübingen, 1966.

"The progressive decline and final collapse of the market for government bonds reflects the inflationary character of the period. The degree of inflation (...) was hardly conducive to the raising of capital from the public by means of fixed-interest securities. Given the expectations of further price rises (...) the prospective real rate of return on such bonds was negative throughout the period. Under the circumstances, the diversion of purchasing-power from the public by conventional means became increasingly difficult".

As far as the market for corporate bonds is concerned, the author writes:

"The challenge of inflation was particularly serious in view of the structure of the Tel-Aviv market; the Palestinian market had been based primarily on fixed-interest-bearing securities, i.e. bonds and preferred stocks. The extent of the inflationary impact on the securities-market can be best gauged from the fact that between 1950 and 1954 no long-term conventional bond issues by Israel corporations, nor were there any public issues of conventional preference shares".

Between 1950 and 1954, new equity-financing in Israel accounted for 80 per cent of the number of new issues and for 70 per cent of the nominal value, the remaining part being conventional bond-financing.

That the index-linkage of bonds contributed to the revival of the market for this asset is undoubtedly confirmed by the figures :

the importance of an asset-market in the lending-borrowing process may be validly measured by the net new issues, i.e. issues net of redemptions, over a yearly period.

The next table (1) compares net new issues of securities with domestic capital formation in Israel during the years 1950 to 1953.

	Net new issues of securities in £ millions	Net new issues as per cent of capital formation
1950	12	9.6 }
1951	21	11.1 }

(1) SARNAT, op. cit., p. 56.

1952	6	2.1	
1953	6	1.9	
1954	5	1.3	
1955	9	1.7	— index-linkage
1956	40	6.5	
1957	44	5.8	
1958	44	4.6	
1959	100	9.5	— equity-financing
1960	57	5.1	
1961	177	12.1	
1962	229	11.6	
1963	447	21.0	

The years 1950-1951 show high net issues,which were made by the government essentially.

From 1952 to 1955, there is quite clearly a tremendous slow-down in the activity of the bond-market, this reflects the inability of the market to cope with the inflation of this period.

The index-linkage was introduced in 1955 and the following years (until 1959) show a sharp increase in activity of the market.

The years after 1959 show a sharp increase in activity as well, but this is due to an increase in equity-financing.

The revival of the bond-market after 1955 seems to be due to an important shift of savings from the black short-term markets where nominal rates of interest were considered as being at usury levels, to the long-term bond markets.

3.5. CONCLUSION

In our chapter II, we found out why monetary assets are not all equally affected by the risk-effect of inflation.

Indeed, as soon as the expected rate of inflation tends to be reflected in nominal rates of interest and consequently in current market-prices of monetary assets, the market-value of monetary assets tends to vary with the rate of expected inflation. Moreover, the lager the maturity of monetary assets the wider the range of those fluctuations.

We showed as well that, if there is no relation between the nominal rates of interest and consequently, the market-prices of monetary assets, and the rate of anticipated inflation, all monetary assets become equally risky in times of inflation and "equal candidates" for an index-linkage.

In this chapter, we came to the conclusion that in the latter case - where no relation exists between nominal rates of interest and the expected rate of inflation - the index-linkage will make index-tied monetary assets more risky than they would have been otherwise.

By contrast, we came to the conclusion that in the former case - when a definite relation exists between the expected rate of inflation and nominal rates of interest - the index-linkage will effectively reduce the risks entailed by inflation, if real rates of interest display smaller variations than nominal rates of interest in response to changing rates of expected inflation.
Moreover, index-tied assets proved to be entirely risk-free with respect to inflation, if real rates of interest do not change with the expected rate of inflation, i.e. if nominal rates of interest do fully compensate for the expected rate of inflation.

The index-linkage of interest-bearing monetary assets reduces the risk-effect of inflation on monetary assets, because adjustments in nominal rates of interest are operated through expected increases in value of the index, rather than through changes in market-prices of monetary assets.

One ought to notice, finally, that the index-linkage removes the risk-effect of inflation only. It may very well be that real rates of interest and consequently, current market-prices of monetary assets, fluctuate over a given span of time for many other reasons than the rate of inflation. This risk will be equally shared, of course, by nominal and index-tied monetary assets.

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Chapter IV

THE INDEXATION OF FINANCIAL ASSETS : ITS IMPLICATIONS AND ADVANTAGES FOR MONETARY POLICY

Introduction

This chapter is divided into two parts.

The first part discusses the necessary implications of the index-linkage of financial assets for monetary policy. It shows how the index-linkage experiment must be managed, if it is to be successful.

The second part discusses the usefulness of index-tied financial assets for specific purposes of monetary policy.

4.1. THE DESTABILIZING CHARACTER OF THE INDEX-LINKAGE OF FINANCIAL ASSETS

Introduction

In our chapter I, we showed why monetary assets become all more risky in times of inflation.
In our chapter II, we came to the conclusion that, if some of those monetary assets are to be linked to the index, they should be long-term rather than short-term monetary assets.
Moreover, in our chapter III, we demonstrated that, if there is no such a priori reason to link long-term rather than short-term monetary assets to the index (i.e. in the case where no relation exists between the anticipated rate of inflation and the nominal rate of interest), the index-linkage of monetary assets makes them more risky, as their market-value will sharply fluctuate with the expected rate of inflation.

Consequently, if any perverse effect of the index-linkage is to be considered, it must be done under the assumption that the index-linkage has been applied to long-term monetary assets, all other monetary assets being ordinary = i.e. non-index-tied = assets.

Now, our proposition is that the main perverse effect to be expected from the index-linkage of long-term monetary assets is a strong destabilizing mechanism, which tends to impede the natural adjustment of an economy trying to live with a new rate of expected inflation.

This destabilizing mechanism refers to the dynamics of the adjustment process, which we considered from a static point of view in our comparative statics analyses in chapters II and III.

The problem with the comparative statics technique is that it concerns only the signs of variations in endogeneous economic variables in response to a change in an exogeneous parameter, which for our purposes is the expected rate of inflation. This technique is essentially qualitative and it fails to take account of the dynamics of the adjustment process.

The point, however, is to know whether or not a new equilibrium will be effectively reached and what the conditions of the adjustment process are likely to be.
This goes clearly beyond the level of knowledge that we may acquire from a compartive statics analysis.

If we want to get insight into what we call the destabilizing character of the index-linkage of financial assets, we must compare the dynamics of adjustment of two economies : the first having no index-linked bonds, the second having its bonds tied to the index.
Any difference in the dynamics of adjustment will result from the index-linkage applied to long-term monetary assets.
In other words, we now try to apply Samuelson's Principle of Correspondence, which states that the qualitative knowledge acquired from a comparative statics analysis should be completed by further information on the dynamics of adjustment.

In fact, the models built in our chapters II and III should be entirely worked out in all their dynamic relations. We shall not go so far. We shall reconsider the causal relations between the economic variables and simply mention the main dynamic properties to be taken into account. This seems to be justified, as far as a specification of dynamic relations, i.e. functional relations between variables, that change in value over time, may be arbitrary from a theoretical point of view and dictated by the needs of mathematical convenience, while such a specification should be based in fact on empirical observations.

In this first part of our chapter, we aim at analyzing briefly some of the perverse effects of the index-linkage so as to discuss the appropriate policy measures that should be taken to avoid them.

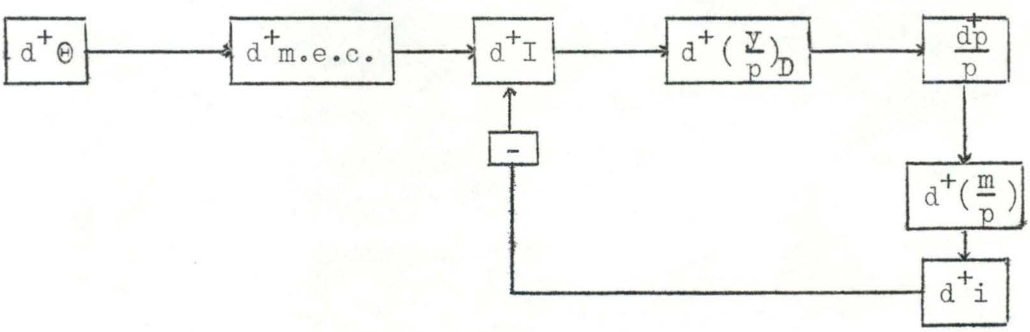
In the second part of our chapter, we discuss the positive merits of the index-linkage of financial assets in connection with monetary policy.

4.1.1. A schematical comparison of the causal relations in the static adjustment

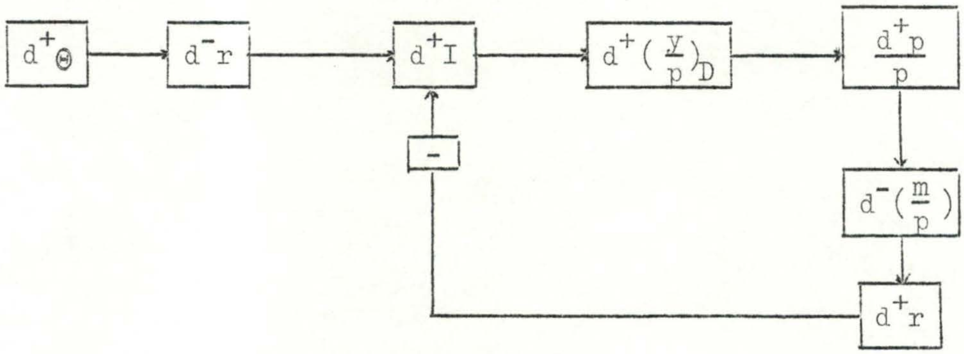
In an economy with no index-tied bonds, the adjustment of nominal rates of interest to the expected rate of inflation will follow the next causal sequence :

> An increase in the expected rate of inflation implies an increase in the marginal efficiency of capital and increases the demand for productive capital. This increase in demand has a multiplied effect on total real income demanded. As real income produced is assumed to be at its full-employment level, the increase in total demand results into increases in the general price-level. This in turn reduces the value of total real liquidity supplied and increases nominal rates of interest. Nominal rates of interest will increase up to the point where they choke off the inflation-induced excess-demand.
> At this point, nominal rates of interest will be equal to the now higher marginal efficiency of capital.

The causal sequence of this adjustment may be represented as follows:



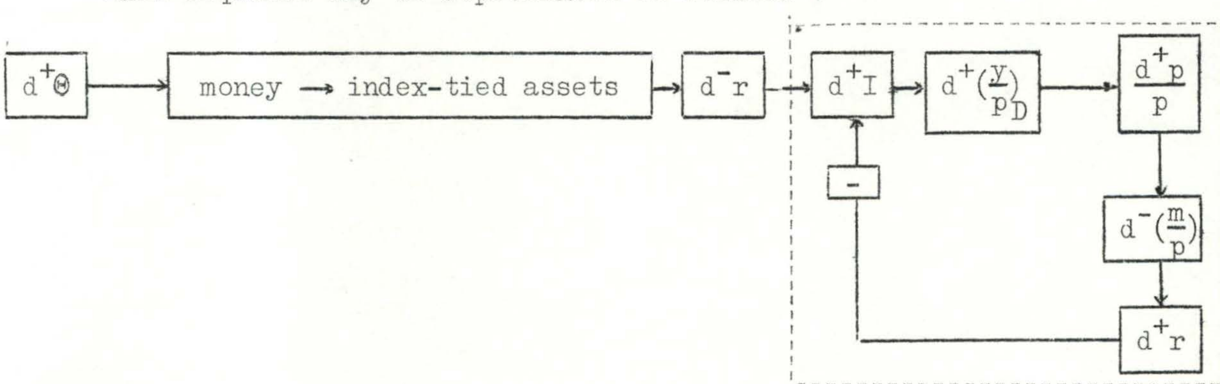
As in equilibrium, the m.e.c. must be equal to the nominal rate of interest, we may rewrite this as an adjustment of real rates of interest, so we have :



In an economy where long-term monetary assets have been linked to the index, the adjustment to the expected rate of inflation will follow a much similar causal sequence.

An increase in the expected rate of inflation leads to a fligh from nominal monetary assets i.e. money and short-term monetary assets, into the long-term index-linked monetary assets. This increases the price of those assets and lowers the real rate of interest, which stimulates final demand. Real income produced being at its full-employment level, the increase in demand results into increases in the general price-level and lowers the level of real liquidity supplied. The real rate of interest - on nominal monetary assets and on index-tied bonds - increases.
Equilibrium will be reached, as soon as real rates of interest are back at their previous level.

This sequence may be represented as follows :



So far there does not seem to be much difference in the channels of adjustment in economies where long-term monetary assets are respectively nominal, i.e. not tied to any index, and real, i.e. linked to a representative index.
Actually, that part of the sequence surrounded by the broken line is the same in both economies.

We shall now pay some attention to the dynamic effects that are likely to take place throughout this sequence and more precisely we shall emphasize the main differences in those effects that arise from the index-linkage of long-term monetary assets.

4.1.2. Some elements of the dynamics of the adjustment process in connection with the index-linkage of long-term monetary assets

4.1.2.1. The dynamics of inflationary expectations

In our static models, the rate of expected inflation was assumed to be an exogeneous parameter and no attention was devoted to the dynamics of this particular parameter.

In the empirical part of the research, however, the use of past rates of inflation as a proxy for the anticipated rate of inflation turned out to be successful and the distribution of weights of those past rates displayed a very plausible shape : the weights decreased very smoothly in value with the distance of past rates of inflation in time.
Hence we may validly conclude from this that such a distribution gives the right picture of the way people come to expect a certain rate of future inflation : the expected rate of inflation tends to be functionally extrapolated from past rates of inflation and the immediate past is likely to tell them more about the future than the more distant past.

No account of such a mechanism was taken in the adjustment process, which we analyzed from a purely static, i.e. timeless, point of view.
But, as we will show, this relation entails a major difference in the dynamics of adjustment in two economies that respectively do and do not apply the index-linkage to their long-term monetary assets.

Indeed, as we showed in the adjustment sequences, a change in the expected rate of inflation induces a change in total aggregate demand, which at full-employment results into effective changes in the general price-level. From a static point of view, the necessary change in the price-level will be small if the nominal rate of interest is initially high and the demand for money interest-inelastic, whereas the effective change in the price-level must be large if the nominal rate of interest is initially low and the demand for money interest-elastic.

Now, the point is to know what happens to the adjustment process, if those effective price variations are extrapolated so as to yield new inflationary expectations and more precisely what the effects of the index-linkage of long-term monetary assets is likely to be in this respect.

One central feature is that the relevant time-horizon of inflationary expectations will be different in economies that respectively do and do not have their long-term monetary assets linked to the index.

Indeed, in the case where no index-tied monetary assets are available, the nominal rate of interest on nominal monetary assets is adjusted to what should essentially be considered as a long-term rate of inflation. Nominal rates of interest will finally allow for the anticipated rate of inflation, provided that this rate of inflation is expected to prevail over the whole lifetime of the capital-goods that entrepreneurs envisage to buy with the funds raised in the bond-market.

If the rate of inflation is expected to depart provisionally from its long-term level, they may not find it worthwhile to increase their current investment and nominal rates of interest will increase to a negligible amount.

The relevant-time horizon of those inflationary expectations being long, the expected rate of inflation is likely to be a function of a long series of past rates of inflation : the rate of inflation that one may expect to prevail over, for instance, the next decade will be a function of the rates of inflation during the last decade rather than of the rate of inflation experienced during the current year. To put it in Hickxian terms, the elasticity of expectations will be rather low and possibly close to zero.

This means that the effective price-variations will probably have a small influence on inflationary expectations.
The strength of this influence, if any, depends positively on the elasticity of expectations and the speed of adjustment of the general price-level in response to excess-demand.
It depends negatively on the speed of adjustment of the real rates of interest.

By contrast, in an economy where long-term monetary assets have been linked to the index, the relevant time-horizon for inflationary expectations will be much shorter.
Indeed, ultimate wealth-holders may very well shift their monetary assets from nominal forms into index-tied forms in response to a change in the rate of inflation expected to prevail over a short span of time. They may find it profitable to hold index-tied monetary assets, if they expect a higher rate of inflation for the next year or even the next quarter.
This implies that the elasticity of expectations, i.e. the coefficient by which inflationary expectations are adapted to current levels of inflation, will be much higher and possibly close to unity. This means in turn that effective rates of change in the price-level will be "immediately" extrapolated so as to yield new rates of expected inflation.

As we demonstrated, however, that the index-linkage should be applied to long-term rather than to short-term monetary assets, an increase in demand for index-linked assets will increase their price and decrease the long-term real rates of interest.
Moreover, by contrast to an economy where no monetary asset is linked to the index, this decrease in the real rate of interest is certain and not dependent on inflationary expectations.
This decrease in the long-term real rate of interest will stimulate total aggregate demand and lead to effective increases in the general price-level. The elasticity of expectation being high, inflationary expectations will be adapted and induce new changes in aggregate demand and in the price-level.

We may now validly conclude on the dynamic effects of the index-linkage of long-term monetary assets :

The real rates of interest on long-term monetary assets will tend to vary with short-run inflationary expectations.
Those changes in real rates of interest will induce changes in total aggregate demand, which at full-employment result into effective rates of change in the general price-level.
As the elasticity of expectations will be close to unity effective changes in the price-level will affect inflationary expectations.
As a result, the index-linkage has a destabilizing character in the sense that total aggregate demand and the general price-level will be subject to sharps fluctuations of a cyclical kind.

4.1.2.2. The dynamics of the term structure of monetary assets demanded

The index-linkage of long-term monetary assets entails another destabilizing effect inherent in the dynamics of the demand for monetary assets with various maturities of different length.
This effect may be analyzed, once again, on a comparative basis.

Let us first consider an economy where long-term monetary assets have not been linked to the index.
If, in this economy, the general public has come to learn the positive relation that exists between the expected rate of inflation and the level of nominal rates of interest, an increase in this expected rate of inflation will induce the ultimate wealth-holders to get rid of their long-term monetary assets, as they expect capital-losses to occur on those assets. Consequently, there will be a general decrease in demand for long-term monetary assets, which results into an immediate increase in the long-term rate of interest. Consequently, the adjustment of nominal rates of interest to the expected rate of inflation is at least partly automatic, owing to a pure mechanism of expectations.

In an economy where long-term monetary assets have been tied to the index, there will be no such automatic adjustment of long-term rates of interest. Indeed, an increase in the expected rate of inflation - as we showed above, even over a short span of time - results into a shift in demand for assets into the opposite direction : portfolios are reallocated from short-term nominal monetary assets into long-term index-tied monetary assets. This results into a decrease in the real rate of interest on index-linked monetary assets and stimulates aggregate demand.

We thus come to the conclusion that the index-linkage of long-term monetary assets induces people to reallocate their asset-portfolio in the wrong way from the point of view of the natural adjustment of the economy. The same principle may be applied to foreign long-term capital-flows.
If no index-linkage is applied to long-term monetary assets, there will be an outflow of long-term capital, whenever an increase in the expected domestic rate of inflation leads to expected capital-losses.
If long-term monetary assets have been linked to the index, there will be an inflow of foreign capital, whenever the rate of expected inflation increases. This has, of course, the wrong influence on domestic rates of interest.

4.1.3. The implications for monetary policy in connection with the index-linkage of long-term monetary assets

a) A more active open-market policy

The few considerations on the dynamic implications of the index-linkage have led us to the conclusion that changes in expected rates of inflation generate changes in demand and resulted into effective changes in the price-level. Those effective changes in the price-level affect in turn inflationary expectations, as, owing to the index-linkage, the elasticity of expectation will be close to unity.
This fluctuating character of the adjustments may, of course, be avoided by an active monetary policy on the open-market.

In this case, any increase in the expected rate of inflation would require from the monetary authorities to supply the index-tied monetary

assets demanded and to demand the nominal monetary assets supplied by the market.
Conversely, any decrease in the expected rate of inflation that induces the market to increase its demand for nominal assets and its supply of index-tied assets should bring the monetary authorities along to supply the market with nominal debts and to increase their demand for index-tied debt-instruments.

Those operations on the open-market would allow the adjustment to be achieved through changes in the nominal stock of money rather than through changing price-levels.
The merry - go - round of changes in price-levels and inflationary expectations as well as the perverse behaviour of long-term real rates of interest would be prevented to a great extent by such a very active debt-management policy.

b) Decycling the index

If an inflationary economy is subject to cyclical variations in demand, there is no doubt about the occurrence of the perverse dynamic effects of the index-linkage.
Indeed, the factual rate of inflation will be positively correlated with the cycle and so will the inflationary expectations.
In times of booms, the factual and the anticipated rate of inflation will be beyond their trend-value, while in times of depression they will be below their trend-value.
This will entail all the dynamic effects expected, which, however, may be successfully counteracted by an appropriate debt-management in the way suggested above.

We think, however, that the task of the monetary authorities may be largely facilitated by appropriate techniques of computing the value of the index.

One of such techniques should be a decycling of the index.

Indeed, assume that factual rates of inflation may be decomposed into a constant trend-value and a cyclical component.

Mathematically, we could have a very simple expression as :

$$\frac{dp}{pd\,t} = a + b\sin\left(\frac{\pi}{2}t\right) \qquad \text{where } \frac{dp}{pd\,t} = \text{rate of inflation}$$

$$a = \text{constant trend-component}, \quad a > o$$

$$b\sin\left(\frac{\pi}{2}t\right) = \text{cyclical component}, \quad b > o$$

Assume that at t = o, the rate of inflation is at its trend-value, we have :

$$\left(\frac{d\,p}{pdt}\right)_{t=o} = a$$

Similarly, assume that at t=1 it is at its "boom-value", we have :

$$\frac{d\,p}{pdt} = a + b$$

The evolution of the index through time would be given by the expression :

$$\int_o^t \frac{dp}{p} = \int_o^t \left(a + b\sin\left(\frac{\pi}{2}t\right)\right) dt$$

$$\log\frac{p(t)}{p(o)} = a\,t - b\,\frac{2}{\pi}\cos\left(\frac{\pi}{2}\,.\,t\right)$$

hence $p(t) = p(o)\ c^{\,at-b\,2/\pi\,\cos(\pi/2\,t)}$

A decycling of the index would mean that it increases permanently at a rate equal to the trend-value of the rate of inflation.
Mathematically, we would have :

$$\frac{d\,I}{Idt} = a = \frac{d\,p}{pdt} - b\sin\left(\frac{\pi}{2}t\right).$$

We may now find the numerical weights that should be given to rates of inflation observed in different phases of the cycle.
Those weights are given by the ratio of the increase in the index, which is restricted to increase at the rate a, and the rate of inflation effectively observed.

Mathematically, we have :

$$w(t) = \frac{\frac{dI}{Idt}}{\frac{dp}{pdt}} = \frac{\frac{dp}{pdt} - b \sin \frac{\pi}{2} t}{\frac{dp}{pdt}} = 1 - \frac{b \sin \frac{\pi}{2} t}{\frac{dp}{pdt}}$$

as $\frac{dp}{pdt} = a + b \sin (\frac{\pi}{2} t)$, the series of "decycling weights" is given by

$$w(t) = 1 - \frac{b \sin (\frac{\pi}{2} t)}{a + b \sin (\frac{\pi}{2} t)}$$

Consequently, if one knows the trend-component and the cyclical component of the rate of inflation, a series of numerical weights may be computed so as to give less numerical importance to rates of inflation in times of booms and depression and, as a result, to avoid the perverse portfolio-shifts from index-tied assets into nominal assets and vice-versa.

A second technique that might still better perform would consist into "inversing" the cycle of the index with regard to the cycle of the rate of inflation.

This would imply that in times of booms, the rate of increase of the index would be below the trend-value, while in times of depression it would be above the trend-value.

If, for instance, the cycle of the rate of increase of the index is a negative mirror-picture of the cycle of the rate of inflation, we would have :

$$\frac{dp}{pdt} = a + b \sin (\frac{\pi}{2} t)$$

$$\frac{dI}{Idt} = a - b \sin (\frac{\pi}{2} t).$$

The series of weights to be given to the rates of inflation effectively observed would be equal to :

$$\frac{\frac{dI}{Idt}}{\frac{dp}{pdt}} = \frac{a - b \sin (\frac{\pi}{2} t)}{a + b \sin (\frac{\pi}{2} t)} = w(t).$$

This technique would induce portfolio-reallocations and fluctuations in the real rate of interest, that go into the direction of adjustment.

Indeed, in a boom, the exprcted increase in value of the index decreases. As a result, there will be less demand for long-term index-tied monetary assets and an increase in demand for short-term nominal monetary assets. Consequently, the long-term real rate of interest will rise, which has a countercyclical effect on aggregate demand.
Conversely, in a depression, the expected increase in value of the index increases. This stimulates the demand for long-term index-tied assets and lowers the long-term real rate of interest, which has a positive effect on aggregate demand.

Those techniques do not diminish in any way the risk-free properties of index-tied long-term assets, on the contrary, the first technique allows the adjustment to be operated smoothly on the open-market, while the second technique tends to even the cyclical fluctuations in the economy, which is likely to stabilize the rates of interest and hence the asset-prices.

c) The spreading of the index-linkage

In order to avoid the speculative and destabilizing portfolio reallocations, one might as well envisage to spread the index-linkage so as to apply it to short-term monetary assets.

This, however, is likely to reinforce the inflationary forces. Whatever the spreading of the index-linkage may be, there will always remain some assets that have not been tied to the index.
One limit to the spreading of the index-linkage over the range of monetary assets is inherent in the indexation-technique itself : money cannot be tied to the index.

Consequently, if all monetary assets are linked to the index with the exception of money, fluctuations in the expected rate of inflation will result into fluctuations in the demand for money in a cycle-reinforcing sense.

All things considered, this may not be regarded as an appropriate measure.

d) Acting upon the liquidity of index-tied monetary assets

To prevent the destabilizing portfolio reallocations between nominal and index-tied monetary assets, another arrangement could be possibly resorted to.
Monetary authorities could envisage to decrease institutionally the liquidity of long-term index-tied assets.
One possible way to achieve this decrease in liquidity would be to confine the index-linkage to large amounts of purchasing-power transfered only. This would force the average investor to choose between holding a large part of his assets in index-tied forms and holding his entire portfolio in nominal forms.
Similar measures could be taken for financial intermediaries.

With such an increase in the share of index-tied assets in the average portfolio, one may expect marginal portfolio reallocations induced by the slightest changes in the expected rate of inflation to be avoided.

Whatever may be done in this respect, the monetary authorities should aim at segmentizing the markets for nominal and index-tied assets so as to avoid the undesirable shifts in demand from one market into another with all the destabilizing effects involved.

4.1.4. Conclusion

We may conclude from this that, once it has been decided to apply the index-linkage to long-term monetary assets, the monetary authorities should be concerned with a permanent management of the markets for both nominal and index-tied debts.
This interference may be achieved in a number of ways.
The perverse dynamic effects of the index-linkage seem to point to the desirability of active interest-rate policies and the monetary authorities' task could be facilitated by a few measures concerning the computing techniques of the index.

4.2. THE BENEFITS ASSOCIATED WITH INDEX-TIED DEBTS IN MONETARY POLICY

Introduction

So far we have laid stress on a few elements that point to the monetary authorities'increasing concern with the financial well-being of an economy, once some of the financial assets have been tied to the index. In the absence of such an active financial management, the perverse effects may very well outweigh the benefits associated with the indexation of financial assets.

There are, however, some specific benefits associated with index-tied debts that point to a greater effectiveness of monetary policy in times of inflation. Consequently, monetary authorities will partly safeguard the necessary conditions for the indexation to be successful and partly resort to the indexation for more specific purposes of monetary policy.

In this part of our analysis, we shall consider some aspects of monetary policy that will definitely gain by the index-linkage of financial assets.

4.2.1. The effectiveness of debt-management in inflationary economies

The aim of a debt-management policy is to control aggregate demand by acting upon the demand-price of real capital. If the demand-price for real capital is high compared to the current supply-price, there will be a strong tendency to invest and an expansion of total aggregate demand. If the demand-price is low compared to the supply-price, there will be a decrease in current investment and a contraction in total aggregate demand.

The demand-price for capital goods is negatively related to the rate of interest : if one knows in advance the value of current flows of goods and services produced by a capital-good, its demand-price, which is equal to the present-value of future net profits, is high for low rates of interest and low for high rates of interest.
Consequently, monetary authorities may act upon the demand-price for real capital by varying the rate of interest.

As the government cannot act as a buyer of real capital or equity, it must determine the price and the rate of interest on its own debt.
This rate of interest will determine the demand-price of real capital, if government debt is a more or less close substitute to real capital.
In other words, in case of an excess-demand for real capital, the government may counteract this excess-demand by an increase in supply of its own debt, the extent of which depends on a rate of substitution between real capital and government-debt.
If this rate of substitution is high, a small increase in supply of government-bonds may suffice to stop the excess-demand for real capital.
Large amounts will be necessary, if the rate of substitution is low.
An increase in supply of government-debt would even stimulate the demand for real capital, if government-bonds and real capital are complementary to each other. Consequently, the effectiveness of debt-management in checking aggregate demand is heavily dependent on this rate of substitution.

In our chapter I, we used a general model of demand for assets.
We deduced the inflationary effects on the demand for assets from the Hickxian equations between the rates of substitution and the second-order conditions for a maximum.
The whole analysis, however, was conducted under the assumption that inflation leaves the rates of substitution between assets unchanged.
But, as in the aggregate the rates of substitution have a crucial significance from the point of view of monetary policy - the more debt-instruments are substitutes for real capital, the more debt-management will be effective - we are to consider the effects that inflation is likely to have on those rates of substitution and the importance of the index-linkage in this respect.

It is intuitively obvious that inflation will not leave the rates of substitution unchanged. Indeed, rates of substitution between assets will be function of the covariances between the rates of return on those assets and inflation affects at least some covariances between assets.
The more the rates of return on two assets tend to covary, the more they will be substitutes for each other : an increase in the real rate of return

on the first asset will imply an increase in the real rate of return on the second asset. Consequently, from the point of view of variations in real rates of return, those assets will be good substitutes to each other and it does not decrease total risk on a portfolio to diversify wealth between those two assets.

Conversely, if variations in the real rates of return on two assets tend to be negatively related, i.e. if they tend to covary negatively, those assets will not be substitutes to each other and it will decrease total risk on a portfolio to diversify part of total wealth between those two assets. Actually, the assets will be regarded as complements to each other.

Now, by virtue of what we know about the effects of inflation on the risk-parameters of monetary assets, we may find out what happens to the rates of substitution between assets and to the effectiveness of monetary policy conducted with ordinary nominal debt-instruments in times of inflation.

Assume that ultimate wealth-owners have (m+1) assets at their disposal, m of those assets being monetary assets and the (m+1)th asset being a real asset.

Asset x_{m+1} will be considered as equity; if equity-financing implies investment into real productive capital and, consequently, increasing aggregate demand, the rate of substitution of government-debt to equity measures the effectiveness of debt-management in its control over aggregate demand. This assumption allows to consider the rates of substitution as they really exist for the general public : most people have no choice between a bundle of monetary assets and real productive capital, they may, however, invest in equity.

Hence we consider the bundle of assets $(x_1, .. x_m, x_{m+1})$

Hickx has demonstrated that the rates of substitution between consumption goods are linearly dependent.

Applying the proof to the case of asset-choices in our chapter I, we found:

$$(1) \quad \sum_{j=1}^{m+1} s_{ij} = o \qquad i=1... m+1.$$

Assume that x_1 represents nominal (i.e. non-index-tied) government-bonds that monetary authorities use to influence the demand for real capital and, consequently, aggregate demand.

The change in effectiveness of debt-management must then be measured by a variation in $s_{1,m+1}$, which is the rate of substitution between government-bonds and equity.

Remember that $s_{ij} < o$ means that x_i and x_j are substitutes

while $s_{ij} > o$ means that x_i and x_j are complements.

Writing the first equation of the system above, we have :

(2) $s_{11} + s_{12} + \ldots\ldots + s_{1m} + s_{1\ m+1} = o$

hence :

(3) $ds_{11} + ds_{12} + \ldots\ldots + ds_{1m} + ds_{1\ m+1} = o.$

Now Royama(1) demonstrated that $\frac{\delta s_{ij}}{\delta \sigma_{ij}} < o \quad i \neq j$, which means that the more two rates of return tend to covary, the more their respective assets become substitutes.

This result may be validly used in this model as well, provided that $b < o$

in $s_{ij} = - \left[\frac{1+b}{b} + \sum x_h \left(1 + \bar{g}_h\right)\right] \frac{v_{ij}}{v}$

Applying Roama's differentiation technique, we found that $\frac{\delta s_{ii}}{\delta \sigma_{ii}} < o$

If we may disregard the variation in the rate of substitution between the assets x_i and x_j in response to a variation in the covariances of rates of return in the assets x_k and x_e, with $(k,e) \neq (i,j)$, i.e. if we may assume

$\frac{\delta s_{ij}}{\delta \sigma_{ke}} = o$ for $(i,j) \neq (k,e)$, we may calculate the signs of ds_{ij} $(j=1\ldots\ldots m+1)$

(1) ROYAMA, op. cit., p. 37.

N.B. In fact, we calculated the expressions for $\frac{\delta s_{ij}}{\delta \sigma_{ke}}$ as well, but in this case, we could not reach meaningful results for $d\ s_{1,m+1}$, unless very restrictive assumptions were made on the variations in the risk-parameters in response to a change in the rate of inflation.

Assuming $\frac{\delta s_{ij}}{\delta \sigma_{ke}} = o$ for $(i,j) \neq (k,e)$ amounts to considering the "most relevant" parameter for a rate of substitution s_{ij}, which is σ_{ij}.

Under those assumption $d\ s_{ij}$ may be written as :

$$d\ s_{ij} = \frac{\delta s_{ij}}{\delta \sigma_{ij}}\, d\sigma_{ij}$$

Hence equation (3) becomes :

$$(4)\quad \frac{\delta s_{11}}{\delta \sigma_{11}}\, d\ \ _{11} + \frac{\delta s_{12}}{\delta \sigma_{12}}\, d\ \ _{12} + \dots + \frac{\delta s_{1m}}{\delta \sigma_{1m}}\, d\,\sigma_{1m} + d\ s_{1,m+1} = 0.$$

N.B. There is no reason to expect a change in $\sigma_{1,m+1}$ in response to inflation. The rate of substitution $s_{1,m+1}$, however, must change, as it is linearly dependent on all the other rates of substitution, which are expected to change.

Equation (4) may be written as :

$$\frac{\delta s_{11}}{\delta \sigma_{11}}\, d\,\sigma_{11} + \frac{\delta s_{12}}{\delta \sigma_{12}}\, d\ \ _{12} + \dots + \frac{\delta s_{1m}}{\delta \sigma_{1m}}\, d\ \ _{1m} = -\ d\ s_{1,m+1}$$

Now, we know that

$$\frac{\delta s_{ij}}{\delta \sigma_{ij}} < o \quad \text{for} \quad i \neq j$$

and

$$\frac{\delta s_{ii}}{\delta \sigma_{ii}} < o$$

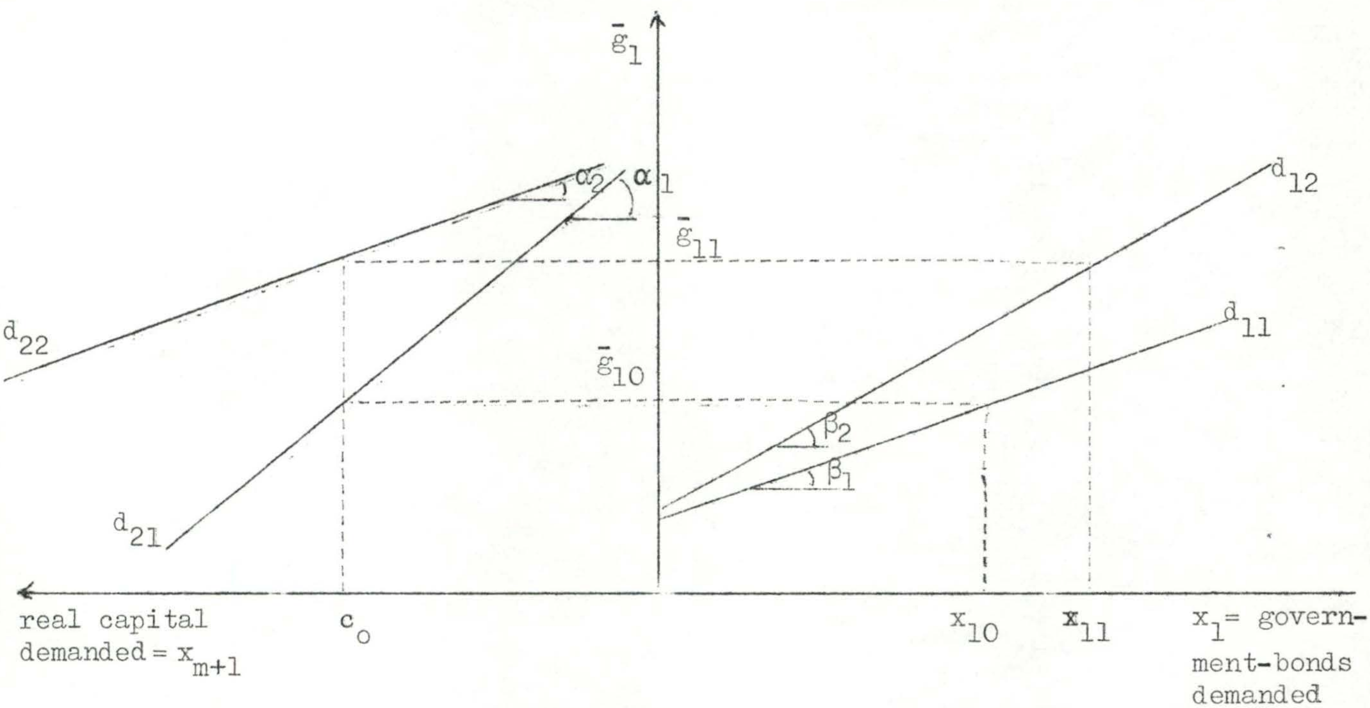
As, on the other hand, all the changes in the covariances of rates of return on monetary assets in response to inflation will be positive, i.e. $d\,\sigma_{ij} > o$ for $i,j = 1 \dots m$, we must have :

$$\frac{\delta s_{11}}{\delta \sigma_{11}}\, d\,\sigma_{11} + \frac{\delta s_{12}}{\delta \sigma_{12}}\, d\,\sigma_{12} + \dots + \frac{\delta s_{1m}}{\delta \sigma_{1m}}\, d\,\sigma_{1m} = -\ d\ s_{1\ m+1} < o$$

As a result, we have $-\ d\ s_{1\ m+1} < o \implies \boxed{d\ s_{1\ m+1} > o}$

In other words, as a result of inflation, monetary assets tend to become closer substitutes to each other, which means that their rates of substitution s_{ij} become more negative.
Since the rates of substitution of one asset with all the other assets must sum up to zero, the rate of substitution of government-bonds with equity decreases in the sense that $s_{1\ m+1}$ becomes less negative or more positive. (Remember that x_1 and x_{m+1} are substitutes, provided that $s_{1\ m+1} < 0$.)

The meaning of the change in this rate of substitution between government-bonds and equity in response to inflation and its implications for the effectiveness of debt-management may be graphically illustrated as follows.



If the demand-curves for government-bonds and real capital may be linearized, the slopes of those demand-curves are given by the values of the rates of substitution.
s_{11} is the slope of the demand-curve for government-bonds expressed as a function of the yield on government-bonds, which in the absence of expected changes in market-value is equal to the rate of interest on government-bonds. $s_{1\ m+1}$ is the slope of the demand-curve for real capital expressed as a function of the rate of interest on government-bonds.

Government-bonds and real capital being taken as substitutes, $s_{1\ m+1}$ must be negative.

In this graphical illustration, we assume x_{m+1} to be real capital rather than equity so as to relate directly the government's operations on the open-market to aggregate demand for investment-goods. As we explained above, this implies no loss of generality, if equity-financing is made for investment purposes.

The demand-curve for real capital as a function of the rate of interest on government-bonds is given by the curve d_{21} in times of price-stability. Its slope, $tg\,\alpha_1$, is equal (on the graph) to the inverse of $s_{1\ m+1}$.
Mathematically, we have : $tg\,\alpha_1 = (s_{1\ m+1})^{-1}$.

The demand-curve d_{22} represents the same demand-relation in times of inflation and its slope is equal to :

$$tg\,\alpha_2 = (s_{1\ m+1} + d\ s_{1\ m+1})^{-1}$$

The demand for government-bonds as a function of their own rate of interest is represented by d_{11} under general price-stability and d_{12} in times of inflation.

The slope of d_{11} is equal to $tg(\beta_1) = (s_{11})^{-1}$

The slope of d_{12} is equal to $tg(\beta_2) = (s_{11} + d\ s_{11})^{-1}$.

If the existing capital-stock is equal to c_o, the community will neither invest nor desinvest, if the rate of interest on government-bonds is equal to $\bar{g}_{10}$ under general price-stability.

For the rate of interest on government-bonds to be equal to $\bar{g}_{10}$, the supply of bonds must be equal to x_{10}.
In times of inflation, the demand-curve for government-bonds shifts from d_{11} to d_{22}, the demand-curve for real capital shifts from d_{21} to d_{22}.
The change in effectiveness of debt-management may now be deduced as follows:

- Suppose that monetary authorities want to maintain their target, as far as aggregate demand is concerned : they want the community neither to run down nor to increase the capital stock, hence c_o must be fixed.

- In times of general price-stability, this implies an outstanding government-debt x_{10} and the amount of interest-payments per period of time is equal to $x_{10}\,\bar{g}_{10}$. If the interest-payments may be capitalized at the rate on government-bonds, their present value is equal to x_{10}.
- In times of inflation maintaining the same policy-target amounts to increasing the rate of interest up to $\bar{g}_{20}$, this may only be achieved through an increase in supply of government-bonds equal to $(x_{11} - x_{10})$. The flow of interest payments is now equal to $x_{21}.\bar{g}_{11}$.
  Capitalizing these flows at the same real rate as under general price-stability, their present value will be equal to $x_{21}\,\bar{g}_{11}/\bar{g}_{10}$.
- Measuring the decrease in effectiveness of debt-management by the increase in social costs associated with the same target, we have :

  $$\text{decrease in effectiveness of debt-management} = \frac{x_{21}\cdot \bar{g}_{11}}{\bar{g}_{10}} - x_{10},$$

  which is in fact equal to the increase in the use of public funds for the same target of monetary policy.

As one may easily see from the graph, maintaining the supply of bonds at x_{10} results into an increase in the rate of interest as well, but this increase will not be large enough to prevent the community from holding a larger capital stock.

This might seem to result from the arbitrariness of the graph, but it does not.

Indeed, the change in slope of the demand-curve for real capital will be much larger than the change in slope of the demand-curve for bonds, as

$$d\,s_{1\ m+1} = -\left[d\,s_{11} + d\,s_{12} + \ldots + d\,s_{1m}\right] = \underbrace{-\,d\,s_{11}}_{>0} \underbrace{-\left[d\,s_{12} + \ldots + d\,s_{1m}\right]}_{>0}$$

Hence $d\,s_{1\ m+1} > -\,d\,s_{11} > 0$

Linking government-bonds to the index amounts to improving their substitution with equity and real capital.

The slightest increase in the expected rate of return on index-linked government-bonds will have a strong restrictive influence on aggregate demand.

The index-linkage of government-bonds will increase their rate of substitution with equity and decrease their rate of substitution with non-index-tied government debt-instruments.
This is, in a more formalized fashion, Tobin's argument for the index-linkage of part of the government's debt (1).

As we demonstrated, the index-linkage applied to government-bonds will in fact maintain the demand-curves in their position of general price-stability. As it costs less to reach the same target under general price-stability, there is a positive social gain associated with the index-linkage. This social gain may be measured by the difference in value of capitalized interest-payments on nominal government-debt in times of inflation and under general price-stability.

The substitution of government-bonds with equity could be still more improved by a generalization of the indexation technique so as to link some debt-instruments to other indexes as well.

Indeed, nominal income currently earned on share-holdings may be divided into two components : a nominal price component, which reflects the rate of inflation and a real income component, which is rather a counterpart of the real value of goods and services effectively produced.
Nominal income earned on shares may be expected to rise through an increase of either of these components.
Consequently, the rate of return on share-holdings is also a function of the expected level of aggregate demand and activity.
This means that in a boom, i.e. when firms expect to run larger real profits, share-prices increase and the rate of interest calculated on shares declines. If monetary authorities want to check aggregate demand through larger supplies of government-debt, they have to pay high rates of interest to compete with the permanent price-increases of share-holdings. If, however, this government-debt is linked to an index that reflects the general

(1) TOBIN, James : "An Essay on Principles of Debt Management, op. cit., p. 202.

increase in activity, the monetary authorities will more easily compete with the share-market, with much lower rates of interest.
Moreover, the government could easily resort to such an index-linkage, as its tax-receipts, i.e. its real income, will show close conformity with the business-cycle of the economy.

4.2.2. The index-linkage of financial assets as one more instrument of monetary policy

Introduction

In the preceding section, we demonstrated that index-tied debt-instruments are likely to perform better as a tool of monetary policy than ordinary debt-instruments through the positive effects on the demand curves for bonds and equity.

In this section, we shall pay some attention to the use of the index-linkage as one more instrument in monetary policy, existing side by side with other debt-instruments.
Indeed, once monetary authorities have decided to resort to the index-linkage of financial assets, the range of possible strategies to reach a given policy-target is enlarged and there may be specific circumstances where a target is more easily reached with an index-linkage of financial assets than with other "traditional" instruments.
In Tinbergen's terms, the implementation of the index-linkage of financial assets means creating one more instrument for the same number of policy-targets and thus enlarging the range of policy choices.

4.2.2.1. An open-market policy with index-tied and nominal government-bonds

Tord Palander (1) paid much attention to what he called the "inter-market policy".
After having developed his theory of the "twin-markets equilibrium", where the rate of interest on nominal bonds is equal to the rate of interest on

(1) PALANDER, op. cit.

real bonds plus the rate of expected inflation, he emphasized the importance of a feed-back effect of the interest-rate differential on the public's inflationary expectations.
In other words, Palander's idea is that once the differential between the rates of interest has been fixed on the market, the public will come to consider this differential as the best indicator of what the future rate of inflation is likely to be.

Palander's inter-market policy simply consists in lending on one market and borrowing in the other market so as to modify the interest rate differential. A modification in this differential will change the public's inflationary expectations.
If the government succeeds in decreasing the public's inflationary expectations, it may expect very stabilizing effects from it.

His theory encounters, we think, a few important objections that would make it uneffective as a policy-instrument.
Firstly, the interest-rate differential will probably not be equal to the anticipated rate of inflation. Indeed, Palander's conclusion on the relation between the rates of interest on nominal and index-tied assets resulted from a partial equilibrium analysis, which does not explain the equilibrium level of asset prices - or rates of interest - from a macro-economic point of view. It only explains an equilibrium relation between rates of interest. We showed, however, that the prices of nominal and index-tied assets were very differently affected by inflation, once the inflationary impact was examined in a macro-model.
In fact, one of the asset-prices was fixed and the other asset-price was to fluctuate with the rate of expected inflation. Consequently, as we showed in our chapter III that there is a definite difference in risk between index-tied and nominal assets, this difference in risk will be reflected in the interest-rate differential as well.

The nominal rates of interest on nominal assets will surpass the rate of interest on index-tied bonds by more than the rate of expected inflation so as to include a risk-premium for the more risky nominal bond-holdings.

Now, Palander's inter-market policy would still work, if this risk-premium is constant for any rate of inflation, but, quite obviously, it will not. Nominal bond-holdings will be more risky for a rate of inflation of, say, 10 % than for a rate of inflation of, say, 3 %.
As a result, the public may find it more difficult to deduce its own inflationary expectations from this interest-rate differential.

Secondly, the interest-rate differential would be nothing but what the market expects the future rate of inflation is going to be. Consequently, extrapolating inflationary expectations from the interest-rate differential amounts to speculating on expectations. The interest rate differential is an indicator of inflationary expectations, it is not necessarily the best indicator of the future rate of inflation.

Thirdly, an inter-market policy aiming at reducing inflationary expectations will be conducted by one single market operator : the monetary authorities. This market-operator may be important by the size of its operations, when compared to other market-operators.
The non-official market-operators, however, may then very well regard the new interest-rate differential as reflecting the inflationary expectations of the monetary authorities. If the market as a whole considers this rate of expected inflation as unrealistic, the new differential between the rates of interest will have little effect upon the public's expectations. This is likely to be true, as soon as the market becomes aware that the monetary authorities seek to reduce the inflationary expectations.

All things considered, the range of effectiveness of such an inter-market policy is likely to be very small.
We think, however, that the open-market policy conducted with both nominal and index-tied debt-instruments may be validly reconsidered in another way.

Assume that the government's target is to stabilize the price-level, i.e. to reach a zero rate of inflation, and that the appropriate policy is an open-market policy.

If the market-operators expect the government to be successful in its stabilization program, they will rush on all the nominal bonds supplied on

the open-market, as they expect to make capital-gains, once the target of a zero rate of inflation has been effectively reached.
Indeed, by that time inflationary expectations will have much decreased, lowering the nominal rates of interest.
As a result, any operation on the open-market with nominal debt-instruments will be successful and the issues can be made at favourable terms.

If the market-operators expect the rate of inflation to be beyond its present level, despite the government's efforts to stabilize the price-level, operations on the open-market will be conducted with index-tied debt-instruments. Indeed, for the same analytical reasons, market-operators now expect to make capital-gains on index-tied debts. The monetary authorities will then resort to index-tied debt-instruments, which may be issued at favourable terms.
If the market expects the rate of inflation to remain unchanged, any operation on the open-market will be conducted with index-tied instruments, as they are the less risky-monetary assets.

Moreover, if the market-operators'inflationary expectations turn out to be wrong ex post, the cost of a stabilization program will be low.
If the market expected the rate of inflation to be left unchanged, while in fact it has been reduced to nihil, the government makes capital-gains on its index-tied debts.

Quite clearly, the government may issue either type of bonds according to the state of inflationary expectations. Moreover, in two of three possible cases, the issues can be made at more favourable terms with index-tied debt-instruments , than with ordinary nominal debt-instruments.

4.2.2.2. The index-linkage of financial assets and the risk of default

A stzbilization program entails undoubtedly a number of far-reaching consequences in the financial sector of the economy, that from many points of view may seem undesirable.

Indeed, in an economy where the nominal rates of interest tend to keep pace with the rate of inflation, a stabilization of the price-level would

be disastrous for debtors. Stabilizing the price-level would amount to increasing the real burden of debt-charges.
As a result, debtors will fear their unability to reimburse and creditors bear a larger risk of default.
Creditors, however, expect a higher real income on their financial assets and possibly a capital-gain, if their assets may be readily sold on the market. As a result, creditors bear a larger risk of default, but are compensated for the increase in risk by prospective capital-gains.
Debtors are definitely worse off after a successful stabilization policy.

Moreover, the government itself must pay for a successful stabilization fo the price-level, as the real burden of the interest-payments on its own debt will have increased as well.

The present value of the increase in real burden of debt-charges will, of course, be positively related to the maturity of the debt. Consequently, creditors experience a large capital-gains on their long-term financial assets on the one hand and large increases in risk of default on long-term holdings on the other hand.

Hence we may assert that debtors will always oppose any policy that aims at stabilizing the price-level, while creditors will oppose such a policy, if they valuate more negatively the increase in risk of default than they valuate positively the prospective capital-gains or gains in real income.

Those financial intermediaries borrowing long and lending long will bear the increased risk of default without being compensated for it by the prospective net gains in capital-value of their assets.
Indeed, they lose on their liabilities what they gain on their assets.

Hence there is much evidence that an index-linkage of financial assets should be given much importance in the framework of a stabilization policy. Debtors will be certain about the real value of their debt-charges and creditors bear the same risk of default as for any rate of inflation.
Wether the index-linkage of financial assets will be finally demanded by creditors depends on their attitude towards risk of default and prospective capital-gains.

This kind of argument points to more occasional implementations of index-clauses in financial markets.

4.2.2.3. The index-linkage of financial assets in connection with banking

In chapter II, we suggested that banks are differently affected by changing rates of inflation when compared to non monetary-financial intermediaries. We came to the conclusion that banks even stabilize the revenue earned through money creation when holding long-term monetary assets.

There are, however, at least two more reasons not to allow banks to hold any of their financial assets in index-tied forms.

Firstly, as long as banks hold their credits and marketable assets in nominal forms, they will not gain by an increase in the rate of inflation, as the increase of their assets through inflation is exactly matched by the decrease in real value of their assets.
If, however, banks were allowed to hold index-tied assets, an increase in the rate of inflation would still imply a higher rate of expansion of their assets without a corresponding decrease in real value of their assets.
As a result, banks may very well adopt an active attitude towards the rate of inflation, pushing it up to the point where they maximize their revenue from money creation in the Friedmanian sense. This is highly undesirable, as it would result into a redistribution of real income, money-holders paying a tax to money-issuers, i.e. to the banks.

Secondly, the index-linkage of the banks'marketable assets removes the lock-in effect induced by increasing rates of inflation.
Indeed, banks hold at least part of their assets in readily marketable forms so as to be in a position to avail themselves with liquidity whenever needed and without going into discount operations with the Central Bank.

If those marketable assets, i.e. not credits, are not index-tied, an increase in the rate of inflation will result into higher nominal rates of interest and falling market-prices of nominal assets.
As a result, it becomes increasingly costly to buy liquid assets outside the Central Bank, so banks will rather resort to discount-operations with

the Central Bank whenever they are in need of liquidity.
Hence the lock-in effect tends to reinforce the power of the Central Bank to influence the financial and real components of the economy at a moment where a more active monetary policy becomes necessary.

The index-linkage of those marketable assets held by banks would remove the lock-in effect. Moreover, the marketable assets would increase in market-value in times of rising rates of inflation and this would entail a decrease in the effectiveness of a discount policy in times of rising rates of inflation.

Consequently, if any acceleration of the inflationary process is to be avoided, banks should not be allowed to hold any index-tied assets.

4.2.3. Conclusion

In this second part of our chapter IV, we emphasized the specific advantages inherent in the index-linkage of financial assets.

Index-tied debt-instruments proved to be better substitutes for equity and real capital, so the index-linkage improves the working-conditions of debt-management in times of inflation.

Index-tied debt-instruments enlarge the range of tools in monetary policy and may be successfully used according to the state of inflationary expectations.

The index-linkage may considerably reduce the social costs associated with a stabilization of the price-level, as it removes much of the risk of default caused by such a stabilization.

Finally, banks should not be allowed to hold any of their assets in index-tied forms to prevent them from accelerating the inflationary process and to maintain the power of a central discount policy.

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CONCLUSION

The analysis has been conducted far enough to formulate an integrated body of rules and principles on the index-linking of financial assets as a measure of monetary policy.

Firstly, investors will certainly appreciate to be given the opportunity to hold any desired part of their wealth in index-tied forms, if they wish so. The extent of this demand for index-tied financial assets depends on a few institutional characteristics that could be formulated as follows.

The range of real assets available that investors may consider to hold in face of a continuous inflation is an institutional parameter that differs from economy to economy.
Any real asset, however, is likely to be more risky and illiquid than monetary assets. These differences in risk and liquidity depend in turn on the perfection and developments of secundary markets for real assets.

Consequently, one may assert that the demand for index-tied financial assets in inflationary economies depends on the range of real assets, i.e. inflation-hedges, available and on the risk and liquidity gap between monetary and real assets. The smaller this range and the larger the risk and liquidity gap, the larger the potential demand for index-tied assets.
A policy oriented towards an extensive development of a market for common stock could be regarded as a possible alternative to the indexation of financial assets, as it enlarges the range of real assets available and closes the risk and liquidity gap. This is why the degree of development of a market for common stock could be taken as one possible measure of the potential demand for index-tied assets in inflationary economies.

Moreover the index-linkage of monetary assets is likely to correct the inflation-caused distorsions in the allocation of productive resources that might result from a general tendency to hedge against inflation by holding real assets.

Secondly, if any monetary asset must be linked to the index, it should be a long-term rather than a short-term monetary asset.

Indeed, although all monetary assets become more risky in times of inflation, they will not become equally more risky.
Many of the interest-bearing monetary assets may change in current market-value. We demonstrated that, if there is some relation between this market-value and the rate of inflation expected, long-term monetary assets become more risky in inflation than short-term monetary assets. This explainsthe gradual disappearence of bond-markets in inflationary economies.

Moreover, as we proved in our chapter III, if one cannot establish such a ranking of monetary assets according to the amount of risk borne in inflation, i.e. when no relation exists between the current market-value of monetary assets and the expected rate of inflation, the index-linkage applied to monetary assets will make them more risky, as their market-value will fluctuate very sharply with the rate of expected inflation.

Thirdly, the implementation of index-tied monetary assets is not expected to be successful in a world where real rates of interest fluctuate more sharply in response to inflation than nominal rates of interest. Indeed, the market-value of index-tied assets, which bear "the" real rate of interest will fluctuate more than the market-value of nominal assets. Consequently, the nominal rates of interest must allow for at least 50 % of the expected rate of inflation.
If nominal rates of interest entirely compensate for the expected rate of inflation, index-tied monetary assets are absolutely risk-free with respect to inflation.

Consequently, the main economic variable to be considered for an implementation of index-tied monetary assets is the real rate of interest in its relation to the expected rate of inflation.

Fourthly, the monetary authorities are concerned with the indexation of financial assets in two respects : on the one hand, index-tied financial systems must be permanently managed so as to preclude the main perverse effects expected from the index-linkage; on the other hand, they may resort to the index-linkage for specific purposes of monetary policy.

The main perverse effects expected from the index-linkage result from the dynamics of inflationary expectations and from undesirable portfolio reallocations induced by changing rates of expected inflation.

Adjustments of the level of real liquidity to new nominal rates of interest and to new expected rates of inflation should be achieved through changes in the money supply rather than through changes in the price-level, which would lead to new inflationary expectations.
In other words, an active open market policy on the markets for nominal and index-tied debts seems to be called for.

Moreover, since the portfolio reallocations between assets of various maturities go into the wrong direction from the point of view of the natural adjustment of the economy to new rates of expected inflation - from short-term assets to long-term assets for an increase in the expected rate of inflation and from long-term to short-term assets for a decrease in the anticipated rate of inflation - the monetary authorities have to manage the term-structure of oustanding government debt as well.

This management could be much facilitated by a segmentation of the markets for nominal and index-tied debts and by a decycling of the index. Linking government-debt to the index is likely to increase the effectiveness of debt-management in inflation, as the public is offered a much closer substitute to equity, which is precisely a major hedge against inflation.

An occasional indexation of financial assets may be thought of as one of the elements that should enter an integrated stabilization program. Stabilizing the price-level amounts to increasing the real charges on outstanding debt.
As a result, debtors always oppose any policy that aims at reducing the rate of inflation.
Creditors oppose such a policy, if they do not feel compensated for the increase in risk of default by the prospective capital-gains.
An index-linkage of financial assets would preclude those social costs associated with stabilizing the price-level.

We demonstrated as well that bancks should not be allowed to hold any of their assets in index-tied forms.

As a conclusion, we may say that the index-linkage of financial assets safeguards the normal functioning of markets for long-term fixed-interest capital, provided that real rates of interest tend to fluctuate less than nominal rates of interest in response to changing rates of anticipated inflation.

Financial indexation assumes an increased concern of the monetary authorities with the financial well-being of an economy and it enlarges the scope and effectiveness of monetary policy in times of inflation.

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