RESEARCH OUTPUTS / RÉSULTATS DE RECHERCHE

Spectral network identification with generalized diffusive coupling

Gulina, Marvyn; Mauroy, Alexandre

Published in:

41st Benelux Meeting on Systems and Control

Publication date: 2022

Document Version Peer reviewed version

Link to publication

Citation for pulished version (HARVARD):

Gulina, M & Mauroy, A 2022, Spectral network identification with generalized diffusive coupling. in A Vande Wouwer, M Kinnaert, E Garone, L Dewasme & GA Pimentel (eds), 41st Benelux Meeting on Systems and Control: Book of Abstract. Presses universitaires de Mons, 41th Benelux Meeting on Systems and Control, Bruxelles, Belgium, 5/07/22.

https://www.beneluxmeeting.eu/2022/uploads/images/2022/boa BeneluxMeeting2022 Web betaV12 withCha irs.pdf>

General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
 You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal?

Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 13. May. 2024

Spectral network identification with generalized diffusive coupling

Marvyn Gulina and Alexandre Mauroy Namur Institute for Complex Systems (naXys) and Department of Mathematics, University of Namur, Rue de Bruxelles 61, B-5000 Namur, Belgium

Email: marvyn.gulina@unamur.be and alexandre.mauroy@unamur.be

1 Spectral identification of networks

Spectral network identification aims at estimating the eigenvalues of the Laplacian matrix L of the network from data. This allows to infer global information on the network structure from local measurements at a few number of nodes **Mauroy2017**.

2 Scalar diffusive coupling framework

We consider a dynamical system defined over a network. The local dynamics of the unit attached to node k are directly influenced by its neighbors through input and output signals. In the particular case of n identical units with diffusive coupling, the dynamics of node k are described by

$$\begin{cases} \dot{x}_k = F(x_k) + G(x_k)u_k & \in \mathbb{R}^m \\ u_k = \sum_{j=1}^n W_{kj}(y_k - y_j) & \in \mathbb{R}^r \\ y_k = H(x_k), & \in \mathbb{R}^r \end{cases}$$
(1)

with the (continuously differentiable) functions $F: \mathbb{R}^m \to \mathbb{R}^m$, $G: \mathbb{R}^m \to \mathbb{R}^{m \times r}$ and $H: \mathbb{R}^m \to \mathbb{R}^r$. Note that the coupling coefficients W_{kj} are the entries of the adjacency matrix $W \in \mathbb{R}^{n \times n}$ of the network. The network Laplacian matrix is given by L = D - W, where D denotes the degree matrix defined by $d_i = \sum_{k=1}^n W_{ik}$. We further assume that the units asymptotically reach a synchronized state $x_1 = \cdots = x_n = x^*$, $i.e. \lim_{t \to \infty} x_k(t) = x^*$.

The Jacobian matrix associated with the whole system (1) and evaluated at x^* writes

$$J = I_n \otimes A - L \otimes BC^T, \tag{2}$$

with $A = \partial F/\partial x(x^*)$, $B = G(x^*)$ and $C = \nabla H(x^*)$. In the previous work **Mauroy2017**, it was proved that the relationship between the spectrum of the Jacobian matrix $\sigma(J)$ and the spectrum of the Laplacian matrix $\sigma(L)$ is one-to-one when r = 1 and satisfies

$$(A - \mu I_m)w = \lambda BC^T w, \tag{3}$$

where $\mu \in \sigma(J)$ and $\lambda \in \sigma(L)$. It follows that one can retrieve the spectrum of L from the spectrum of J. Moreover, states measurements can be used to compute the spectrum of the Jacobian matrix through the DMD algorithm **Schmid2010**.

It is noticeable that the results of **Mauroy2017** are limited to the case r=1 in (1). This corresponds to scalar-valued input and output signals, a condition that is not satisfied in many situations, such as reaction-diffusion networks. If r>1, the rank of the matrix BC^T can be larger than 1 and the one-to-one relation between μ and λ obtained from (3) does not hold. In this context, the main contribution of the present work is to provide a generalized framework for spectral identification, which is valid for the case of vector-valued inputs and outputs (r>1).

3 Generalization of the framework

We will show that the spectral identification problem is feasible under a mild assumption based on the spectral moments of BC^T . More precisely, one can obtain a linear system of equations which allows to compute the spectral moments of L from those of J. Finally the spectrum of L can be recovered from its spectral moments. However, this process might lead to significant numerical errors on the individual eigenvalues and should therefore be avoided.

We will propose an alternative method for solving the spectral identification problem, which is based on the properties of the characteristic polynomial of the generalized eigenvalues problem (3). This method will be illustrated with numerical simulations.