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*Published in:*

Physics of the Dark Universe

*Publication date:*

2012

[Link to publication](#)

*Citation for pulished version (HARVARD):*

Rekier, J & Füzfa, A 2012, 'Cosmic acceleration with cosmological soft phonons', *Physics of the Dark Universe*.

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# Cosmic acceleration with cosmological soft phonons

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(Dated: July 19, 2012)

The dark energy scalar field is here presented as a mean-field effect arising from the collective motion of interacting structures on an expanding lattice. This cosmological analogue to solid-state soft phonons in an unstable crystal network is shown to produce cosmic acceleration while mimicking phantom equation of state. From an analysis of the Hubble diagram of type Ia supernovae, we present constraints on the parameters of the cosmic Lagrange chain, as well as on time-variation of the soft phonon equation of state, before we conclude on new phenomenology associated to this interpretation.

## I. INTRODUCTION

Thanks to many independent observations, including type-Ia supernovae Hubble diagram [1, 2], angular fluctuations of the Cosmic Microwave Background [3] and many others based on large-scale structures properties, Baryon acoustic oscillation [4] and galaxy redshift distortion [5], it is now a well-established experimental fact that the expansion of the Universe is currently accelerating. What causes this cosmic acceleration is still unknown, since this observation requires modifications to either the matter-energy content of the universe and/or to the gravitational physics of the large-scales (through general relativity and the cosmological principle). One can still account fairly for various observations with the help of the famous Einstein's cosmological constant  $\Lambda$  but the so-called fine-tuning and coincidence problems [6] associated to it appear so intricate that numerous alternative explanations have arisen in the past decade. More generally, explaining cosmic acceleration without modifying gravitation requires the inclusion of a new variety of energy in the Universe, dubbed *Dark Energy* (DE), whose peculiar properties modify the background cosmological expansion. The cosmological constant corresponds to perfectly frozen DE, with absolutely no space-time variations, which is often meant as an ideal case.

Among the models of DE proposed, the hypothesis of quintessence provides an exciting framework in which DE is modeled by a scalar field.[7–9] While those theories already have achieved some success, they often lack to provide a sensible interpretation of the nature of this scalar field.

In this work, we develop an interpretation of the dark energy scalar field as a cosmological analogue of the phonon in solid-state physics. While the phonon is an effective scalar field describing the collective motion of atoms in a crystal, a possible cosmological analogue has

to be investigated in the physics of the deformation network of large-scale cosmic structures, under the action of gravitational interactions and background cosmic expansion. A crystal and the large-scale universe are constituted by a large number of constitutive elements (atoms or structures) undertaking long-ranged interactions that can be approximated in the continuous medium approximation with an effective potential energy resulting from all interactions.

However, cosmic structure formation in an expanding universe is a very complex non-linear mechanism, in which the study of an analogue of the phonon as a mean-field behaviour for averaged quantities constitutes an intricate technical challenge. Rather, we propose here a heuristic approach by deriving a toy model for the cosmological phonon in terms of an effective field theory in a Friedman-Lemaître-Robertson-Walker (FLRW) universe. This will allow us to derive a simple model for a cosmological phonon as a massive scalar degree of freedom with very different cosmological dynamics than the one of quintessence. The reason for that is because the cosmological phonon comes from an effective field theory in which cosmic expansion directly changes the phonon mass and velocity. We also show that this simple model can provide cosmic acceleration as well as a phantom equation of state.

In the next section, we derive the toy-model for the cosmological phonon and its dynamics in a homogeneous and isotropic space-time background from an effective lagrangian approach. Differences with usual massive scalar field are emphasized and discussed. In section III, it is shown how cosmic acceleration can be explained with soft phonons and constraints on model parameters are obtained through the Hubble diagram of far-away supernovae. Finally, we review in section V the interest and possible limitations of the model, as well as we propose lines of investigation for additional predictions of the model with large-scale structure physics.

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## II. A TOY-MODEL OF THE COSMOLOGICAL PHONON

### A. Effective lagrangian

To establish the toy-model of the cosmological phonon, we start with a very simple heuristic approach. Let us consider the lagrangian describing a set of masses  $m_a$  undergoing long-ranged interactions in such a way that each mass is locally affected by the potential  $V$  generated by the whole network. By limiting ourselves to the quadratic term we are left with:

$$L = \sum_a \frac{1}{2} m_a \dot{q}_a^2 - \sum_{a,b} \frac{1}{2} k_{ab} (q_a - q_b)^2 + \sum_a k q_a^2 \quad (1)$$

where the  $q_a$ 's are the displacement of the masses  $m_a$  from their unstable equilibrium point. The couplings  $k_{ab}$  and  $k$  represent respectively the harmonic coupling of the point masses  $a$  and  $b$  to one another and the self-coupling of a single mass, namely:

$$k_{ab} = - \left( \frac{\partial^2 V}{\partial q_a \partial q_b} \right) \quad (2)$$

$$k = - \left( \frac{\partial^2 V}{\partial q_a^2} \right). \quad (3)$$

The function (1) can be expressed in term of a scalar field by going to the continuous limit:

$$q_a \rightarrow \phi(x) \quad (4)$$

$$\sum_a \rightarrow \frac{1}{l^3} \int d^3x.$$

where  $l$  denotes the mean distance between two structures and the integral is to be taken over the whole (possibly infinite) volume of the network. By going through the whole process, one can easily find (see e.g [10]) that the effective lagrangian takes the form:

$$L = \int d^3x \frac{1}{2} \left\{ \frac{1}{v^2} \left( \frac{\partial \psi}{\partial t} \right)^2 - [\vec{\nabla} \psi]^2 + M^2 \psi^2 \right\}. \quad (5)$$

The new parameters of this expression can easily be expressed in term of the network parameters:

$$v^2 = \frac{k_{ab} l^2}{m}, \quad M^2 = 2 \frac{k}{k_{ab} l^2}, \quad (6)$$

which can both be either negative or positive depending on the signs of the network parameters. For more convenience, we also have rescaled the field from (4) as  $\psi = \sqrt{\frac{k_{ab}}{l}} \phi$ .

We will now try to write this lagrangian in a form that is suitable for cosmology. First, in accordance with the cosmological principle, we can drop the term proportional

to  $\vec{\nabla} \psi$  in (5). This is equivalent to considering only the spatially averaged scalar field

$$\psi(t) = \langle \psi(\vec{x}, t) \rangle.$$

To go any further, the key observation is to interpret (5) as the lagrangian in the frame comoving with the cosmic expansion. Therefore, physical quantities will be affected by cosmic expansions through the dependency of the chain parameters to the scale factor. By looking at (6) and considering the fact that the comoving distance scales as  $l \sim a(t)l_0$ , one can easily obtain:

$$\frac{1}{v^2} \sim \frac{1}{a^2}, \quad M^2 \sim \frac{1}{a^2}. \quad (7)$$

This rescaling of the lagrange chain parameters will have tremendous impact on the cosmological dynamics of the phonon, differencing it from the usual quintessence scalar field. To write down the dynamical evolution of the Universe in presence of our new field  $\psi$  we have to consider the total action consisting in the time integral of the lagrangian (5) plus the usual Einstein-Hilbert action. The Friedmann-Lemaître-Robertson-Walker (FLRW) metric being

$$ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \quad (8)$$

we proceed by writing the total action as the integral of an effective 1D lagrangian (see [11]):

$$L = -\frac{3}{\kappa} \frac{\dot{a}^2 a}{N} + \frac{1}{2v^2} \frac{a \dot{\psi}^2}{N} - Na \frac{M^2 \psi^2}{2}. \quad (9)$$

As a matter of comparison, the reduced lagrangian for ordinary minimally coupled scalar field  $\Phi$  such as quintessence writes down

$$L = -\frac{3}{\kappa} \frac{\dot{a}^2 a}{N} + \frac{1}{2c^2} \frac{a^3 \dot{\Phi}^2}{N} - Na^3 \frac{M^2 \Phi^2}{2} \quad (10)$$

where the quintessence potential has been taken as a mass term  $V(\Phi) = \frac{M^2 \Phi^2}{2}$  for the sake of example. While quintessence scalar field comes from a fundamental field approach, the cosmological phonon is an effective scalar field whose mass and velocity are not constant but rather rescaled by cosmic expansion (see Eq. (7)). As a consequence, the cosmological dynamics of both fields will be very different, as one shall see below.

### B. Cosmological dynamics

By varying Eq. (9) with respect to the generalised coordinates  $(a, N, \psi)$  and then specifying to the conformal gauge ( $N=a$ ), we get the following equations (taking into account the contribution of matter and radiation):

$$\begin{aligned} \frac{a'^2}{a^2} &= \frac{\kappa}{3} \left( \frac{1}{2v^2} \frac{\psi'^2}{a^2} + \frac{M^2}{2} \psi^2 \right) + H_0^2 \left( \frac{\Omega_m^0}{a} + \frac{\Omega_{rad}^0}{a^2} \right) \\ \frac{a''}{a} &= \frac{\kappa}{6} M^2 \psi^2 + \frac{H_0^2}{2} \left( \frac{\Omega_m^0}{a} \right) \\ \psi'' &= -a^2 v^2 M^2 \psi. \end{aligned} \quad (11)$$

Where the primed quantities are to be interpreted as derivatives with respect to conformal time  $\eta$  (given in terms of the synchronous time by  $dt = a(t)d\eta$ ) and with  $\kappa = 8\pi G$  in natural units. The zero-indexed symbols represent the present values of these quantities (we took  $\Omega_m^0 = 0.25$ ,  $\Omega_{rad}^0 = 7.97 \times 10^{-5}$ ,  $H_0 = 71 \text{ km/s/Mpc}$ ) [2, 3]. Defining

$$\Omega_\psi(a) = \frac{\kappa}{3a^2 H^2} \left( \frac{1}{2v^2} \frac{\psi'^2}{a^2} + \frac{M^2}{2} \psi^2 \right), \quad (12)$$

the Friedmann equation can be put under the form:

$$\frac{a'^2}{a^2} = \frac{H_0^2}{(1 - \Omega_\psi(a))} \left( \frac{\Omega_m^0}{a} + \frac{\Omega_{rad}^0}{a^2} \right). \quad (13)$$

To get insight about which values the parameters  $v^2$  and  $M^2$  should take, one may look at the asymptotic behaviour of the model.

From the cosmological equations (11) written in the synchronous gauge one can easily guess the ratio of the pressure over the energy density of the field  $\psi$  to be

$$w_\psi = \frac{1}{3} \frac{\left( \frac{\psi^2}{v^2 a^2} - M^2 \psi^2 \right)}{\left( \frac{\psi^2}{v^2 a^2} + M^2 \psi^2 \right)}. \quad (14)$$

This equation of state for the effective field  $\psi$  has remarkable properties. Indeed, it interpolates between  $w_\psi = -1/3$  when the field is frozen ( $\frac{\psi^2}{v^2 a^2} \ll M^2 \psi^2$ ) and a cosmological constant  $w_\psi = -1$  in the far future ( $a \gg 1$ ; de Sitter Universe). To show this is an asymptotic solution, we start from the condition  $w_\psi = -1$ , which can be readily integrated for the asymptotic behaviour of  $\psi$  from (14) as

$$\psi = \psi_0 \exp \left( \sqrt{\left| \frac{v^2 M^2}{2} \right|} (t - t_0) \right), \quad (15)$$

with  $\psi_0$  the present-day value of the field. Inserting this result into the acceleration equation and solving for  $a$ , we find

$$a = \sqrt{\frac{\kappa}{6}} \frac{\psi_0}{\sqrt{|v^2|}} \exp \left( \sqrt{\left| \frac{v^2 M^2}{2} \right|} (t - t_0) \right). \quad (16)$$

From a suitable choice of  $\psi_0$  and  $\sqrt{|v^2|}$ , we can always make  $\sqrt{\frac{\kappa}{6}} \frac{\psi_0}{\sqrt{|v^2|}} \approx a_0$ . The result (16) is then to be compared with the usual de Sitter asymptotic solution  $a = a_0 \exp \left( \sqrt{\frac{\Lambda}{3}} c(t - t_0) \right)$ , showing that the asymptotic state is a de Sitter universe (see also Fig. 1).

Therefore, the cosmological dynamics of the phonon effective field is completely different than that of a massive scalar field (quintessence with quadratic potential) for which  $w \rightarrow -1$  when the field is frozen and  $w$  being zero on average (and the field behaving as pressureless matter) asymptotically. The reason for this is the rescaling of the phonon physical parameters with cosmic expansion, which is not the case for quintessence.

### III. COSMIC ACCELERATION WITH LARGE-SCALE STRUCTURE SOFT PHONONS

Let us now constrain the model parameters with the Hubble diagram of type Ia supernovae.

From Eq.16, it is straightforward to identify the quantity representing the effective cosmological constant of our model to be:

$$\Lambda_{\text{eff}} = \frac{3}{2} \frac{|v^2 M^2|}{c^2}. \quad (17)$$

That is, if we measure the value of the cosmological constant on the Supernovae data, then any two values of the parameters  $v^2$  and  $M^2$  satisfying the constraint (17) should reproduce the same asymptotic behaviour.

A condition for the asymptotic solution of Eqs.11 to be de Sitter is that the arbitrary real parameter we wrote  $v^2$  has to be negative (hence the need for the absolute value in 15). The physical meaning of this can be understood easily by looking at the third equation of (11). Positive values of  $v^2$  would imply oscillations of the field through time while negative values describe an inelastic deformation. The later case is more suited to describe the instabilities in the interplay of the large scale structures through gravitational interactions in a dynamical Universe. This case of facts is to be understood as the analogue of the concept of *soft* phonon in solid state physics [12], which represent fastly growing inelastic deformations when a crystal departs from an unstable equilibrium state.

The dynamics of the system (11) can be investigated numerically. We start the integration at the point  $a_i = 10^{-3}$ , corresponding to the beginning of the process of large-scale structure formation through gravitational collapse. Before that time, acoustic oscillations in the primordial plasma does not correspond to the inelastic deformation of a soft phonon. We also had to impose initial conditions on the values of  $\Omega_\psi^i$  and  $\psi_i'$  at this very point. While we chose to start at rest  $\psi_i' \approx 0$ ,  $\Omega_\psi^i$  is a free parameter found to be less than 1% (see below).

We chose to integrate the second and the third equation of (11) and then check the solution by injecting our result in the Friedmann equation (13). Everytime, the constraint is verified up to the machine precision. The integration has been carried out for several values of  $v^2$  and  $M^2$ . The model fits the SNIa data [2] with  $\chi^2 = 1.0018$  (in the simile,  $\chi^2_{\Lambda\text{CDM}} = 0.99$  for the same data set).

Fig. 1 shows the compared evolution of the acceleration parameter ( $q = \frac{\ddot{a}a}{\dot{a}^2}$ ). We see that the phonon model predicts a stronger yet smoother acceleration than  $\Lambda\text{CDM}$ . The fact that the acceleration is stronger at all times for  $\psi\text{CDM}$  should have impact on the physics of large-scale structure formations.

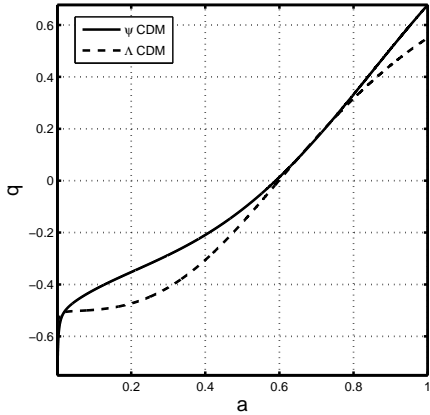


FIG. 1: Evolution of the acceleration parameter ( $q = \frac{\ddot{a}a}{\dot{a}^2}$ ) for the cosmological phonon model ( $\psi$  CDM) as compared to  $\Lambda$ CDM.

Another departure from  $\Lambda$ CDM is to be found in the evolution of the equation of state parameter  $w_\psi$  which is represented on fig. 2. This shows that the energy density of the phonon interpolates between a Nambu-Goto string gas ( $w_\psi = -1/3$ ) at early times and a cosmological constant asymptotically (as found earlier). Quite remarkably, this model evolves with a phantom equation of state ( $w < -1$ ) around today. This is done in spite of the minimal coupling because this model is not covariant since this effective scalar field does not behave like a irreducible representation of the Poincare group: its (squared) propagation velocity is  $|v^2|$  instead of  $c^2$ , and it does not behave like an usual scalar field under conformal transformations for instance. In addition, the rescaling of the network parameters with respect to cosmic expansion (see 7) achieves to give to this scalar field unusual features that allowed him to reproduce cosmic acceleration. One should emphasize that the non-covariance of the present model does not induce any violation of Lorentz invariance at the fundamental level since the concept of phonon is only present on large scales where it accounts for collective motions. This field is an effective one and not a fundamental one, and thus cannot be used to invalidate Lorentz invariance on local scales since it has no existence below the cosmic lattice characteristic length. In addition, the large-scale homogeneous universe is not invariant under global Lorentz transformations, just in the same way a crystal is not either Lorentz invariant. In fact, the solid-state physics phonon itself is not a true scalar field due to the same arguments.

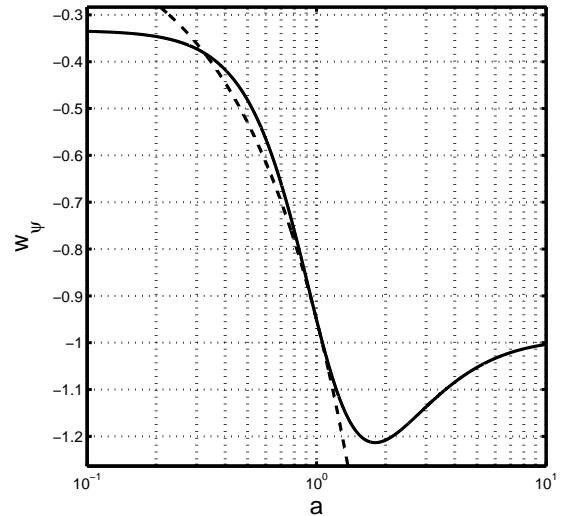


FIG. 2: Evolution of  $w_\psi$  from the CMB to the far future.

The dotted line of fig. 2 represents the linear approximation of the curve around  $a = 1$ :  $w_\psi(a) = w_0 + w_a(1-a)$  [13] with  $w_0 = w_\psi(a_0)$ . On fig. 3 are shown the confidence regions at  $1\sigma$  and  $2\sigma$  level in the plane of parameters  $(w_0, \Omega_m^0)$ . We see that, amongst the compatible models, one recovers a model similar to  $\Lambda$ CDM ( $w_0 = -1$ ) at the limit of  $1\sigma$  level only for lower values of  $\Omega_m^0$ .

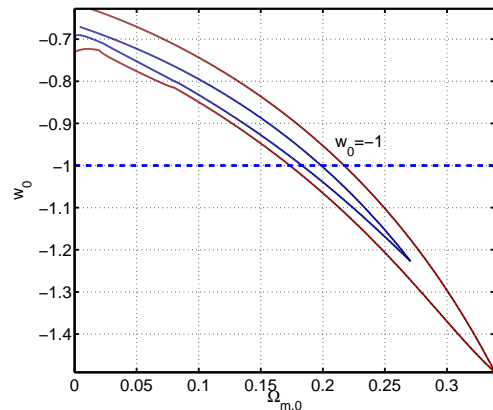


FIG. 3:  $1\sigma$  and  $2\sigma$  level confidence regions in the plane of parameters  $(w_0, \Omega_m^0)$  (data from [2]) ( $H_0 = 71\text{km/s/Mpc}$ )

In table 1, we sum up the best-fitted values of the various cosmological parameters of our model and the boundaries of the first confidence region ( $1\sigma$  level). The value of the best-fit for  $\Omega_m^0$  is small compared to the estimation from [2]. Still the usual constraints on  $\Omega_m^0$  of order 0.25 are compatible with other predictions to  $1\sigma$  level (see e.g. [14]).

Parameter	best-fit $\pm 1\sigma$
$\Omega_m^0$	$0.14^{+0.13}_{-0.14}$
$\Omega_\psi^0$	$0.86^{+0.14}_{-0.13}$
$q_0$	$0.66^{+0.19}_{-0.14}$
$w_0$	$-0.90^{+0.22}_{-0.33}$
$w_a$	$0.77^{+0.36}_{-0.31}$

TABLE I: Best-fit of the cosmological parameters

#### IV. CONCLUSIONS AND PERSPECTIVES

The preceding results show that the phonon model could be a nice alternative to  $\Lambda$ CDM with no more than two free parameters: the initial amount of energy in  $\psi$  and the self-coupling parameter  $M^2$  (the  $v^2$  parameter being constrained by (17)). It reproduces a de Sitter Universe for  $a \gg 1$  and provides an interesting interpretation of the nature of DE without involving any speculative physics. A direct test of the present model could be provided by direct measurements on the equation of state and its time variation. Future directions would include a detailed study of the implication of the cosmological phonon on cosmic structure mobility, higher order-coupling of Dark Matter to the phonon field or other solid-state phonon-inspired observables, instability wavefronts and large-scale collective motions. Another important issue would be to derive the preceding model from conventional cosmological perturbation and statistical analysis in inhomogeneous cosmology. Challenges for numerical N-body simulations of large-scale structure formation would be to estimate the network parameters

from the overall potential energy generated by the inhomogeneous density field. Once this has been done, one can then check whether they can fall in the range predicted by the SNe Ia analysis. Such numerical studies will also allow one to compute the structure network potential energy and the related solid-state phonon-inspired observables. Another interesting question to be addressed by N-body simulations would be to investigate whether collective motions corresponding to the propagation of a soft phonon on large scales can be reproduced.

We would also like to point out the fact that the use of a lattice of structure to model the behaviour of the Universe on large scales has recently been made in [15] in which the authors solve the Einstein equations perturbatively in the presence of a perfect network of structures. An interesting continuation of this work would be to investigate how our soft phonon description might emerge from the introduction of some defects or perturbations in the model of [15]. This further work might provide the tensorial description that our present model is currently lacking.

#### Acknowledgements

The authors warmly thank L.Henrard for the useful discussions from which this paper originates. J.Rekier is supported by an FRS-FNRS (Belgian Fund for Scientific Research) Research Fellowship. This research used ressources of the 'plate forme technologique en calcul intensif (PTCI)' of the University of Namur, Belgium, for which we acknowledge the financial support of the F.R.S.-FNRS (convention No. 2.4617.07. and 2.5020.11)

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