BFO, a trainable and versatile Brute Force Optimizer

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Thanks

- Leverhulme Trust, UK
- Balliol College, Oxford
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- University of Namur, Belgium

How it happened...

- working on an interpolation-based derivative free optimizer for

$$\min_{x \text{ subject to bounds}} f(x)$$

(more on that at the very end)

- needed something quick and dirty to improve its parameter settings
- wrote a "Brute Force" tool ...
- ... which (after some years of tweaking) has turned into BFO

simple ideas + computing power $\stackrel{?}{\Longrightarrow}$ robust/useful tool?

The context

Two common preoccupations in algorithm design/usage:

- For algorithms designers:

  > How to tune the parameters of an algorithm in order to ensure the best possible performance on the *largest possible class* of applications?

- For algorithm/code users:

  > How to tune the parameters of a code in order to ensure the best possible performance on a *specialized class* of applications?

Does achieving the first does help the second?

A way out ?

Some flexibility is needed !

- Provide a tuning methodology which is applicable to many algorithms
- Provide code which allows user-tuning for his/her pet problem class

$\Longrightarrow$ optimization?

- Need to define an objective function
  (how to measure algorithm performance in this context?)
- Need to define the constraints (on algorithmic parameters)
  - simple bounds (algorithm dependent)
  - continuous/integer/categorical variables + mix
    (ex: blocking size, model type, . . .)

Which objective function?

Assume that the (negative) performance perf(params, prob) can be measured by running the considered algorithm with parameters params on problem prob (ex: number of function evaluations).

- First model: optimize the total/average performance (AO, OPAL):

$$\min_{\text{params}} \sum_{\text{problems}} \text{perf}(\text{params}, \text{prob})$$

- Second model: optimize the robust performance (RO):

$$\min_{\text{params}} \max_{\text{perturbed params}} \sum_{\text{problems}} \text{perf}(\text{perturbed params}, \text{prob})$$

where

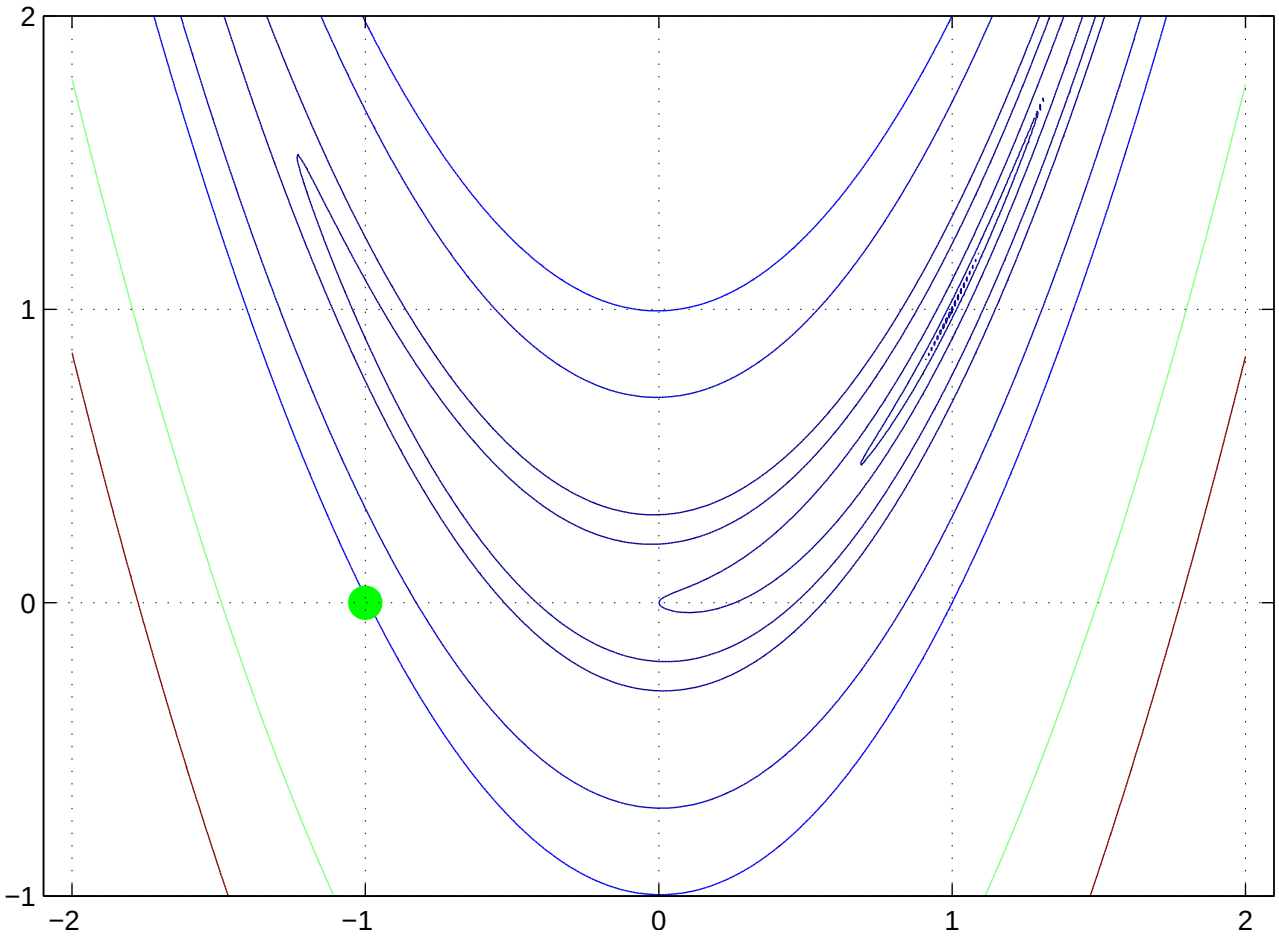
$$0.95 * \text{params} \leq \text{perturbed params} \leq 1.05 * \text{params}$$

A new tool: BFO (the Brute Force Optimizer)

BFO: a new *local* optimization package with

- randomized pattern search methodology
  (does not require continuity of the objective function)
- allows bounds on the variables
- allows continuous/discrete or mixed integer variables
- handles multilevel/equilibrium problems
  (needed for the robust tuning strategy)
- includes self-tuning facilities

The standard compass-search on Rosenbrock's function



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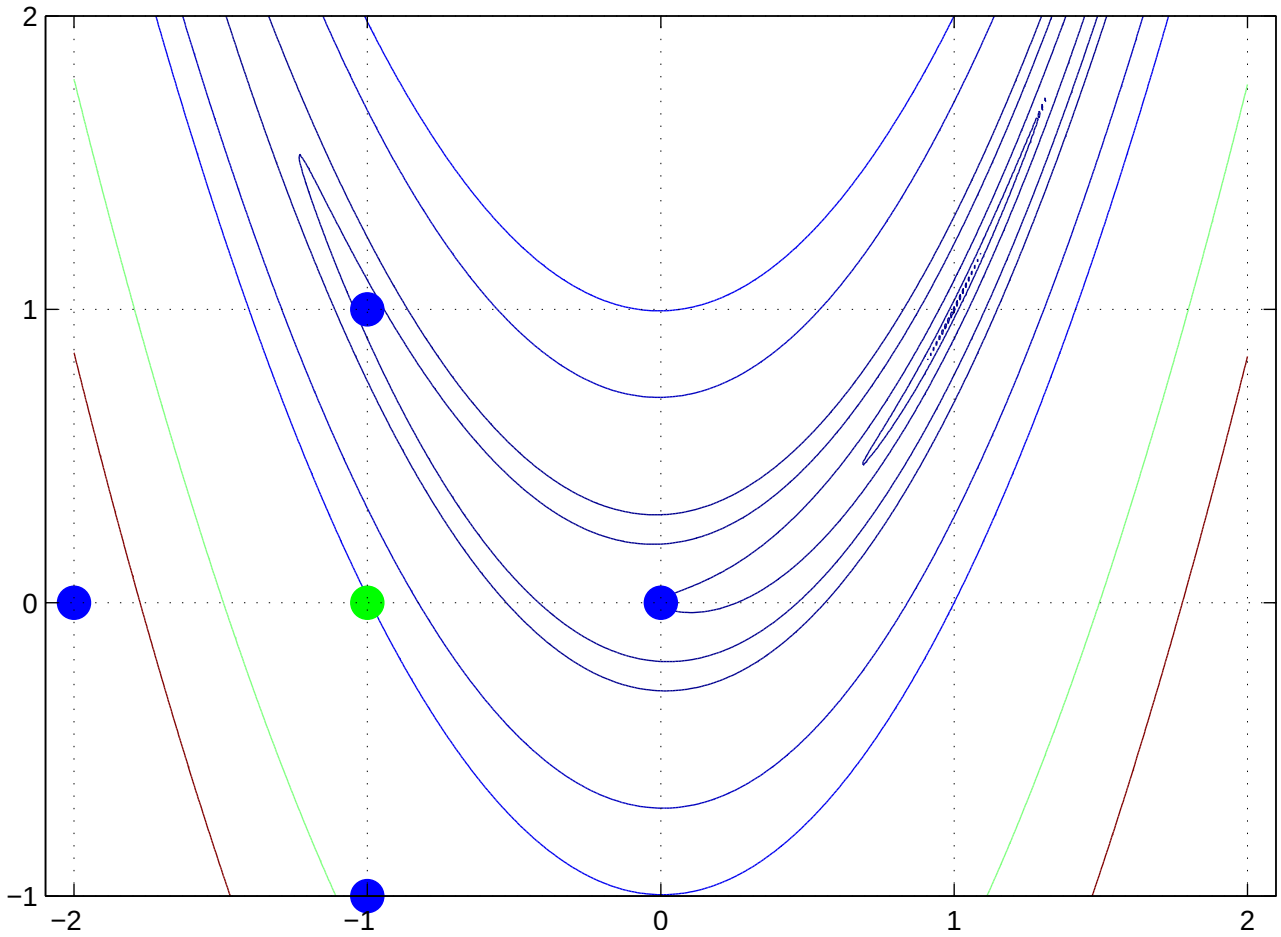

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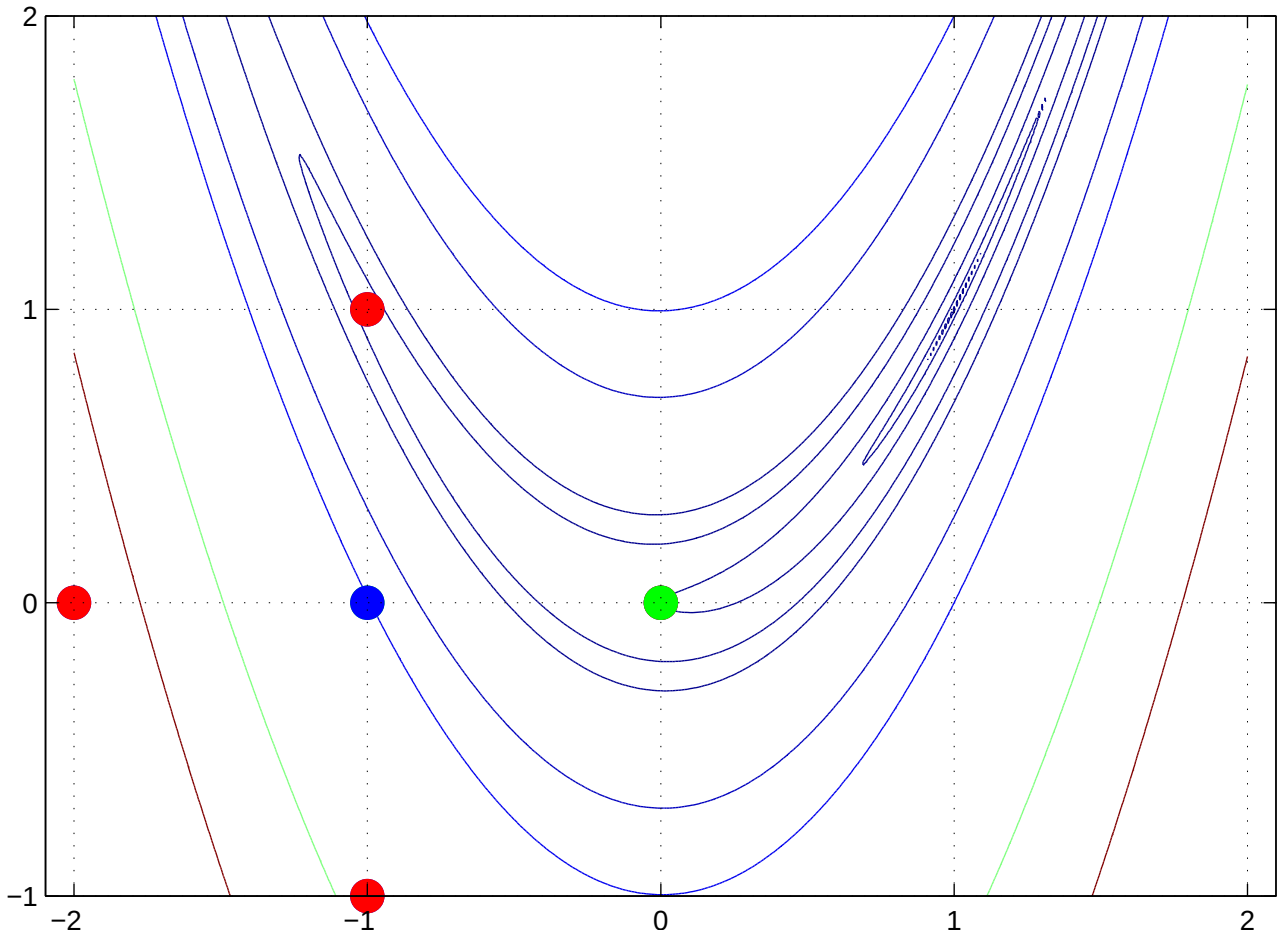

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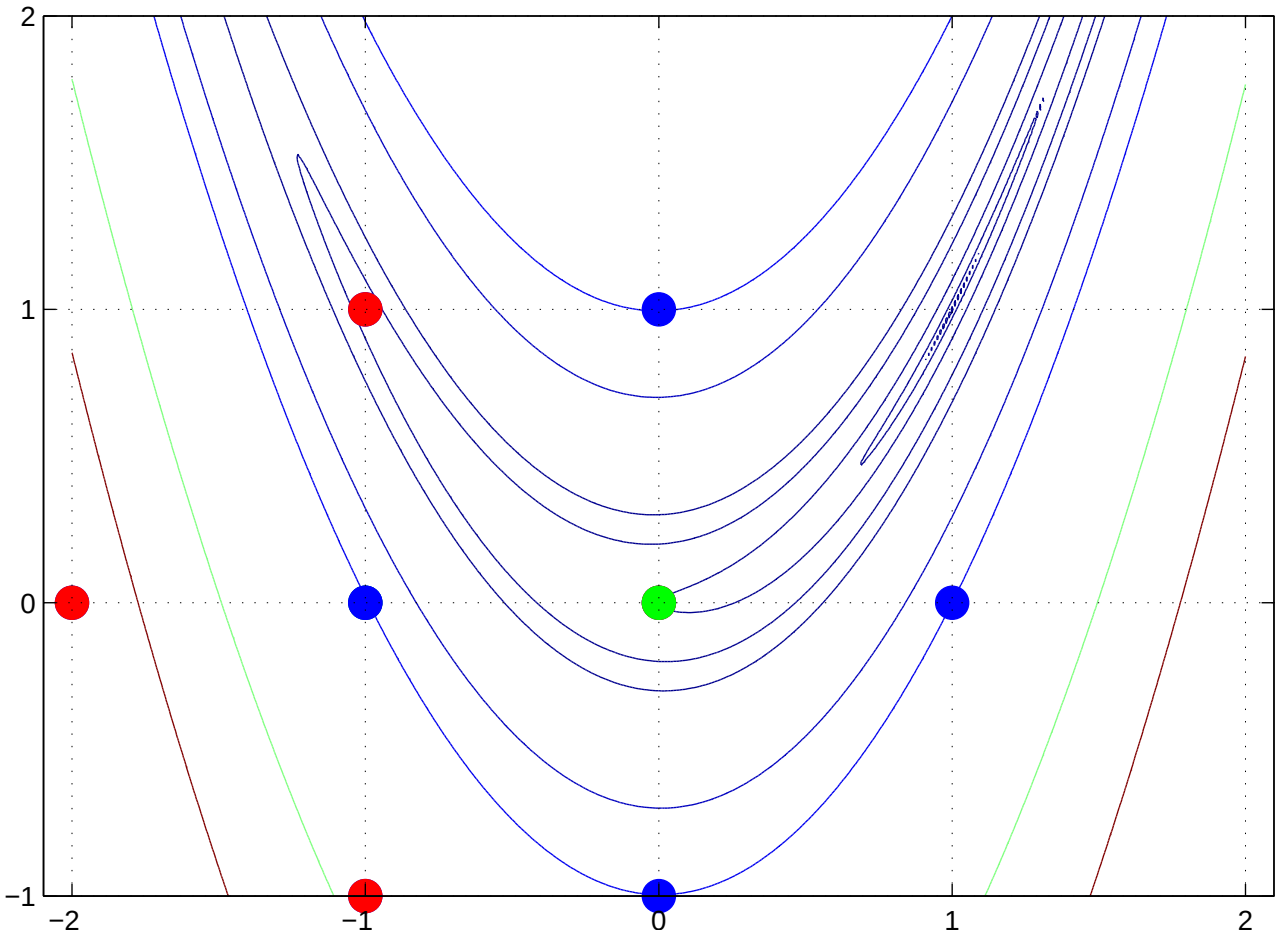

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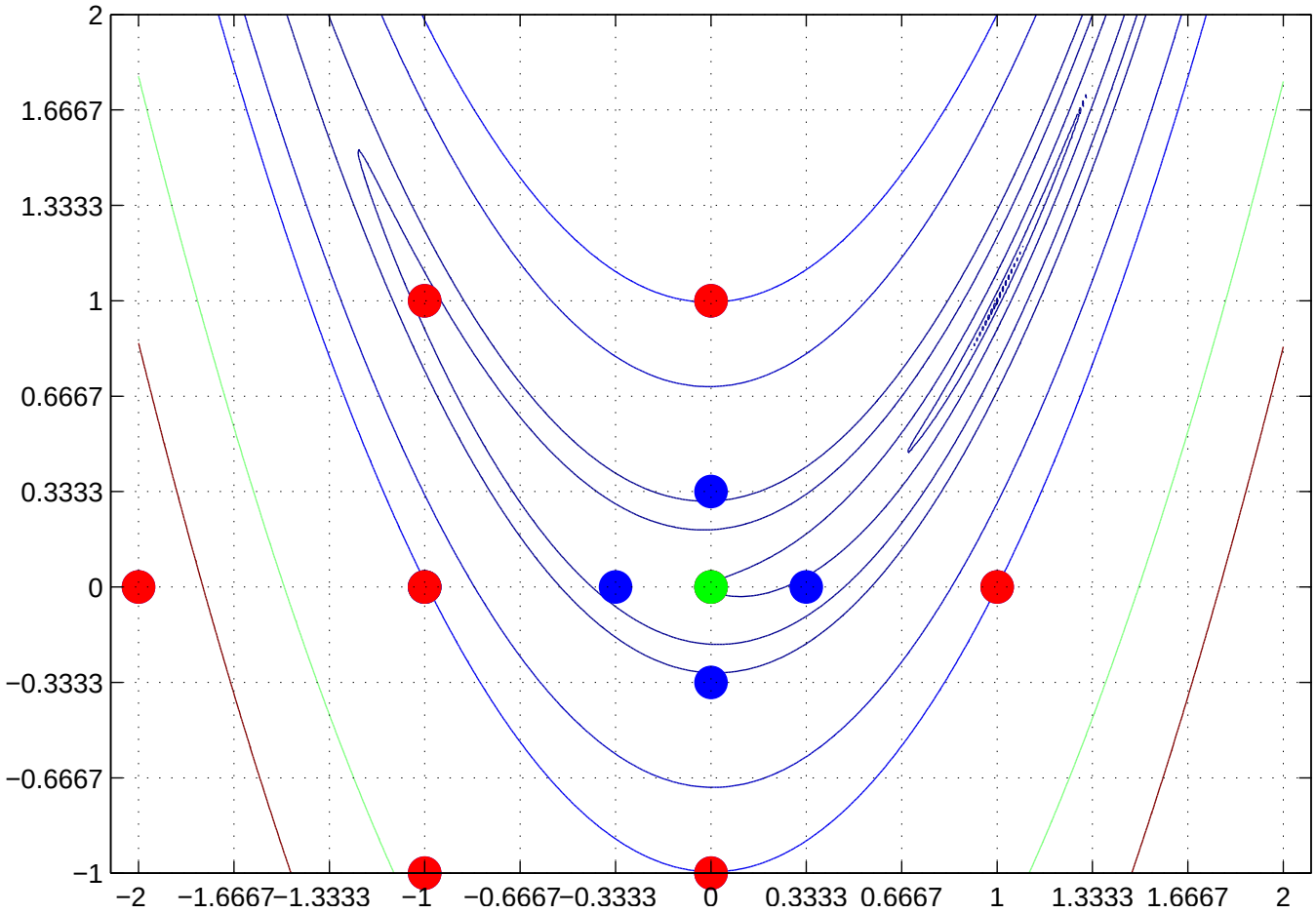

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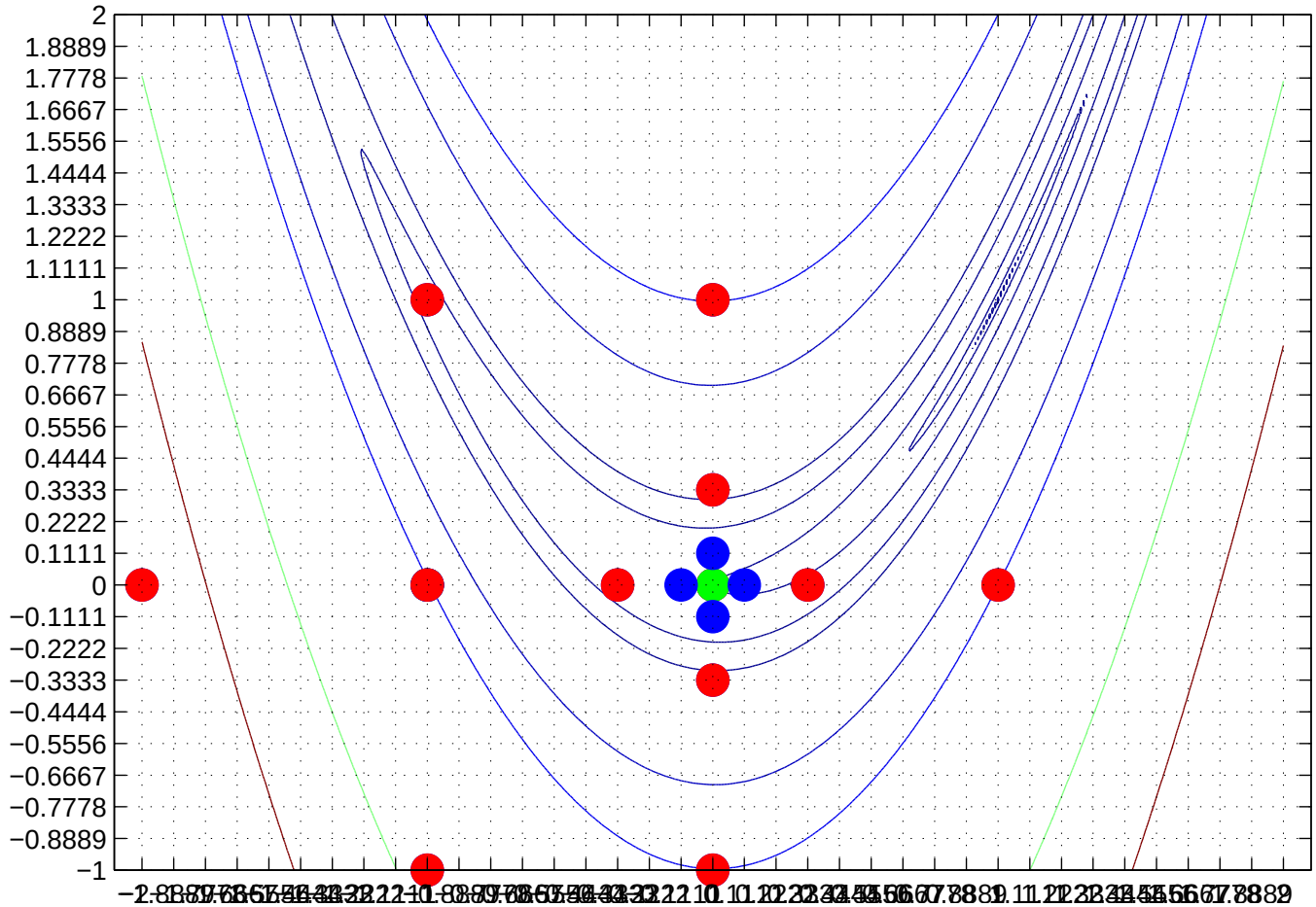

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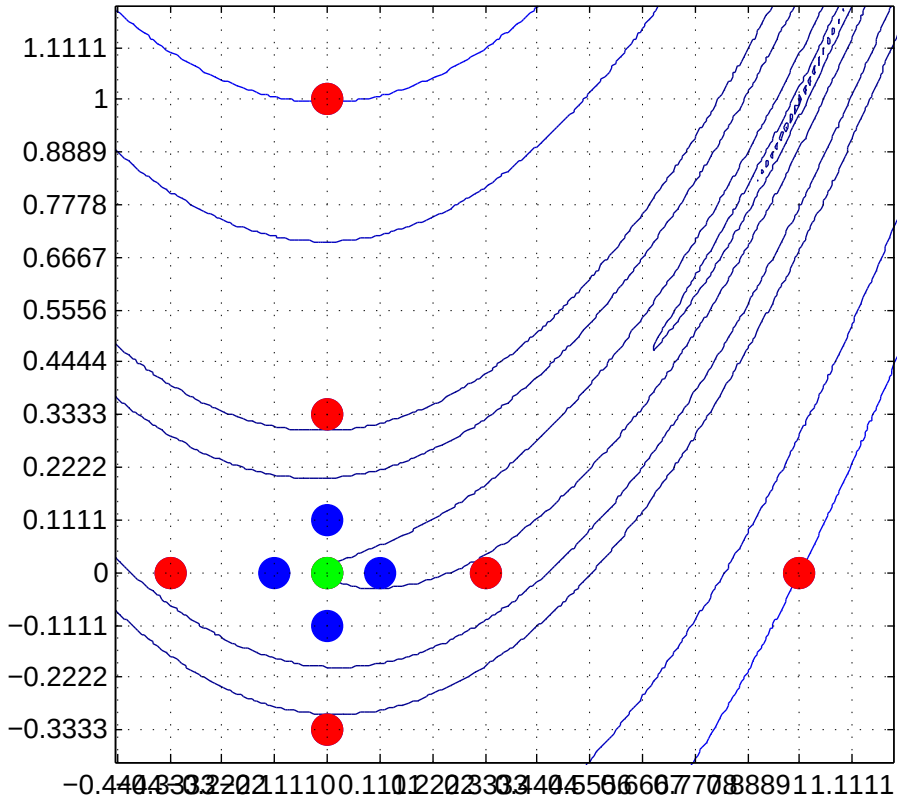

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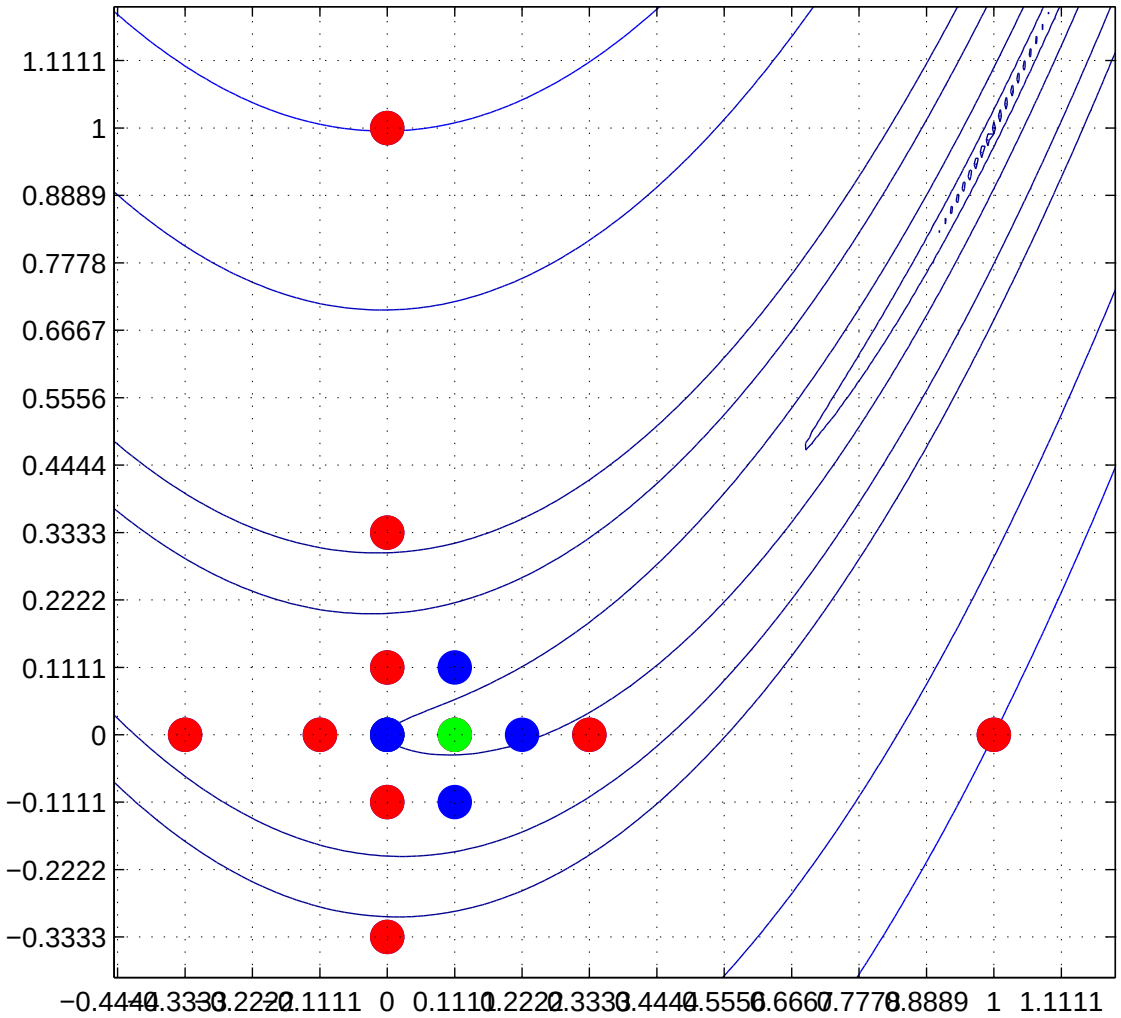

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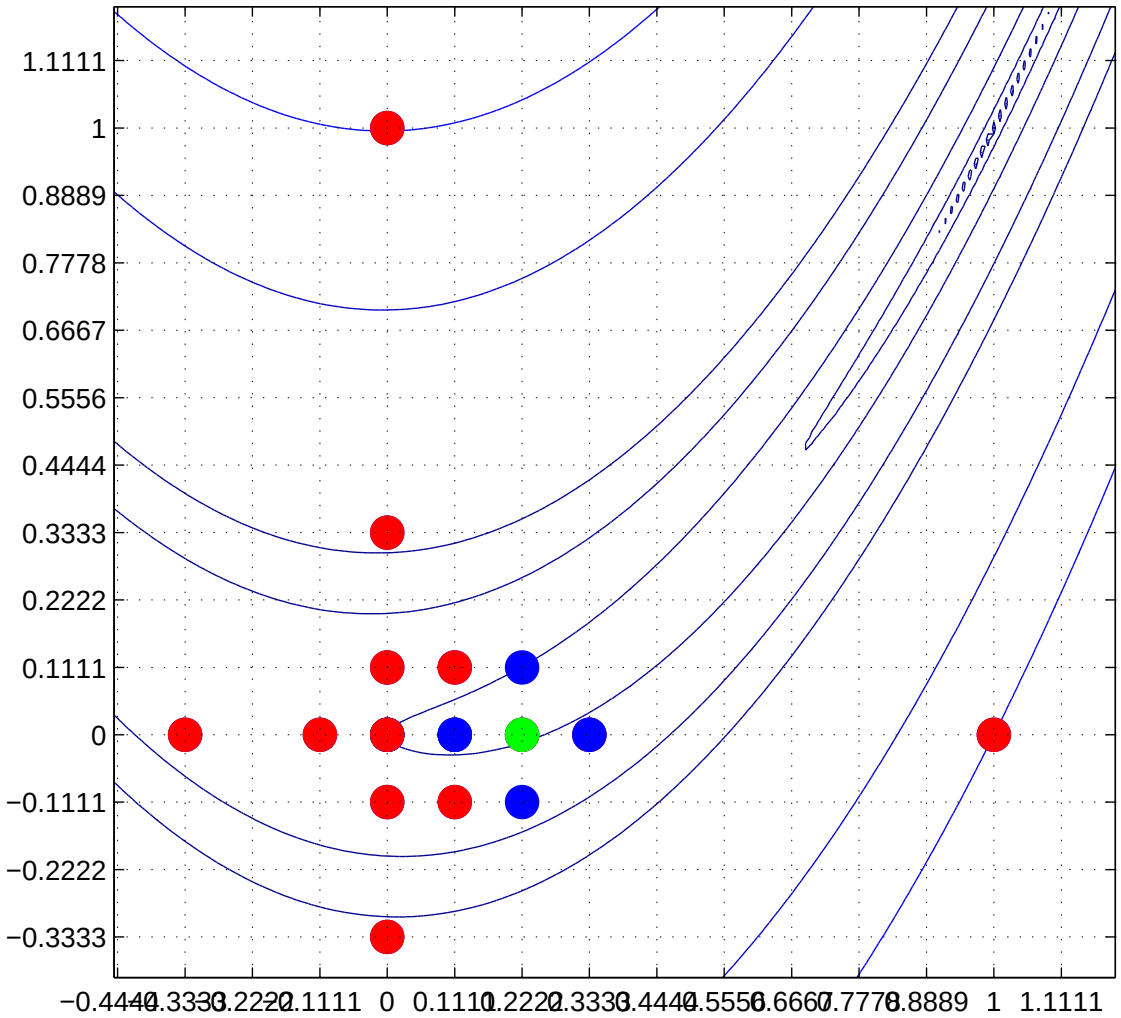

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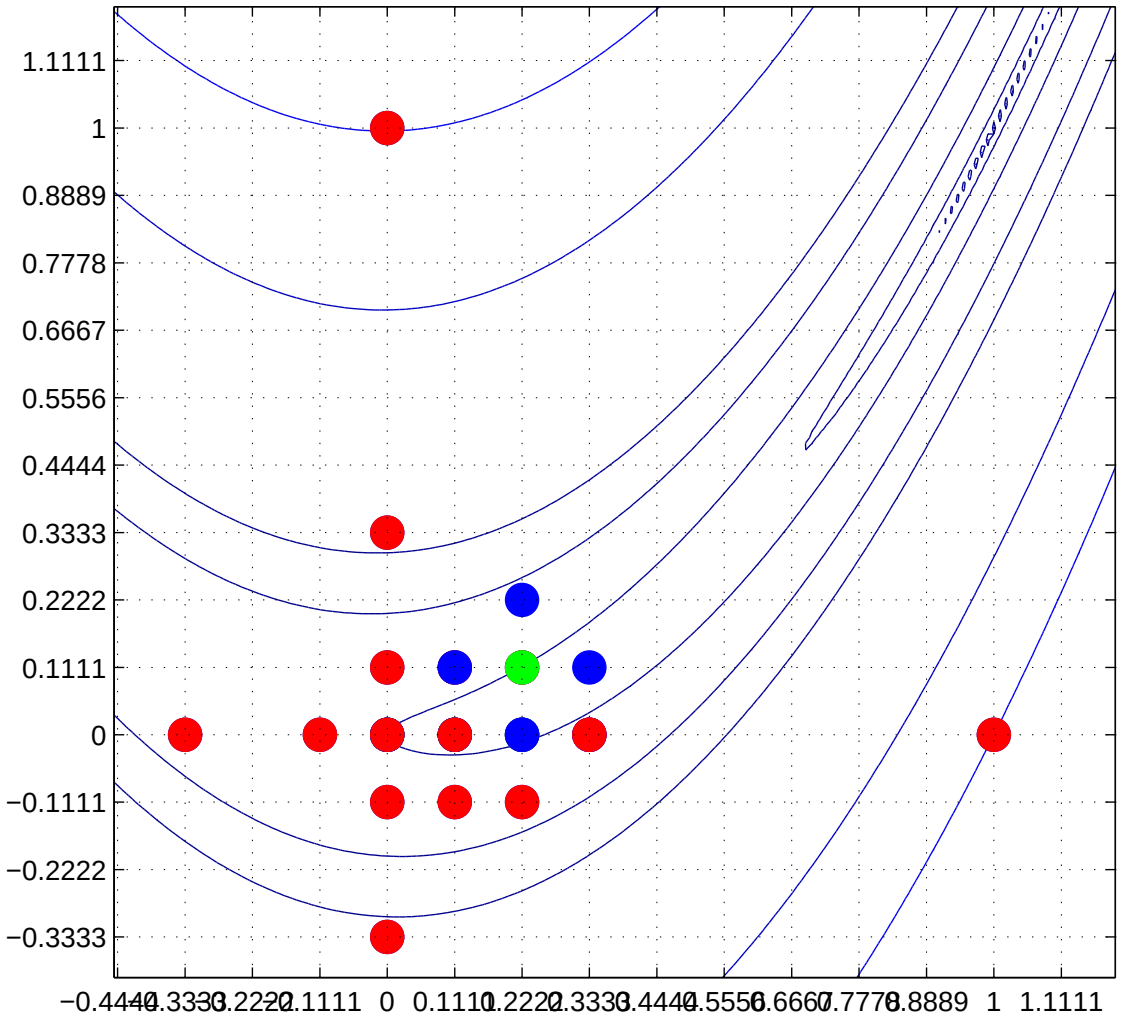

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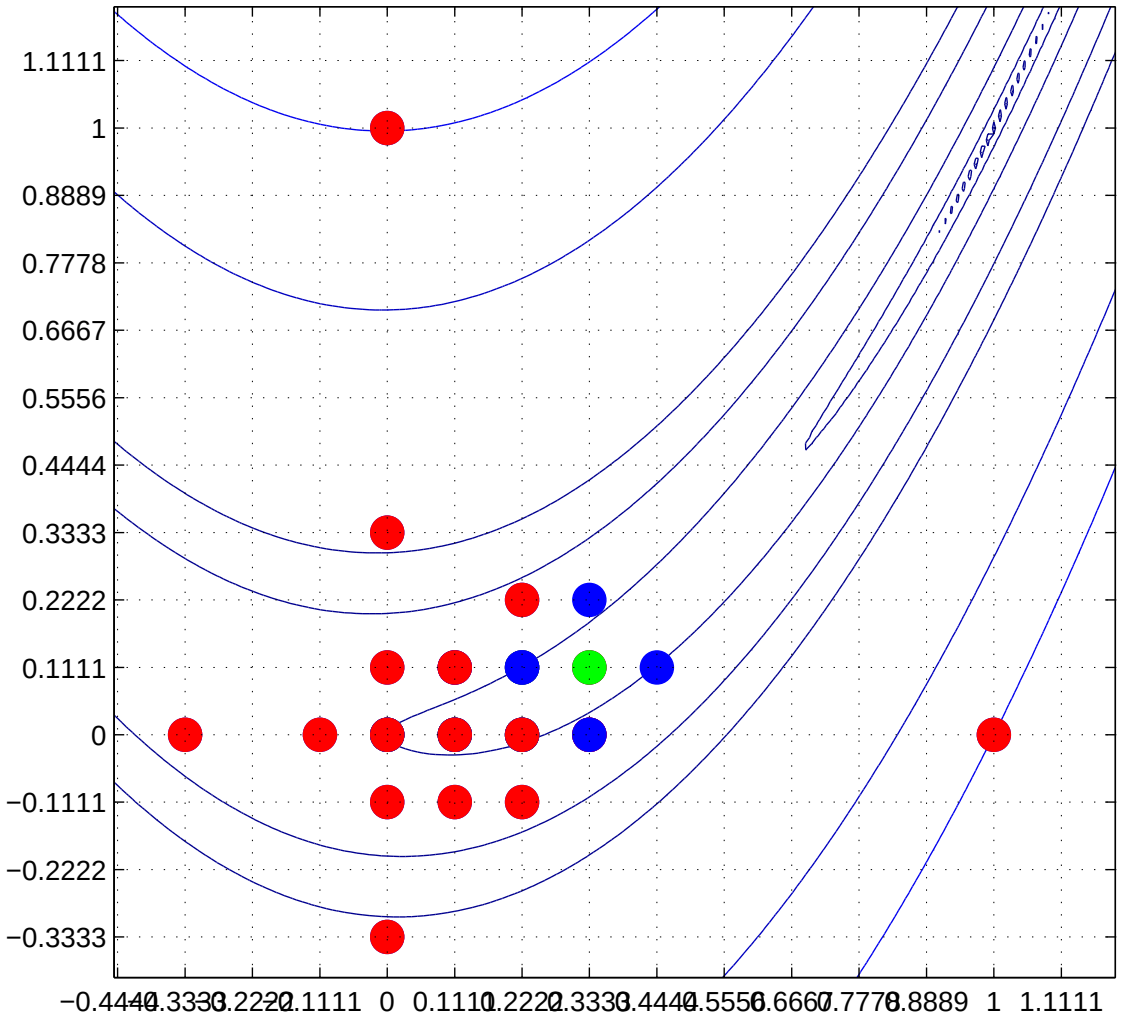

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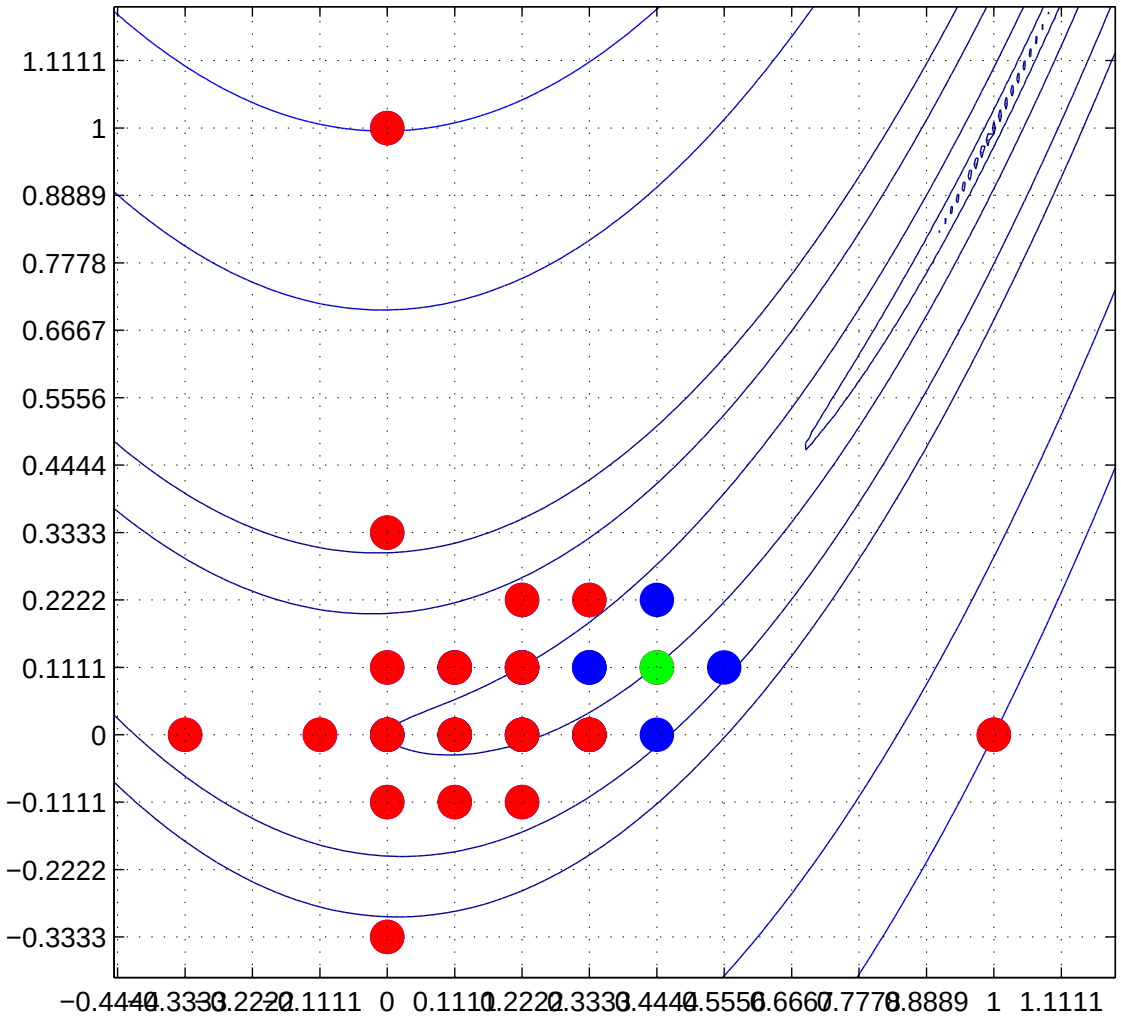

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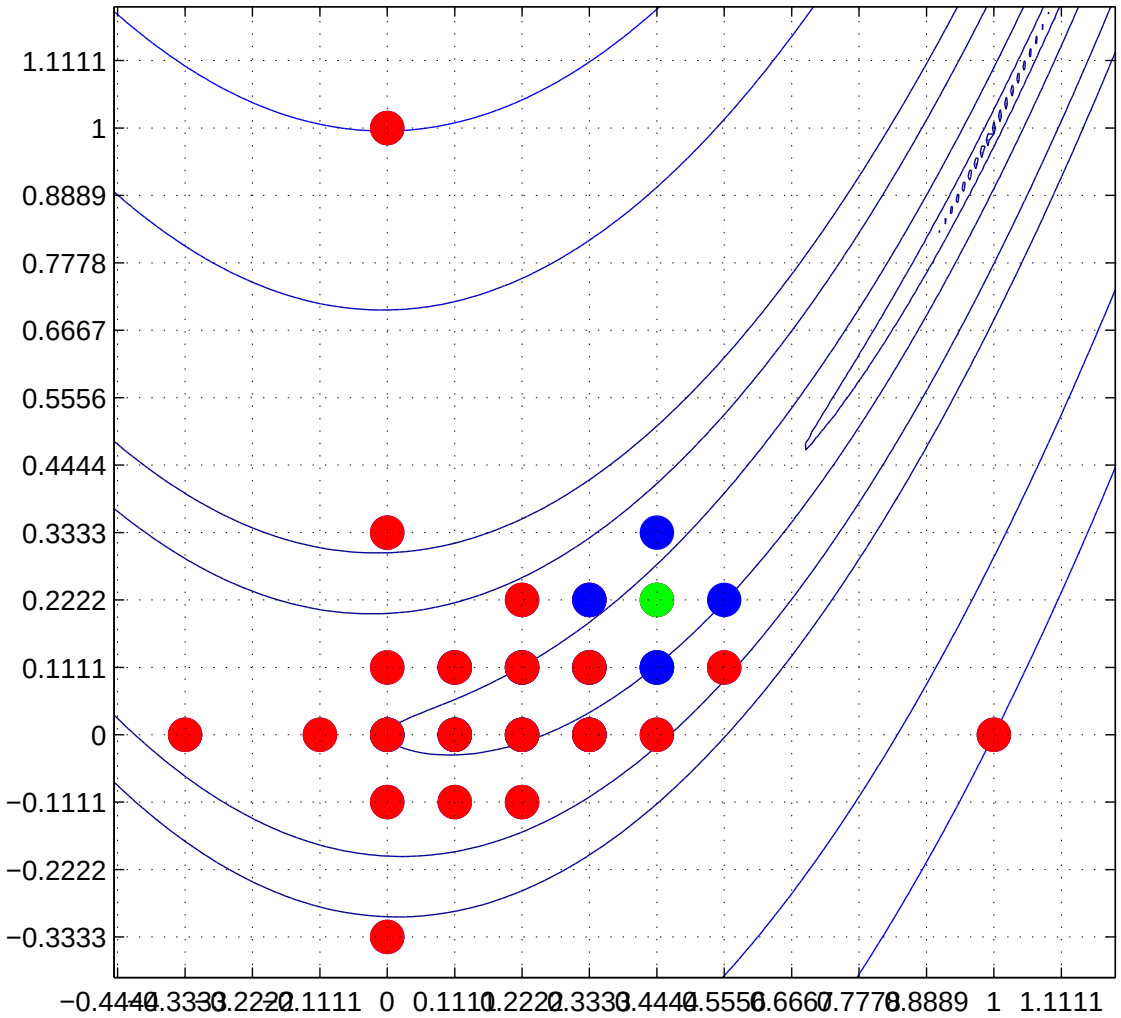

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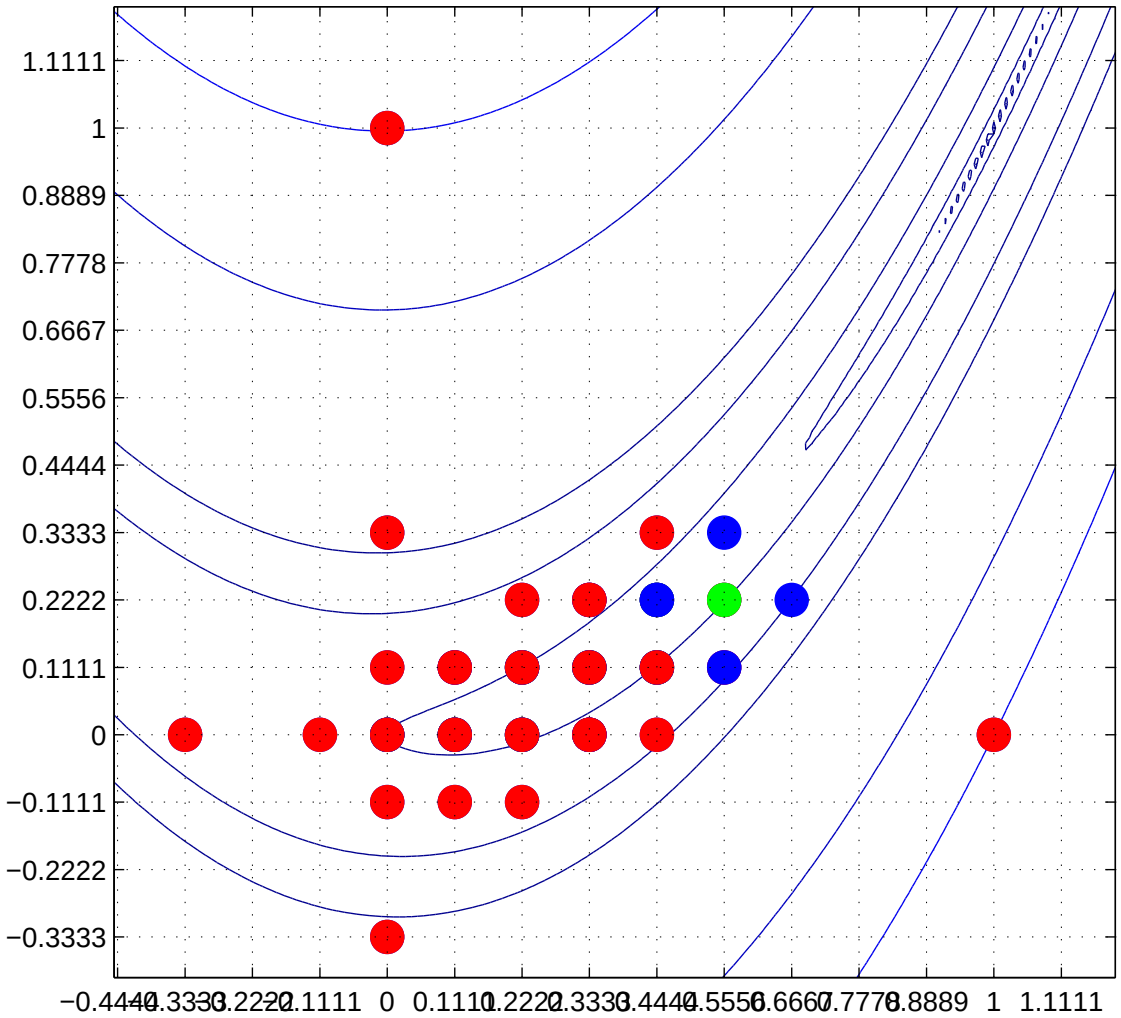

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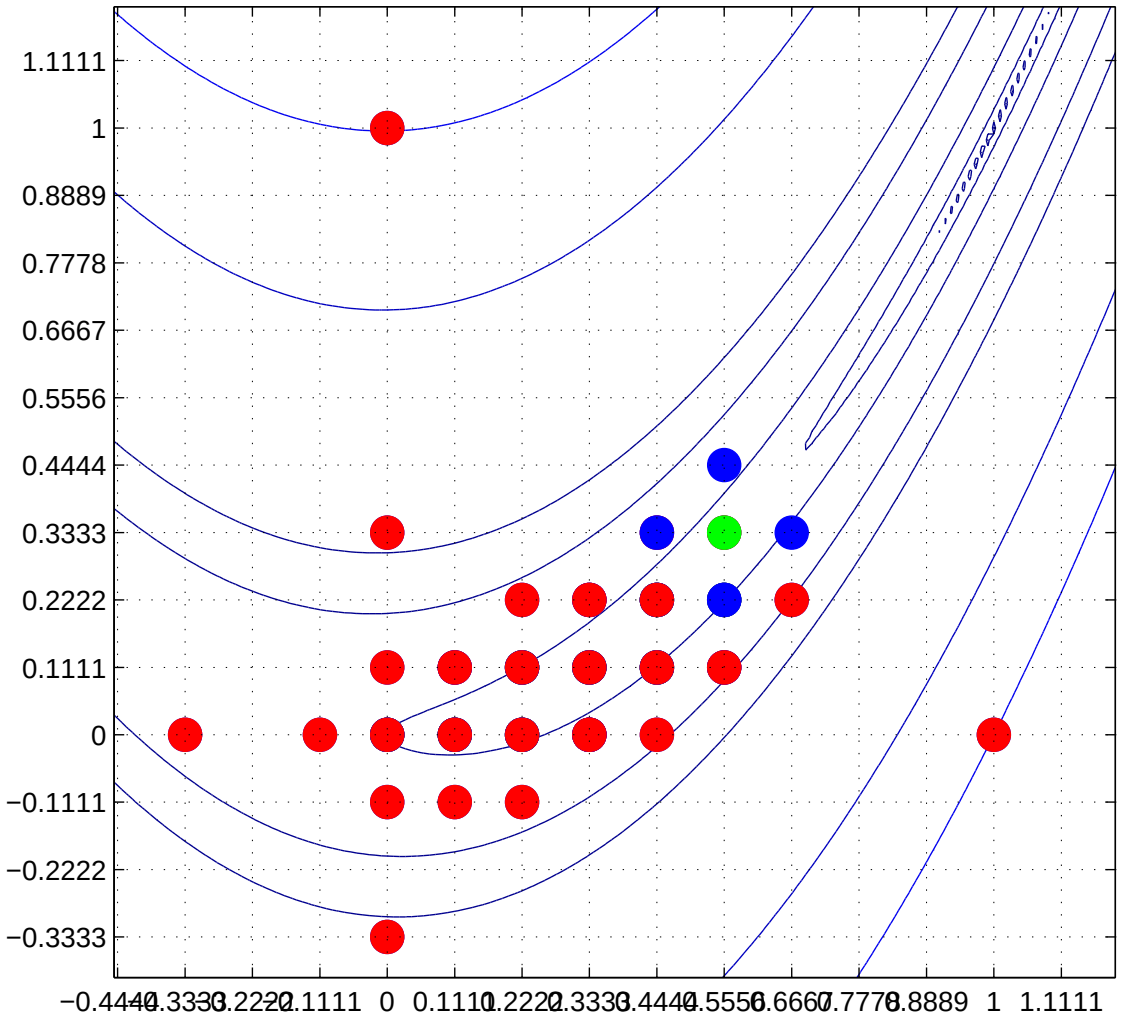

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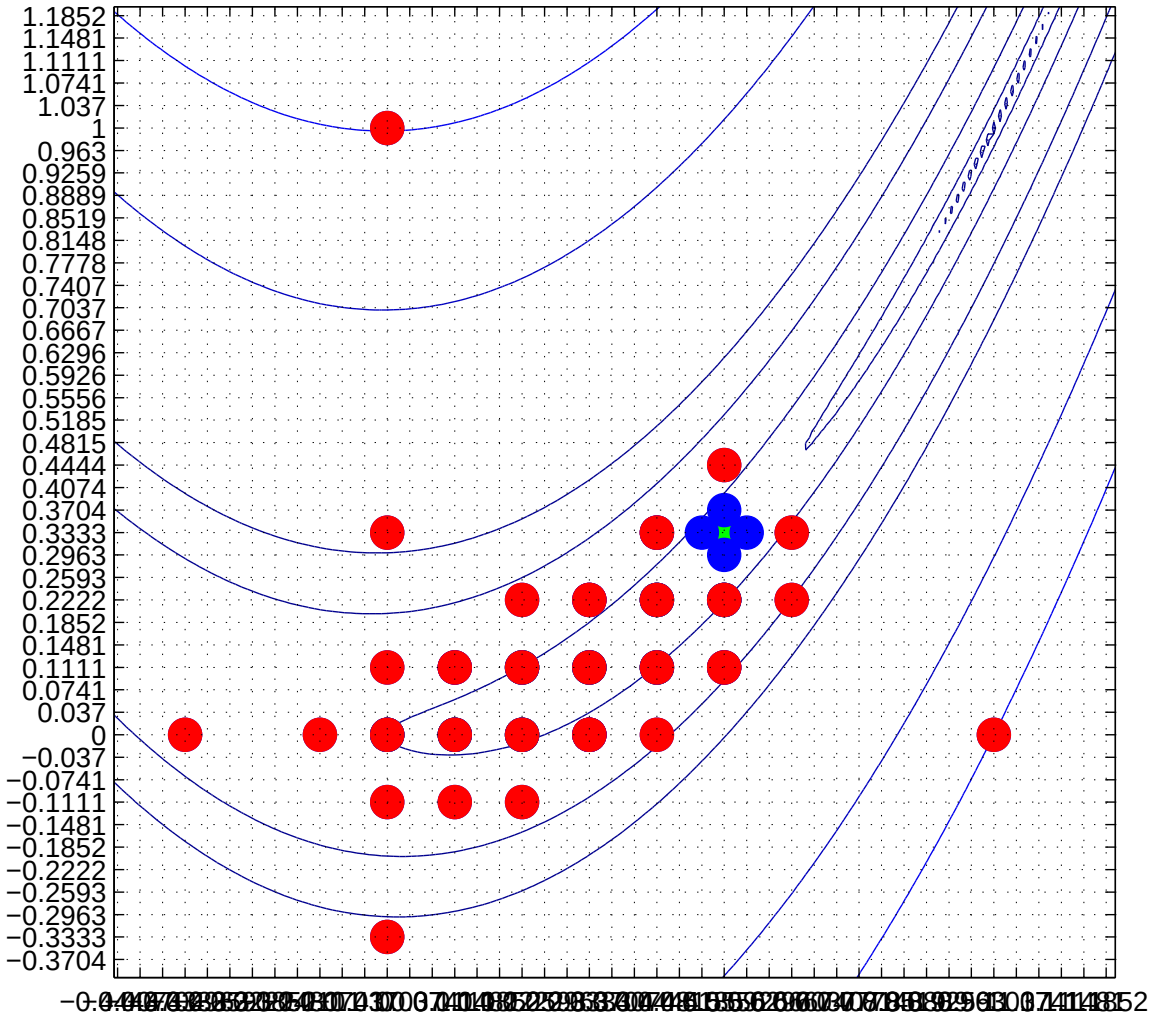

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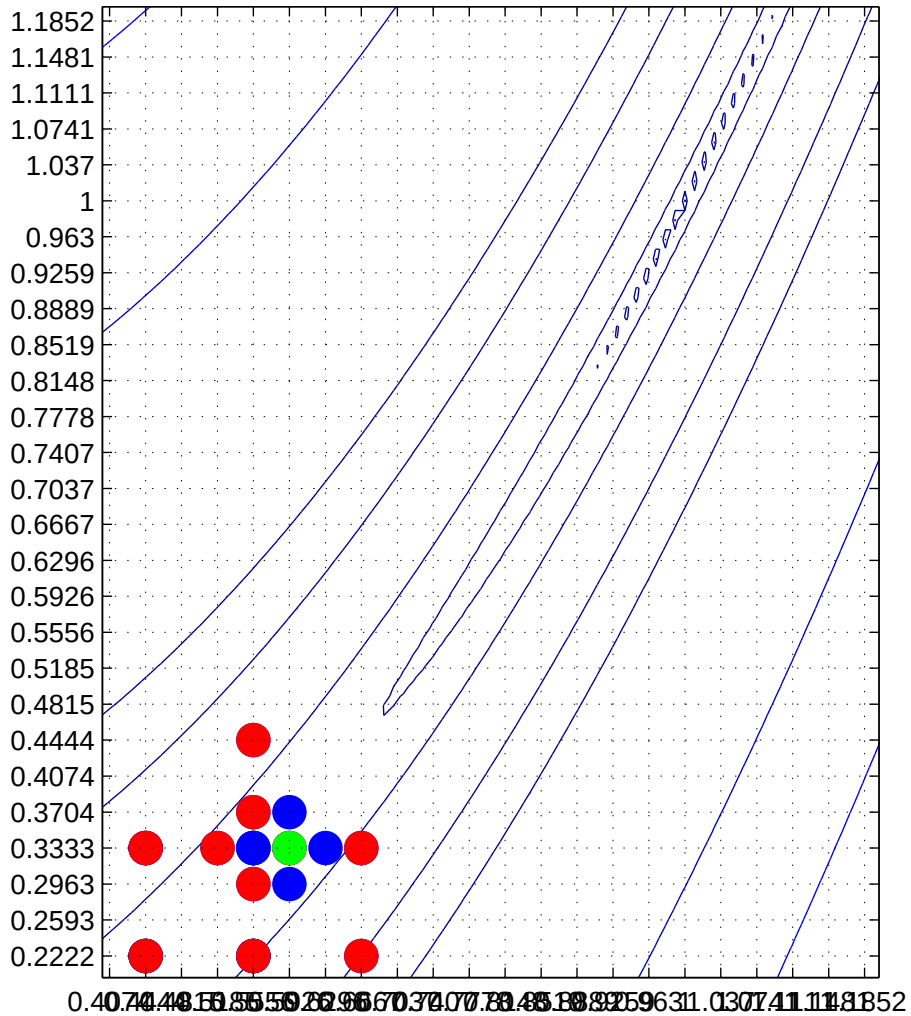

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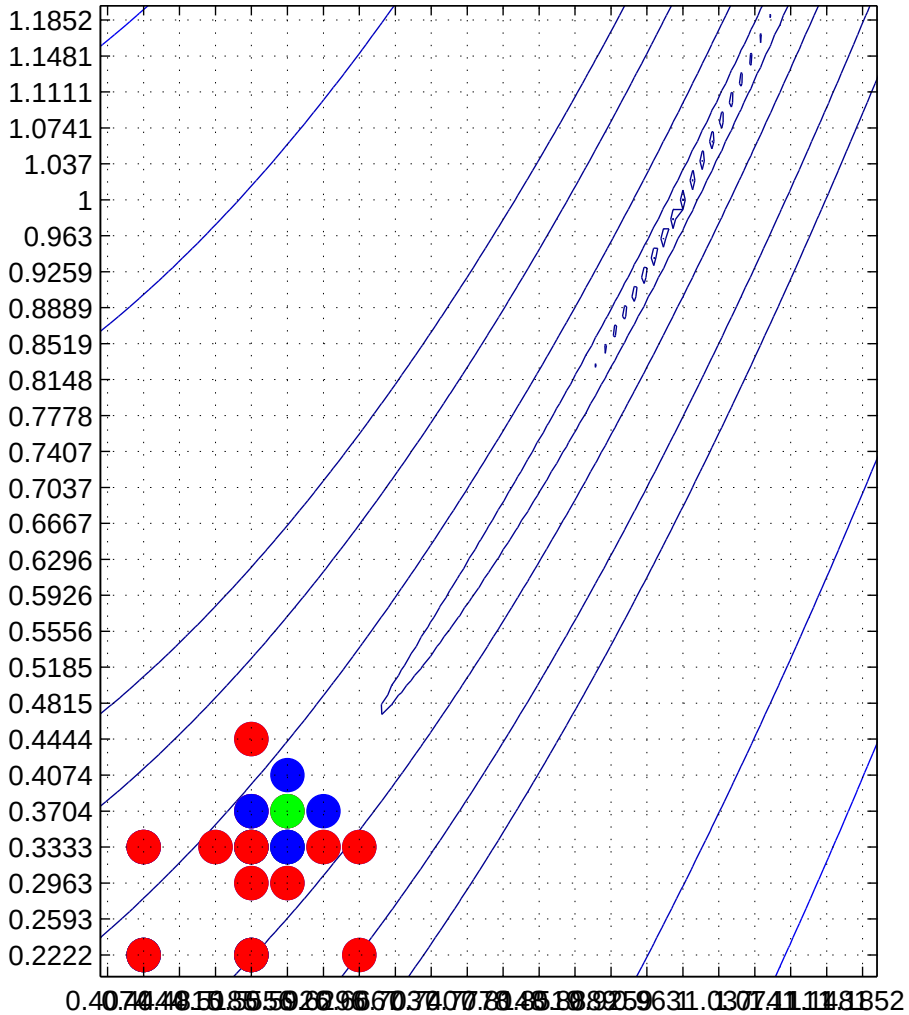

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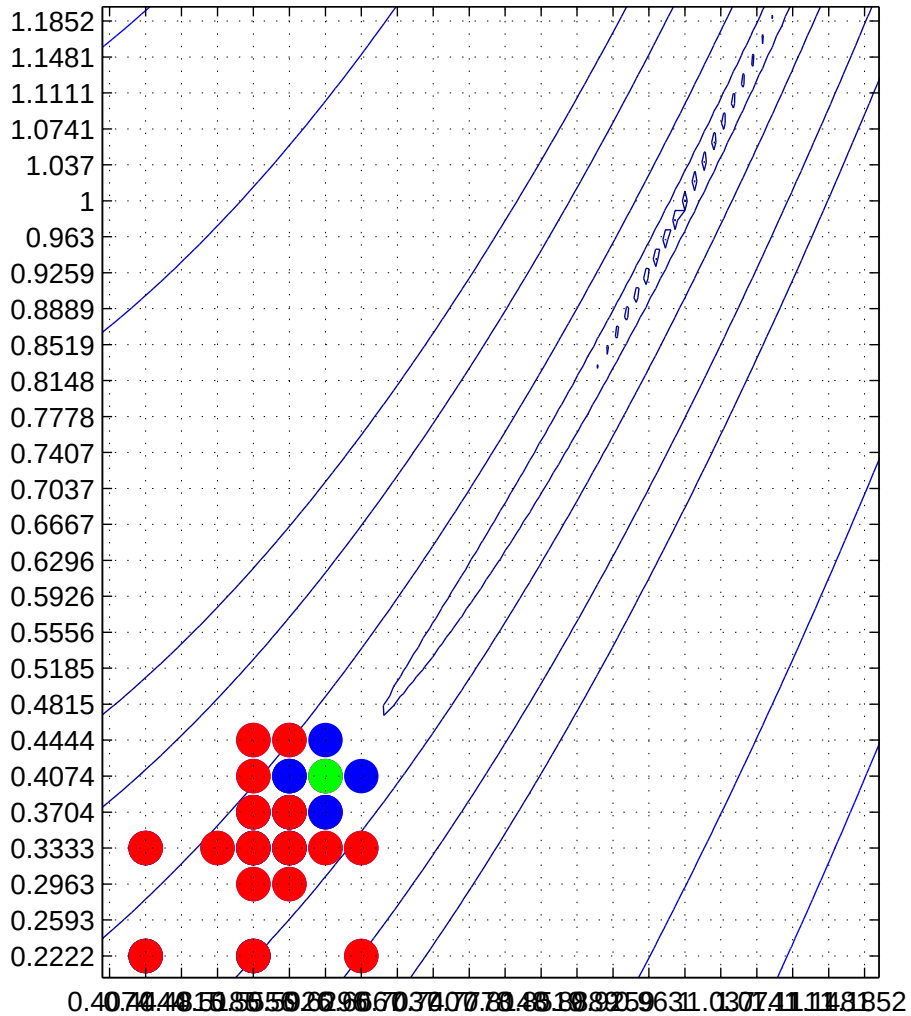

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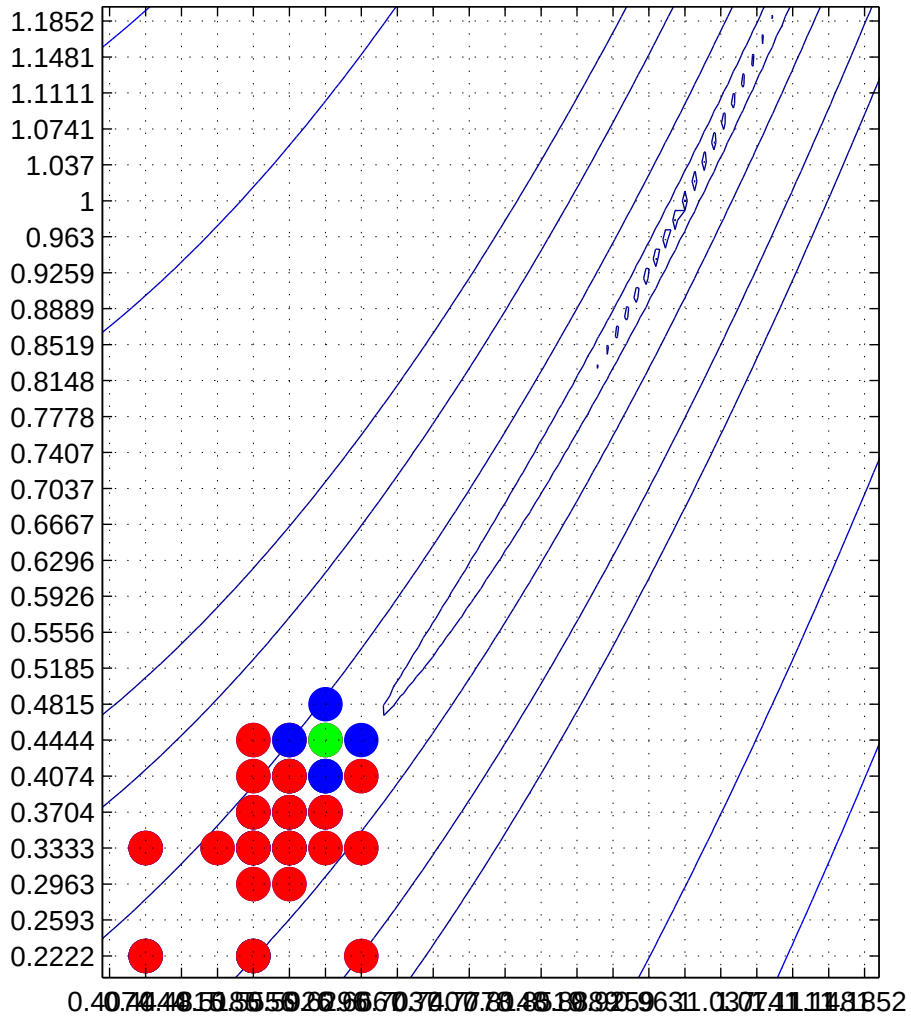

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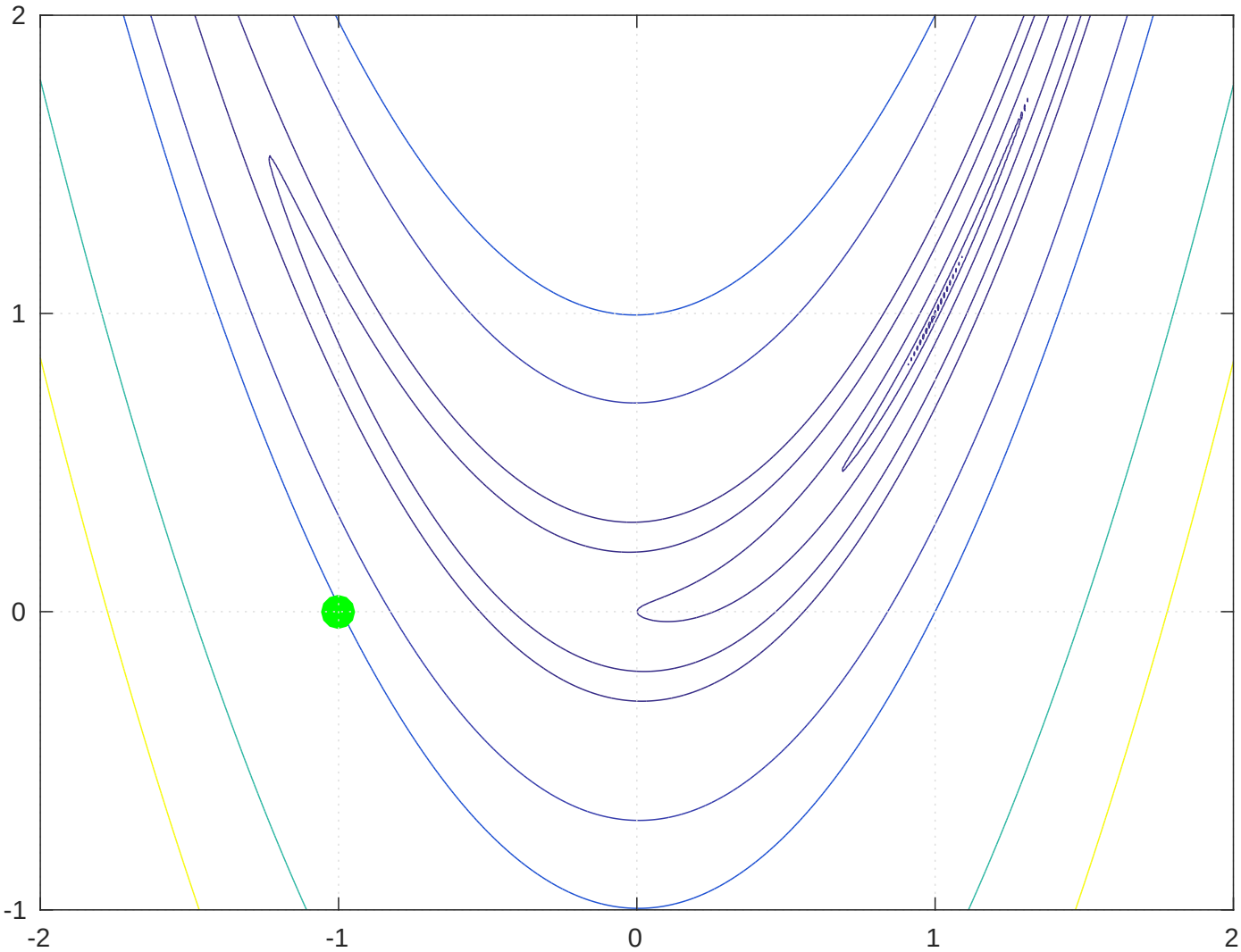

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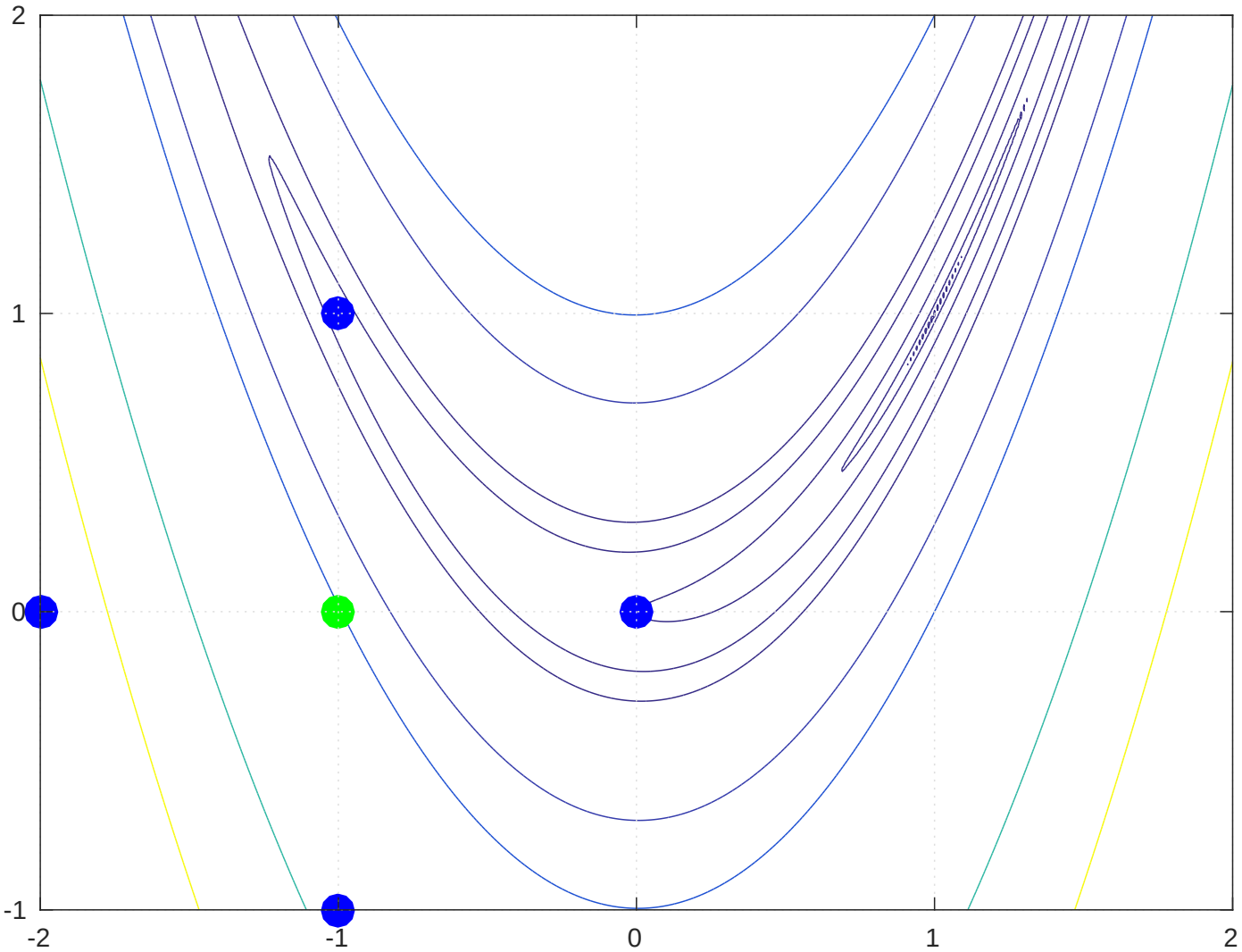

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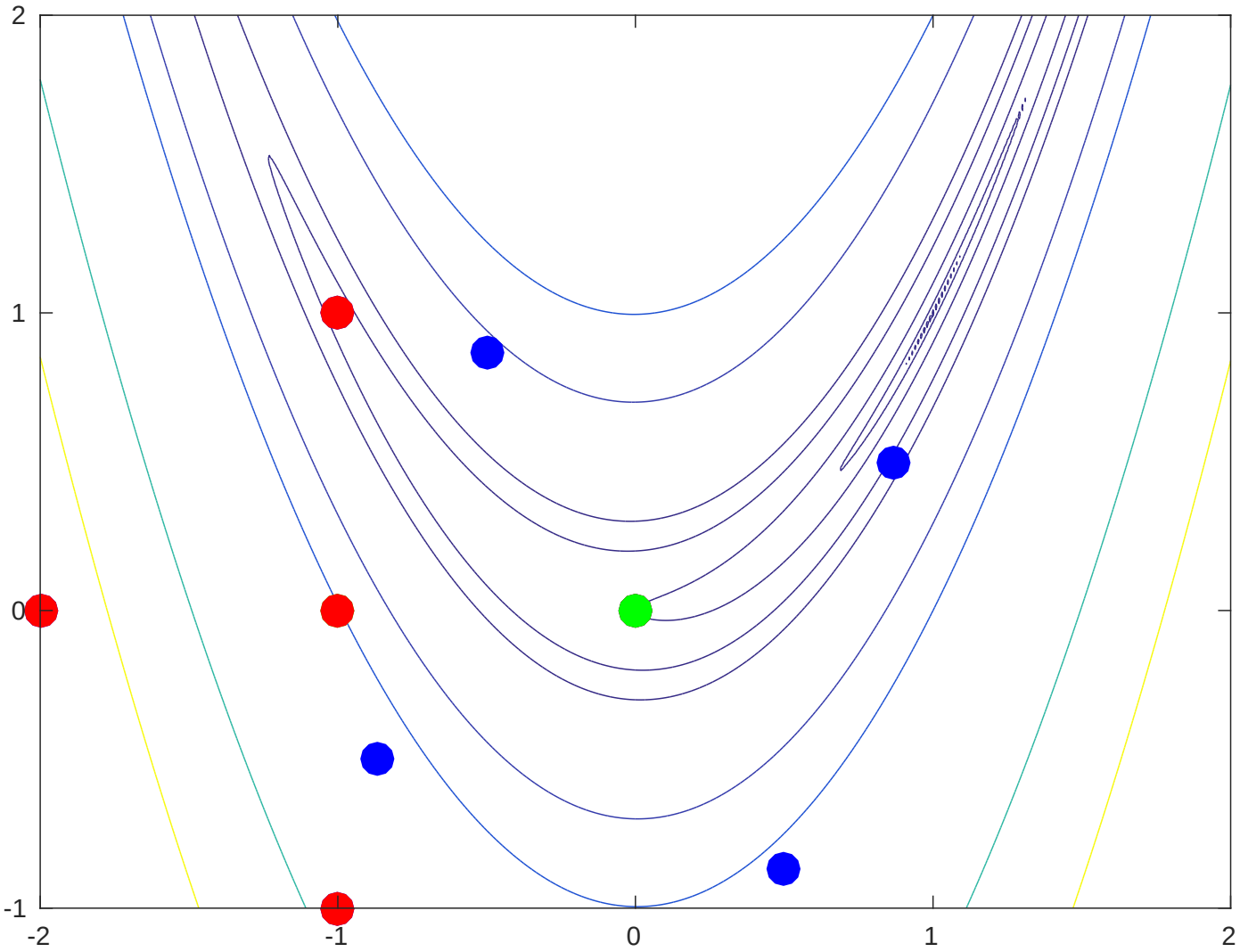

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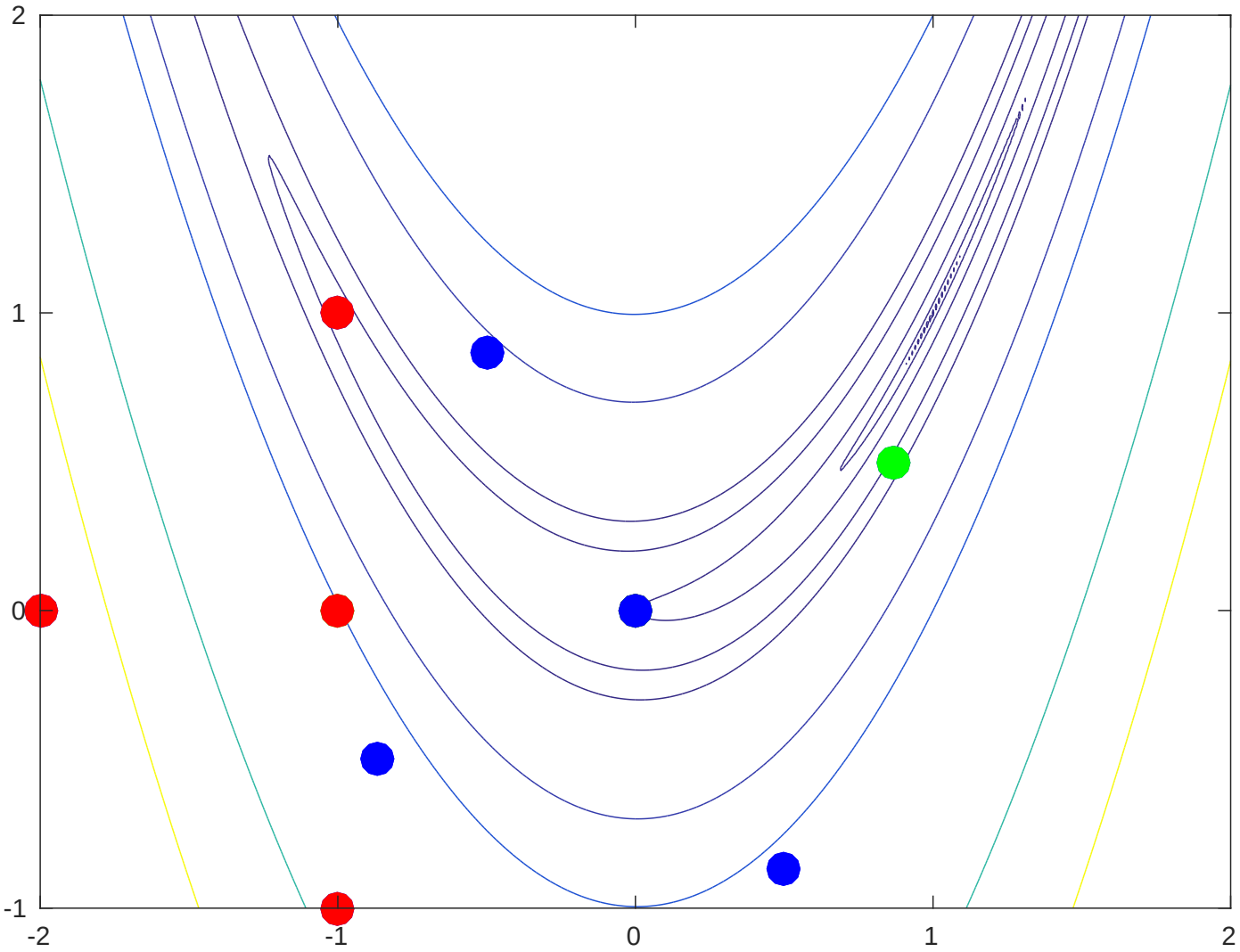

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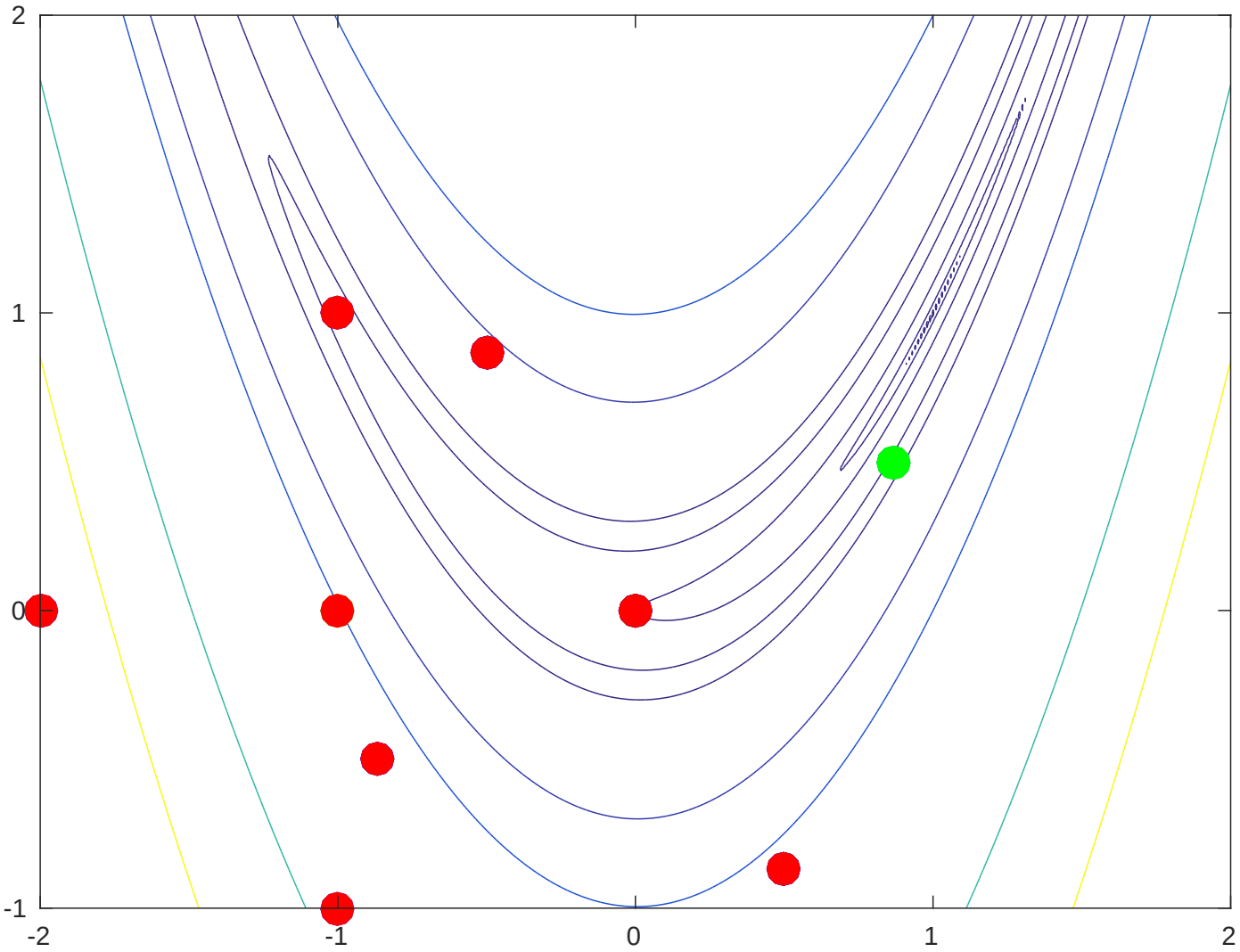

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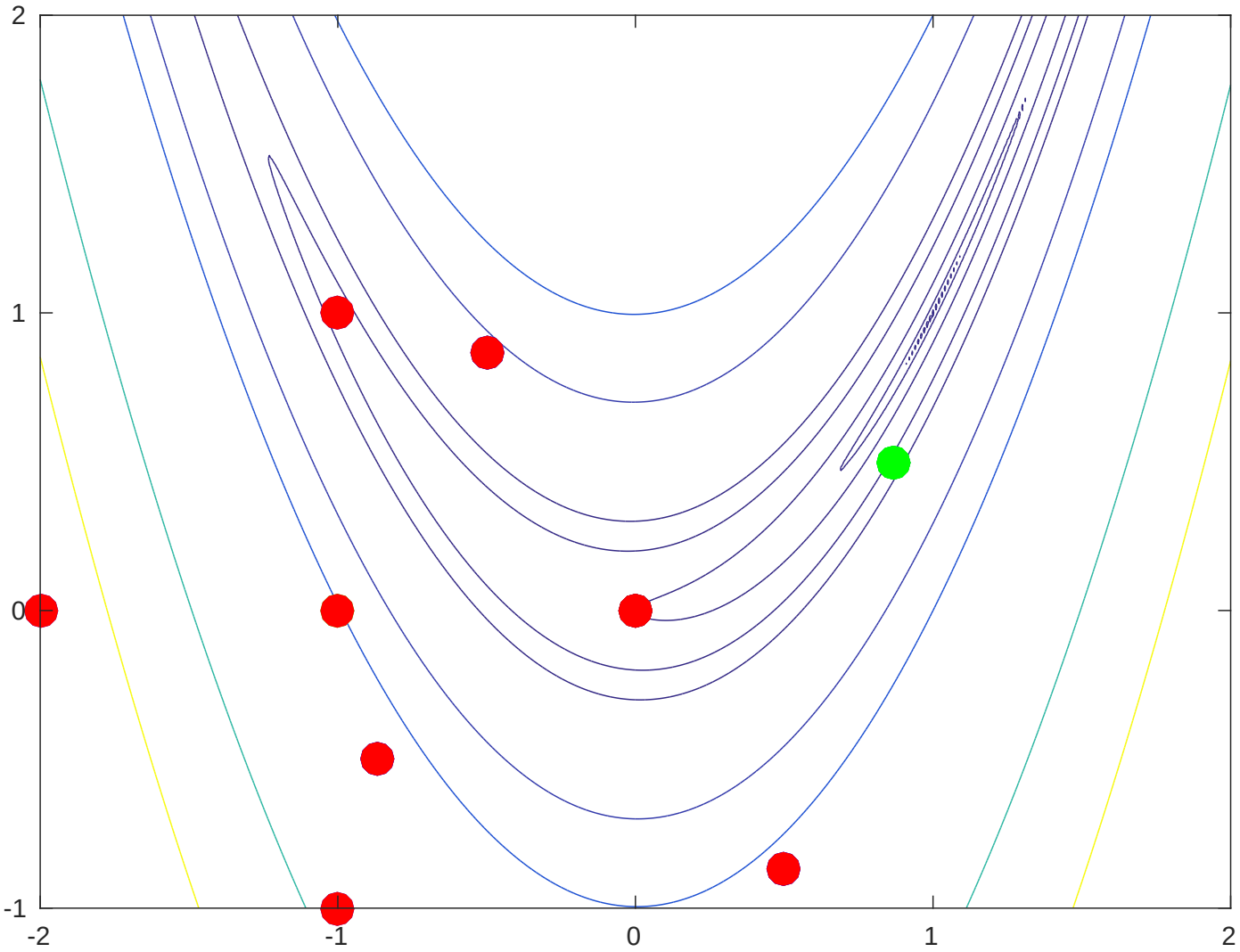

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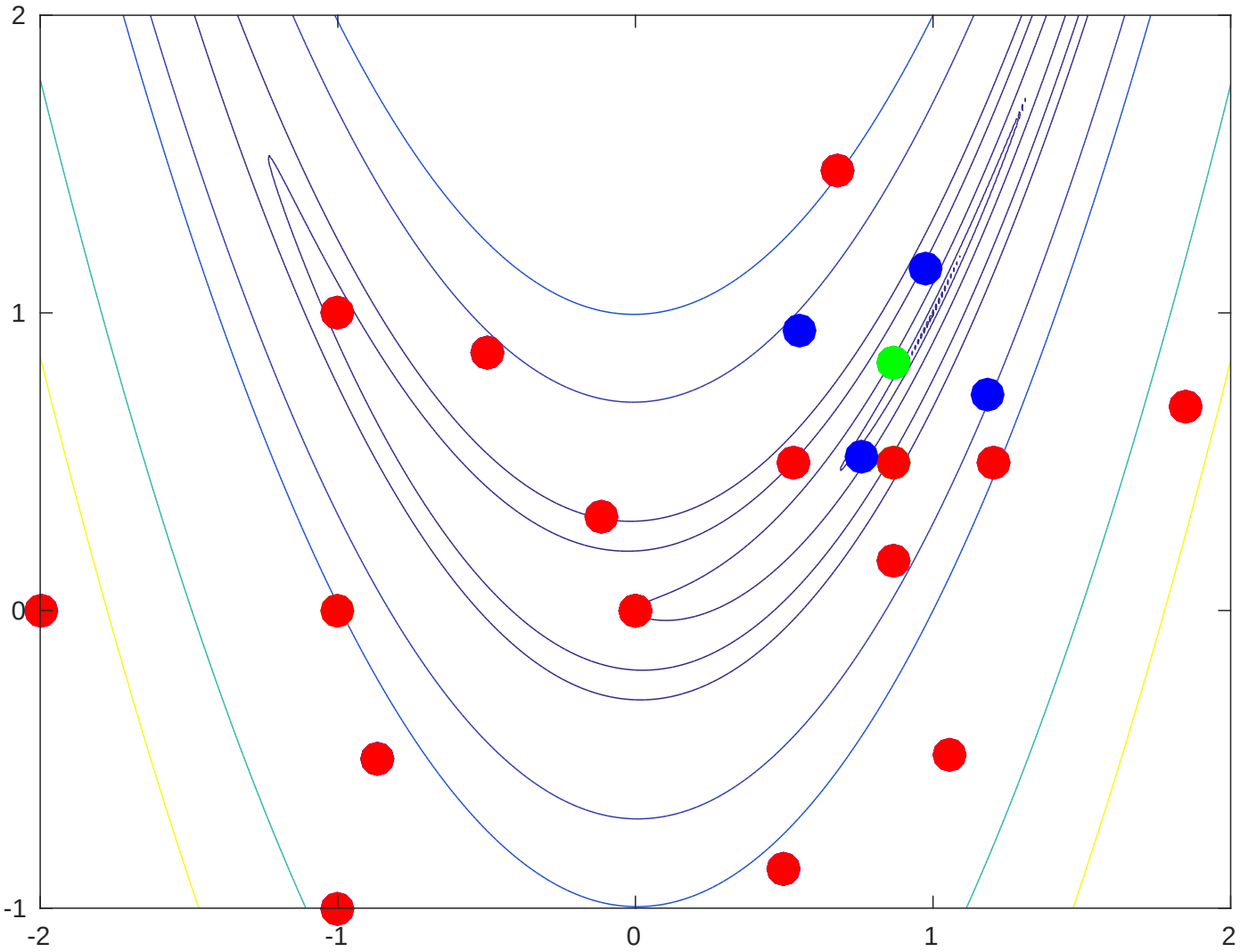


Bound constraints, integer or lattice variables

Bound constraints

- detect which bounds are nearly active
- force their normals to belong to the set of poll directions
- include one-sided or truncated poll search

Integer or lattice variables

- align the initial grid with the integer grid
- avoid shrinking and rotations
- recursively explore a local tree of discrete subspaces
- keep track of record value in each such subspace to avoid re-exploration
- same thing if variables on a user-specified lattice

Additional algorithmic features:

- optional call of user-defined search step using all available objective function values
- accumulate successful descent directions (exploiting "inertia")
- optional user-defined variables' scaling
- provision for multilevel optimization

$$\min_x \max_y \min_z f(x, y, z)$$

with level-dependent bounds (equilibrium/game theory computations)

- incomplete function evaluations (crucial for training)
- flexible termination rules (including objective-function target)
- BFGS finish (for smooth problems)
- allows randomized termination test

Additional implementation features

- check-pointing at user-specified frequency
- allows objective functions with user-defined parameters
- very flexible keyword-based calling sequence
- MATLAB code (single file)
- direct CUTEst interface (for those interested)

Algorithmic parameters

User may specify (amongst others):

- grid shrinking/expansion factors
- inertia for defining progress directions
- initial scale in continuous variables
- local tree-search strategy (depth-first vs breadth-first)

(7 algorithmic parameters in total)

BFO self tuning

BFO has been self-tuned!

- on a large set of test problems (CUTEst) with continuous and mixed-integer variables
- using both the average and robust tuning strategies
- for all 7 algorithmic parameters

Outcome :

- robust strategy slightly better
- gains in performance of
  - 30% for continuous problems
  - 19% for mixed-integer problems

  compared with "intuitively reasonable values"
- *very* competitive with NOMAD (state-of-the-art pattern search algo)

And then...

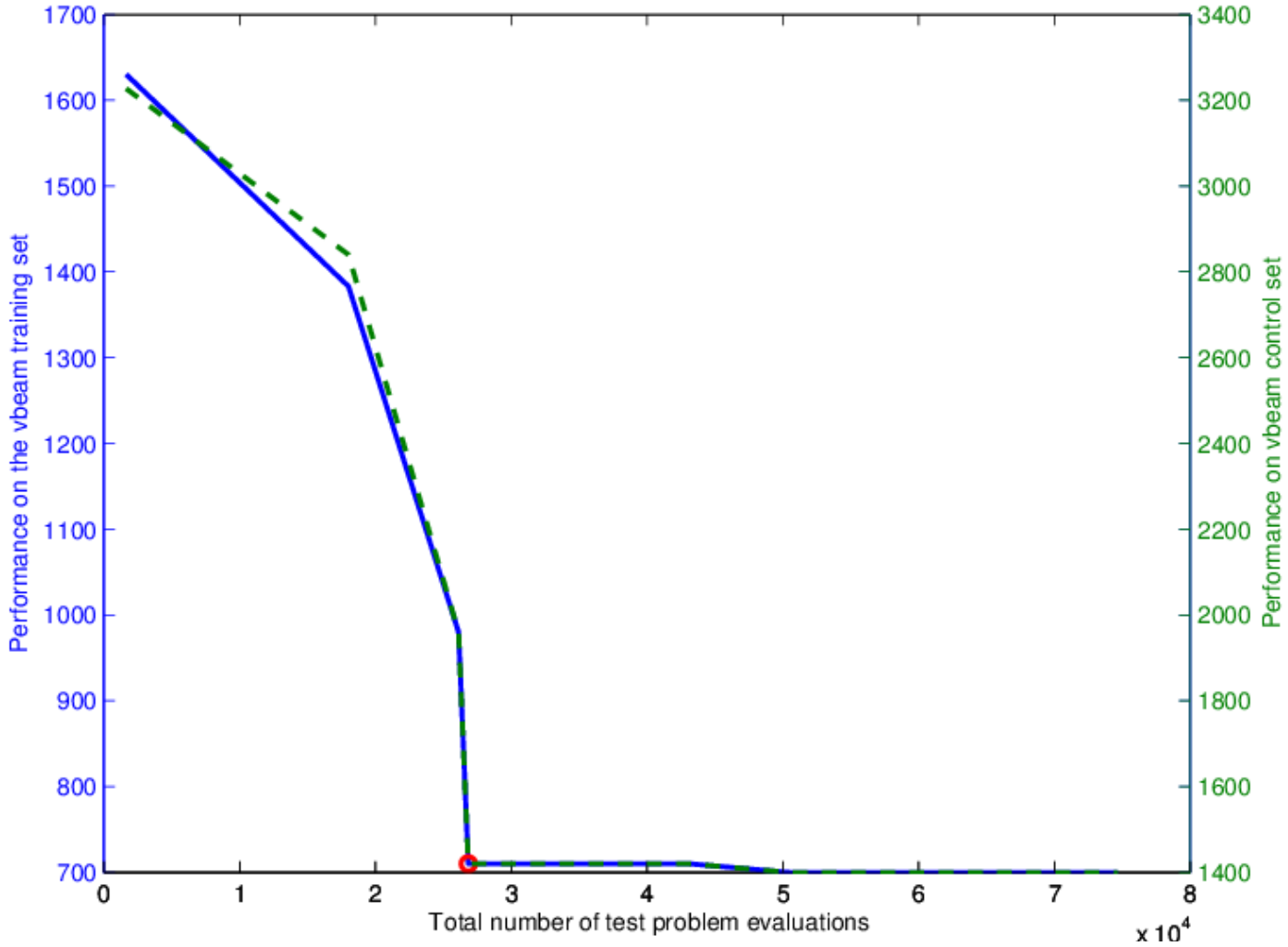
... the algorithm designer is (hopefully) happy !
But what about the user (with his/her own specific problems)?

BFO allows training by the user for specific problem classes

Does this work? Experiment on 2 specific classes of (minimization) problems

- nonlinear nonconvex trajectory tracking least-squares
- nonconvex regularized cubic models

Trajectory tracking:
AO training, medium-low error deviation, low accuracy



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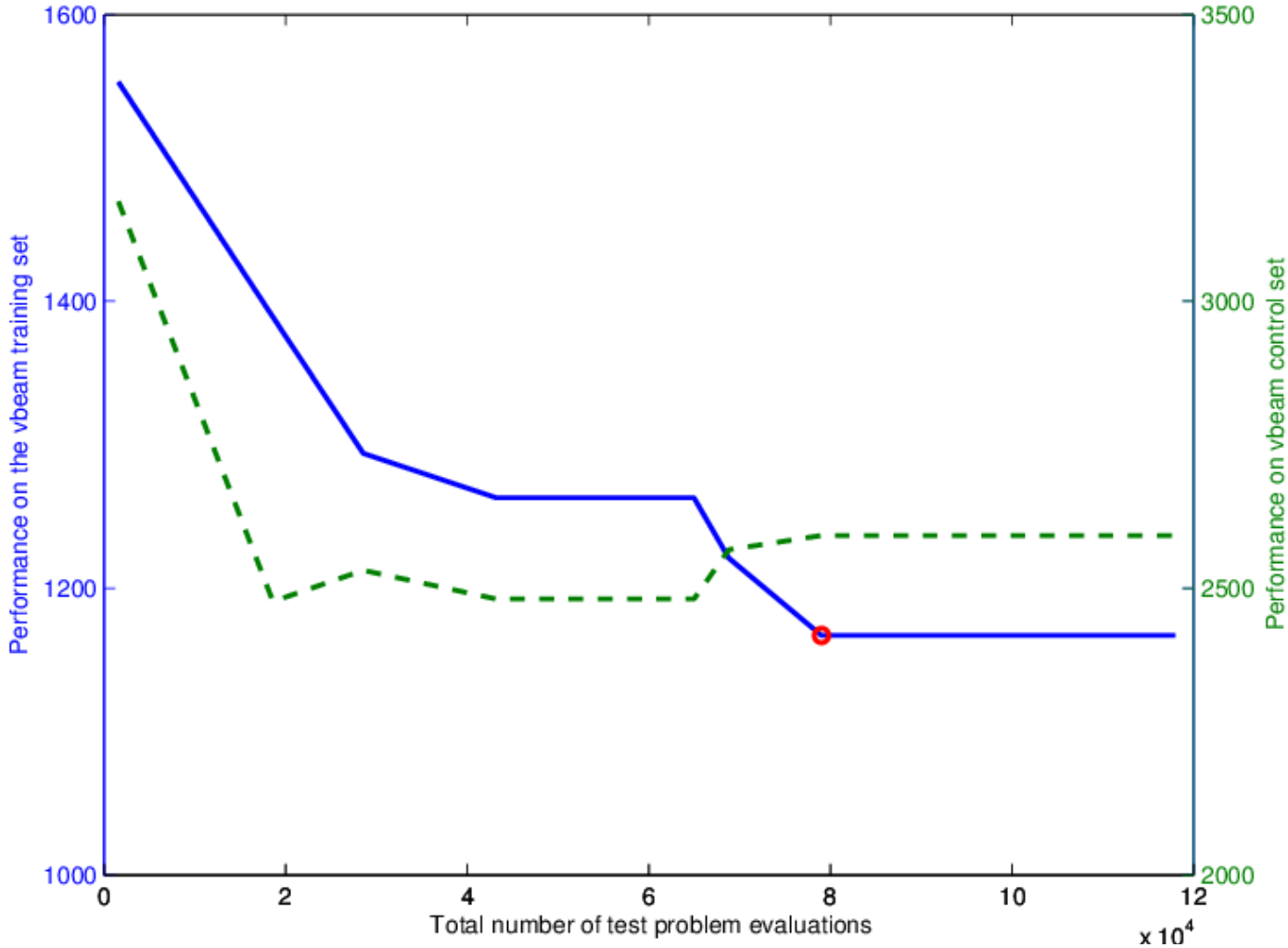
AO training, high error deviation, low accuracy



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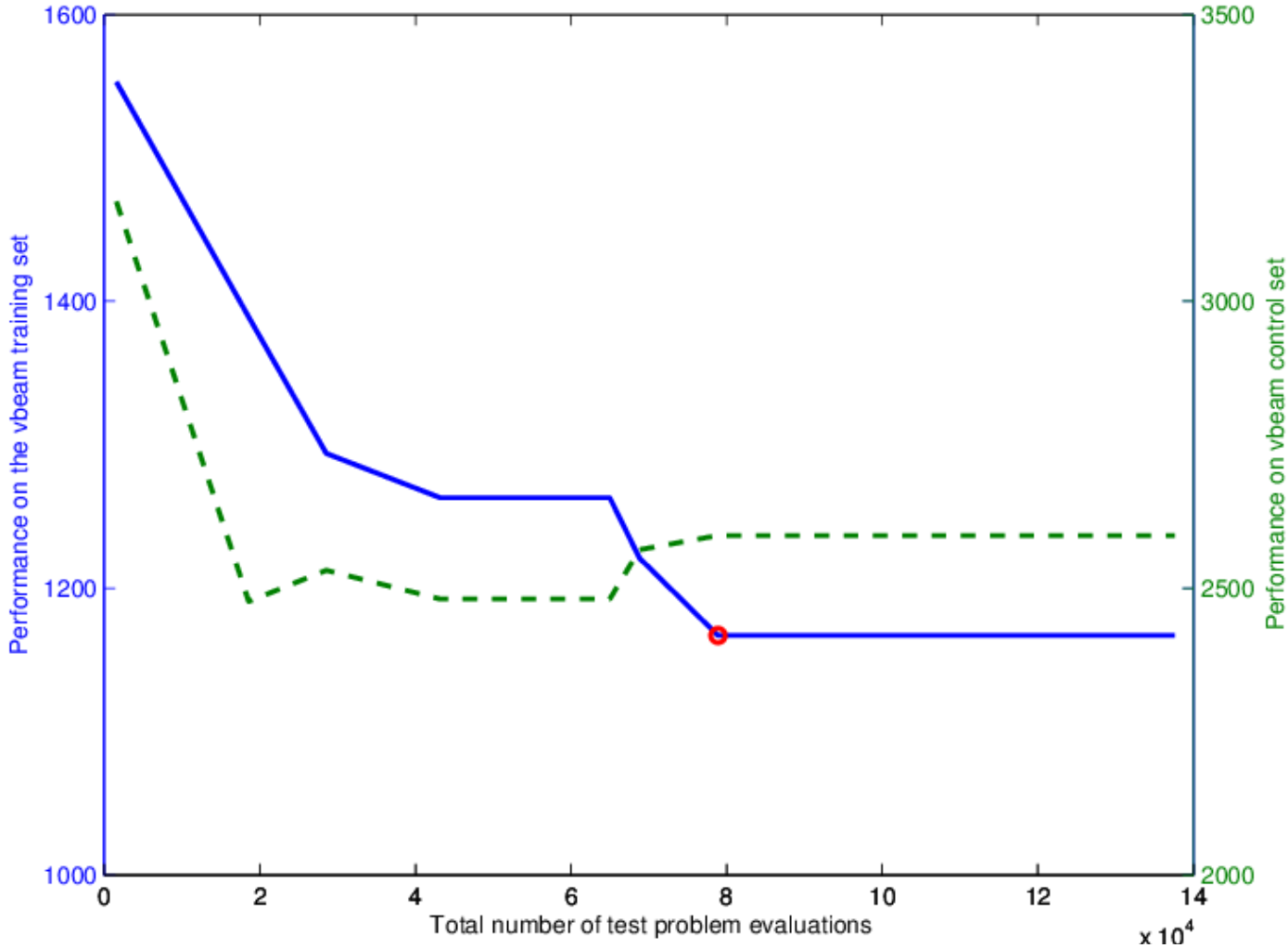
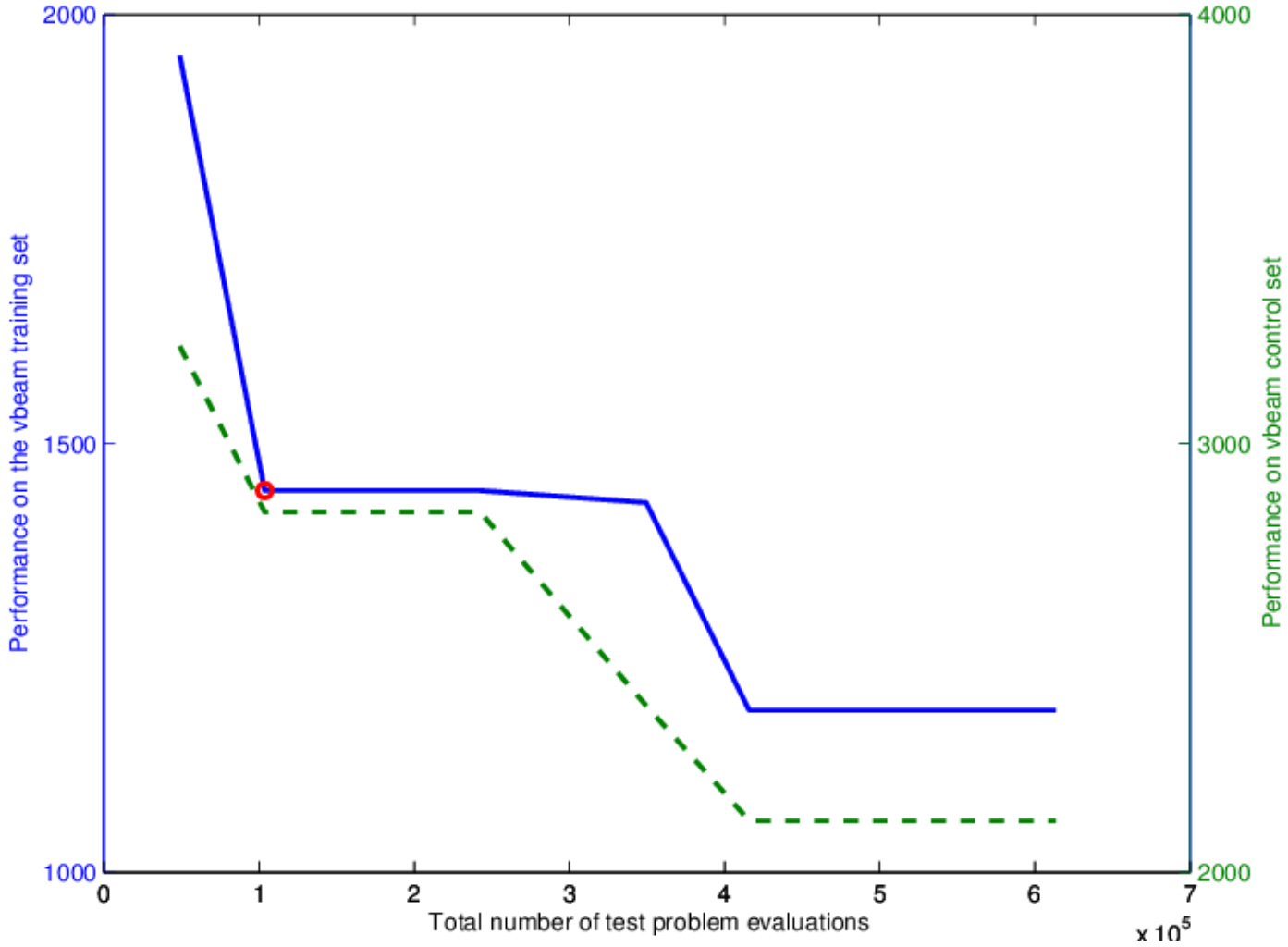


Trajectory tracking:
RO training, medium-low error deviation, low accuracy



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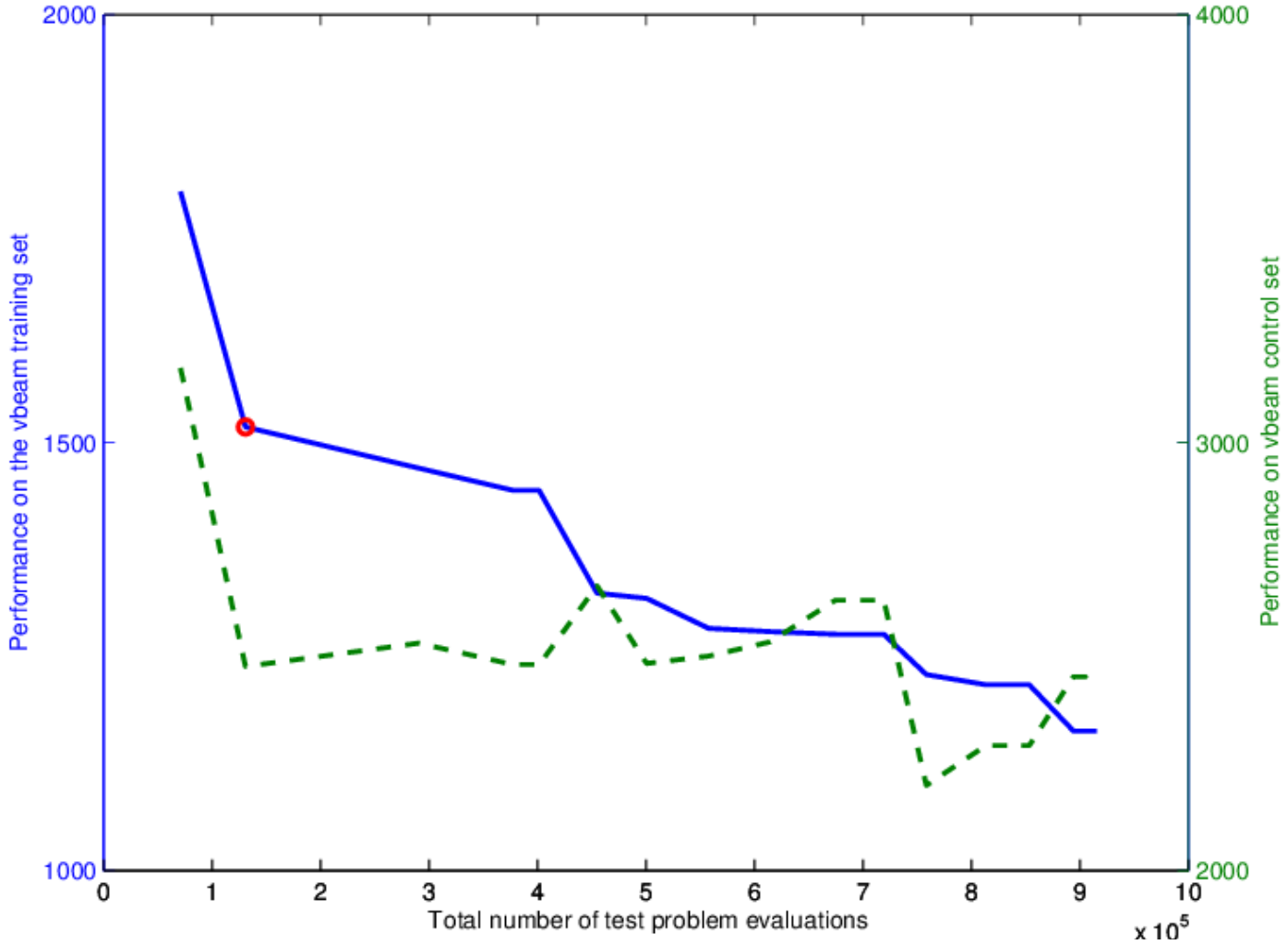

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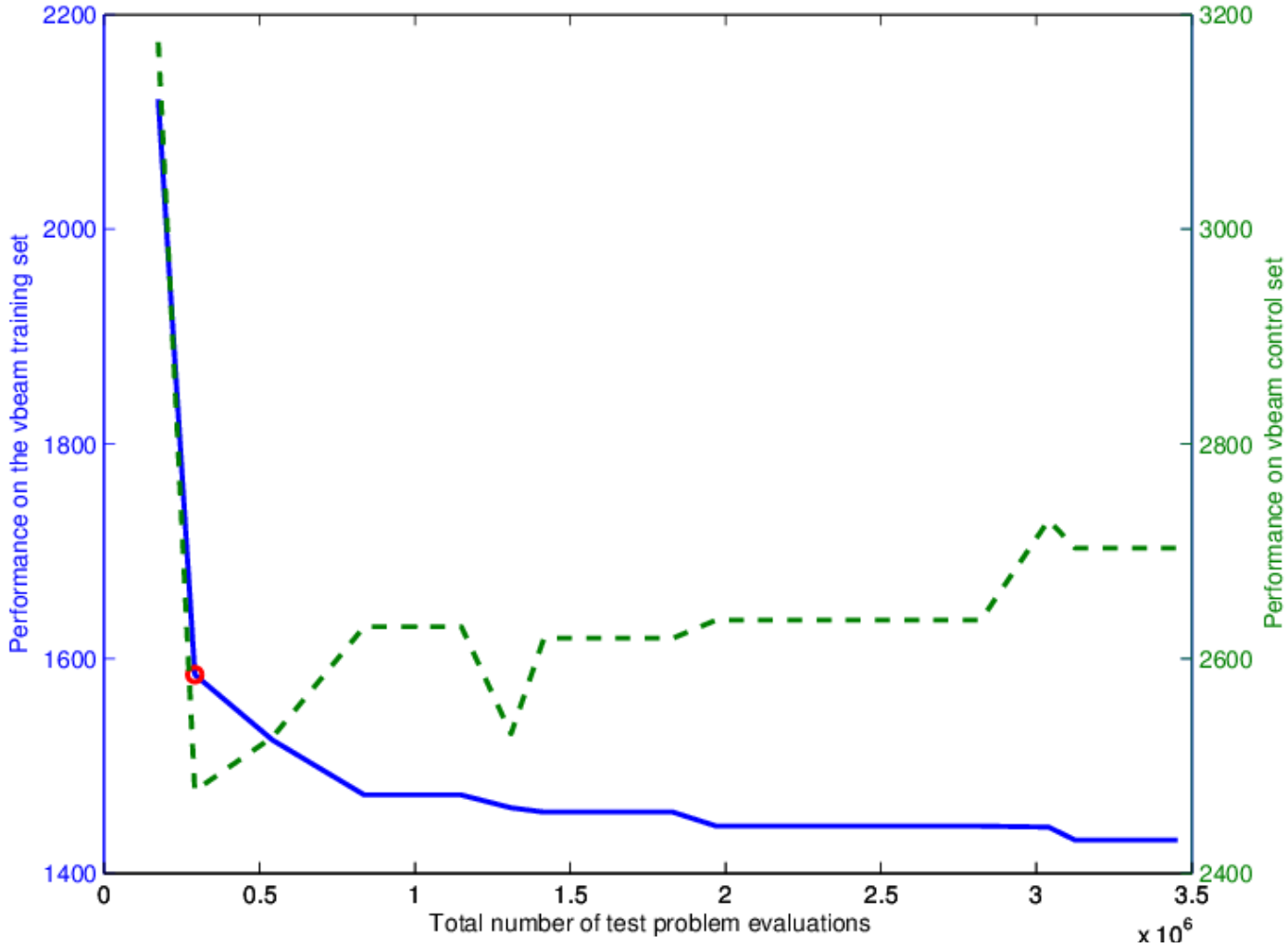
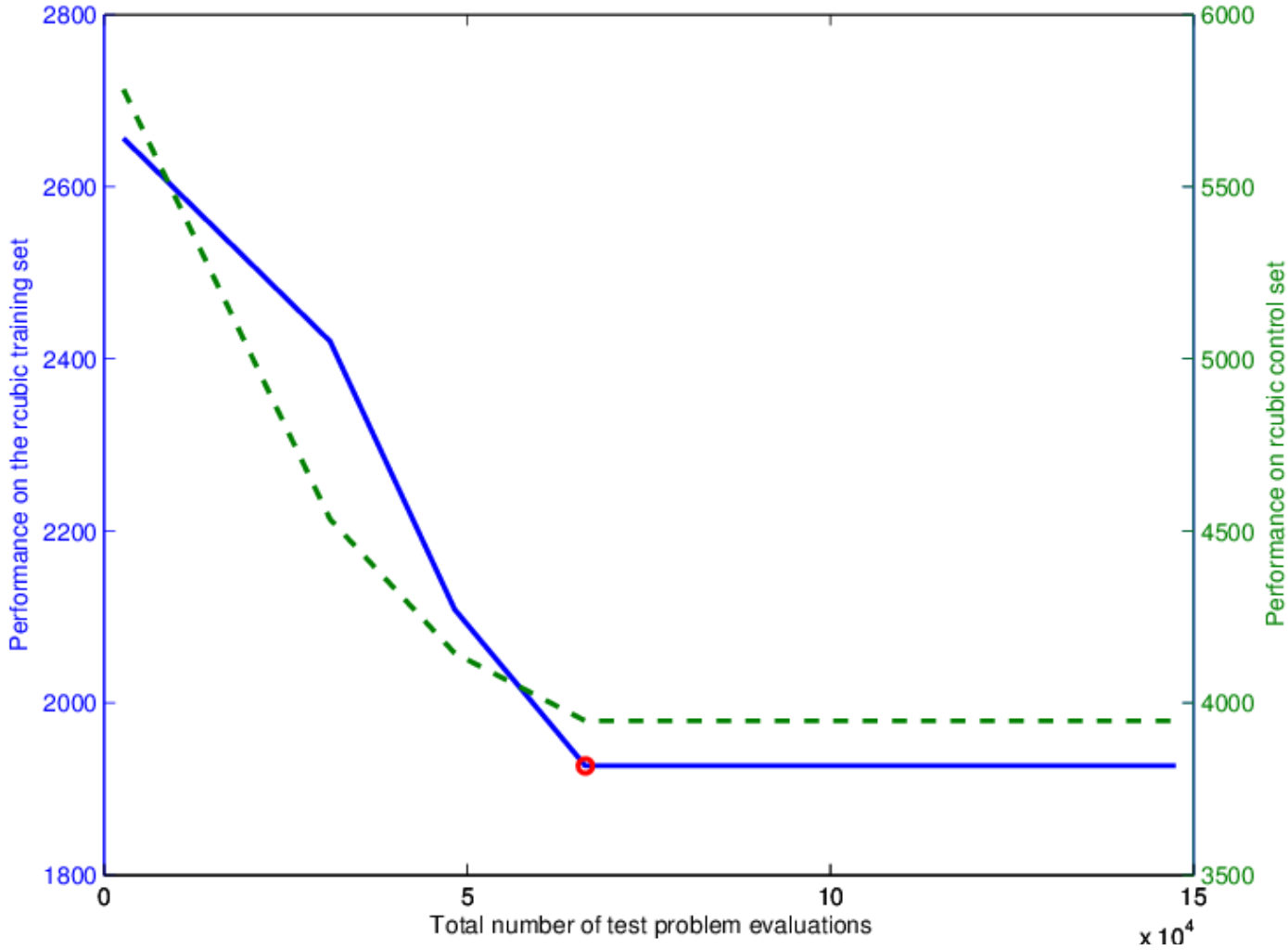


Regularized cubics: AO training, medium-low error deviation, low accuracy



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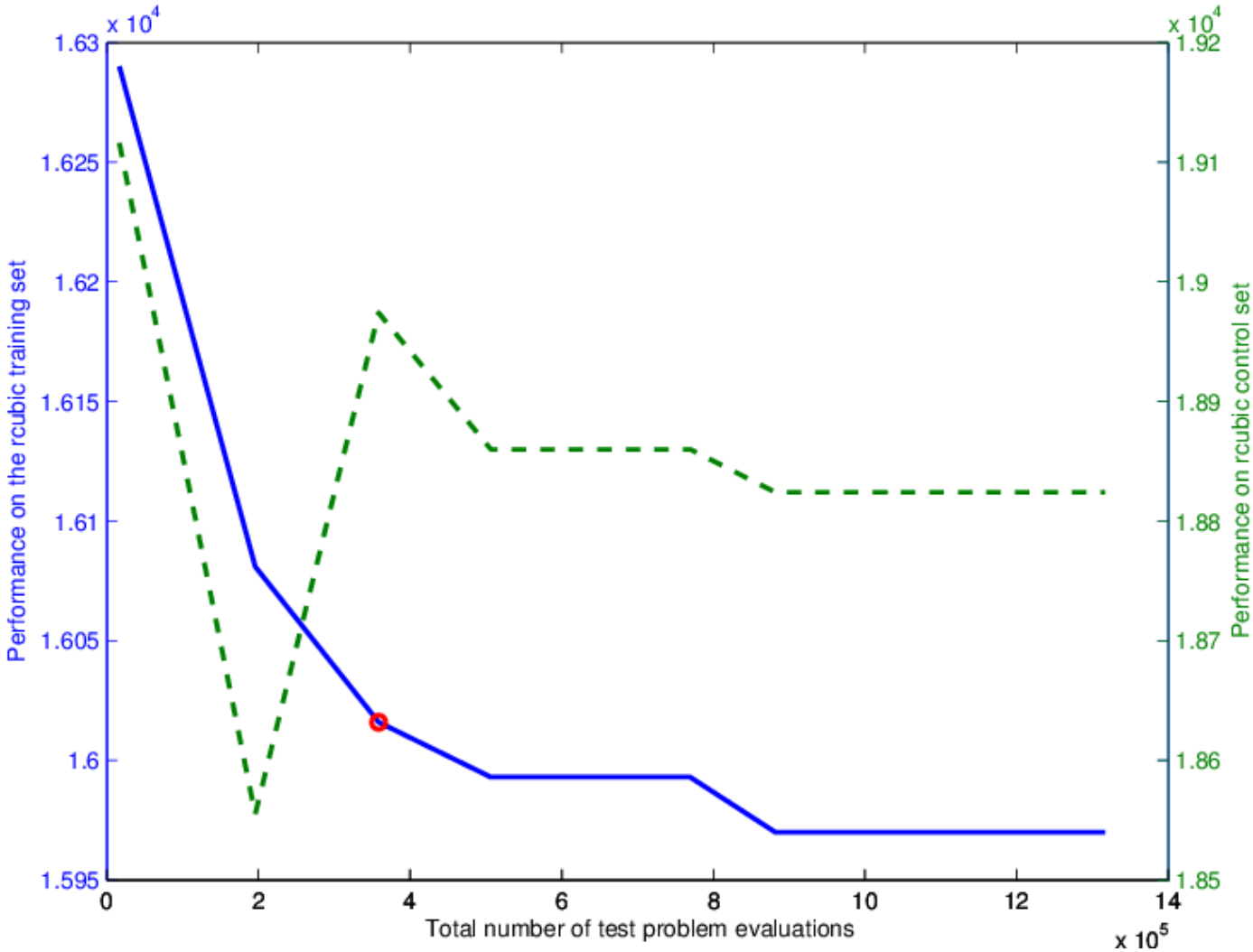

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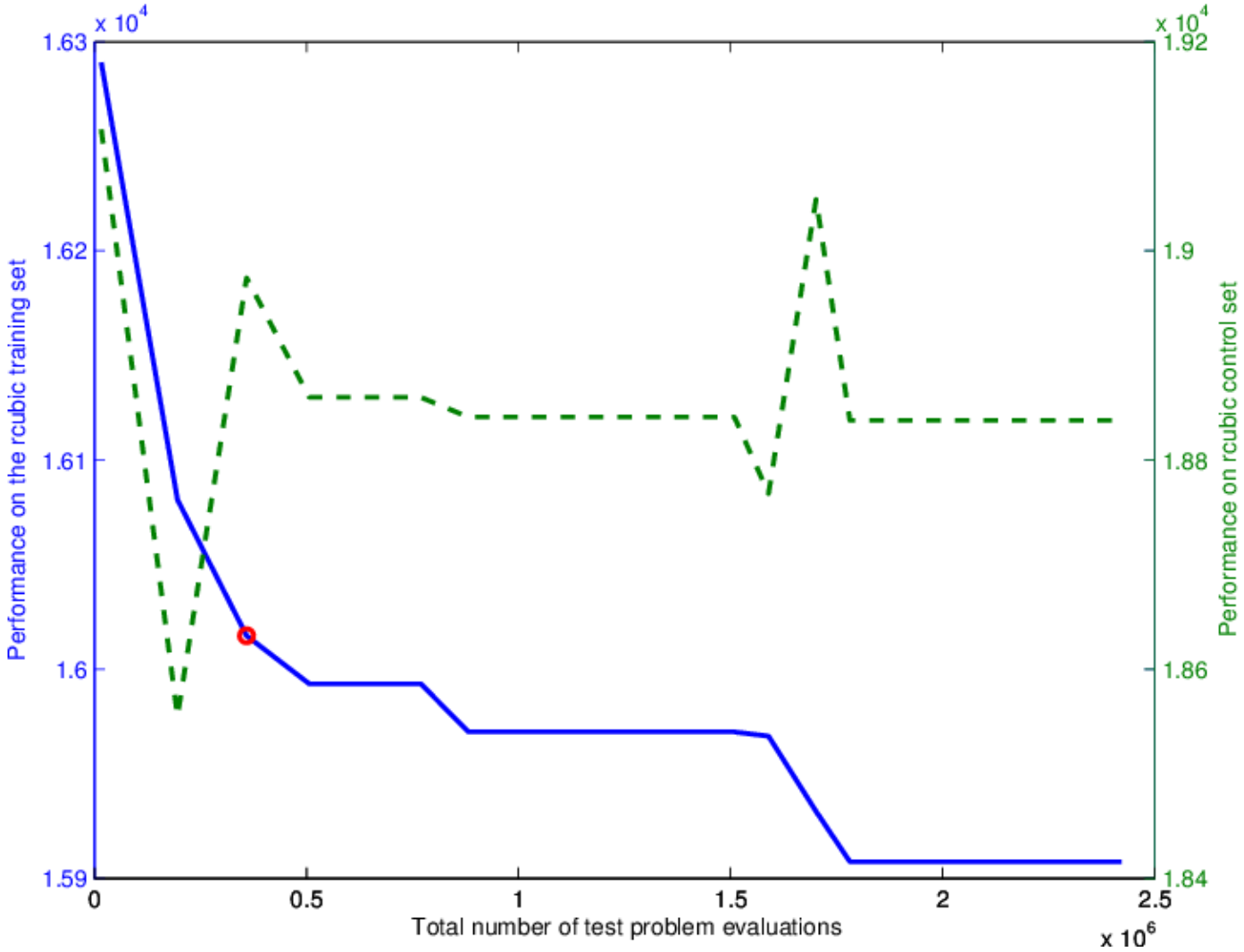

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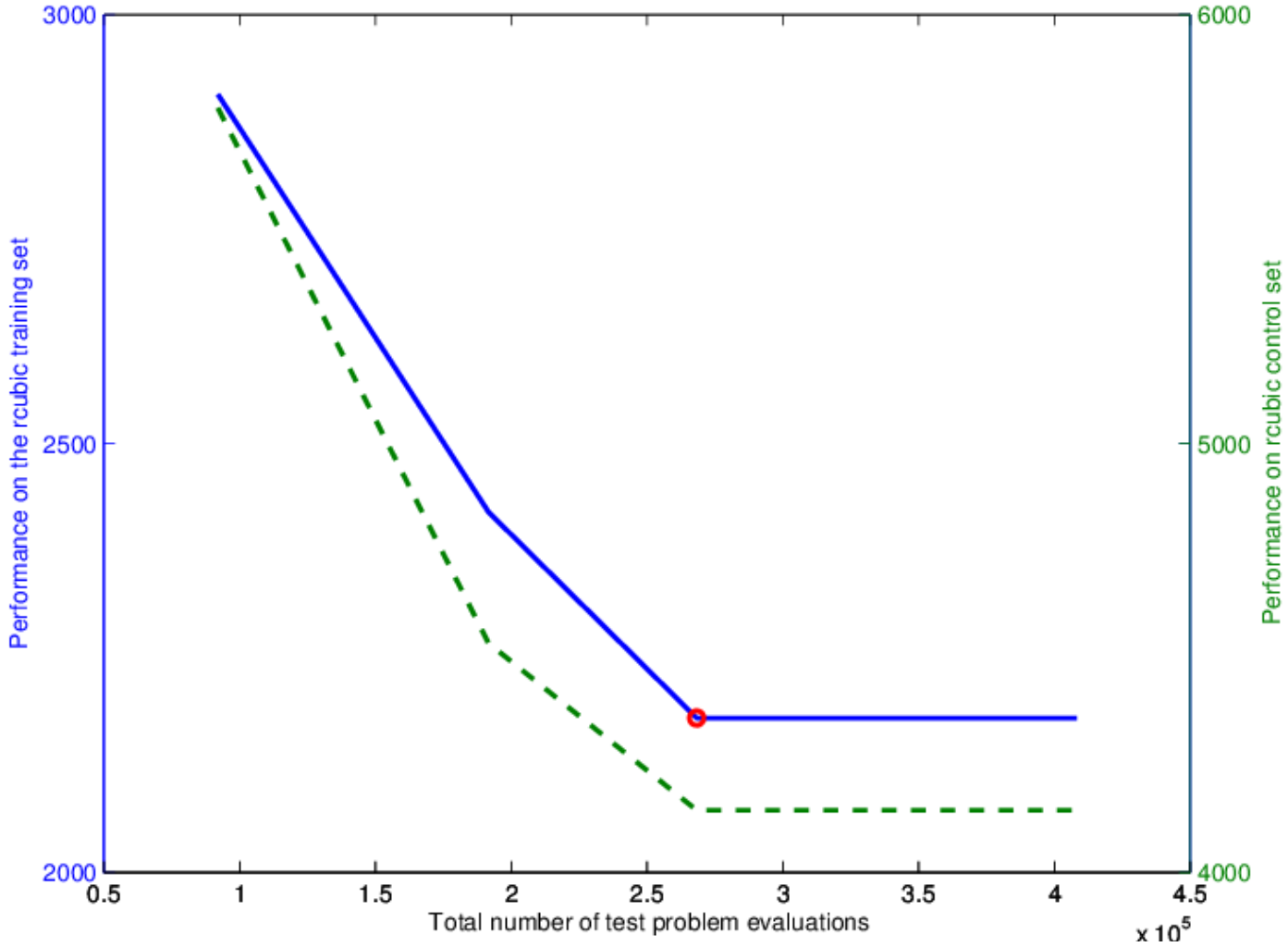

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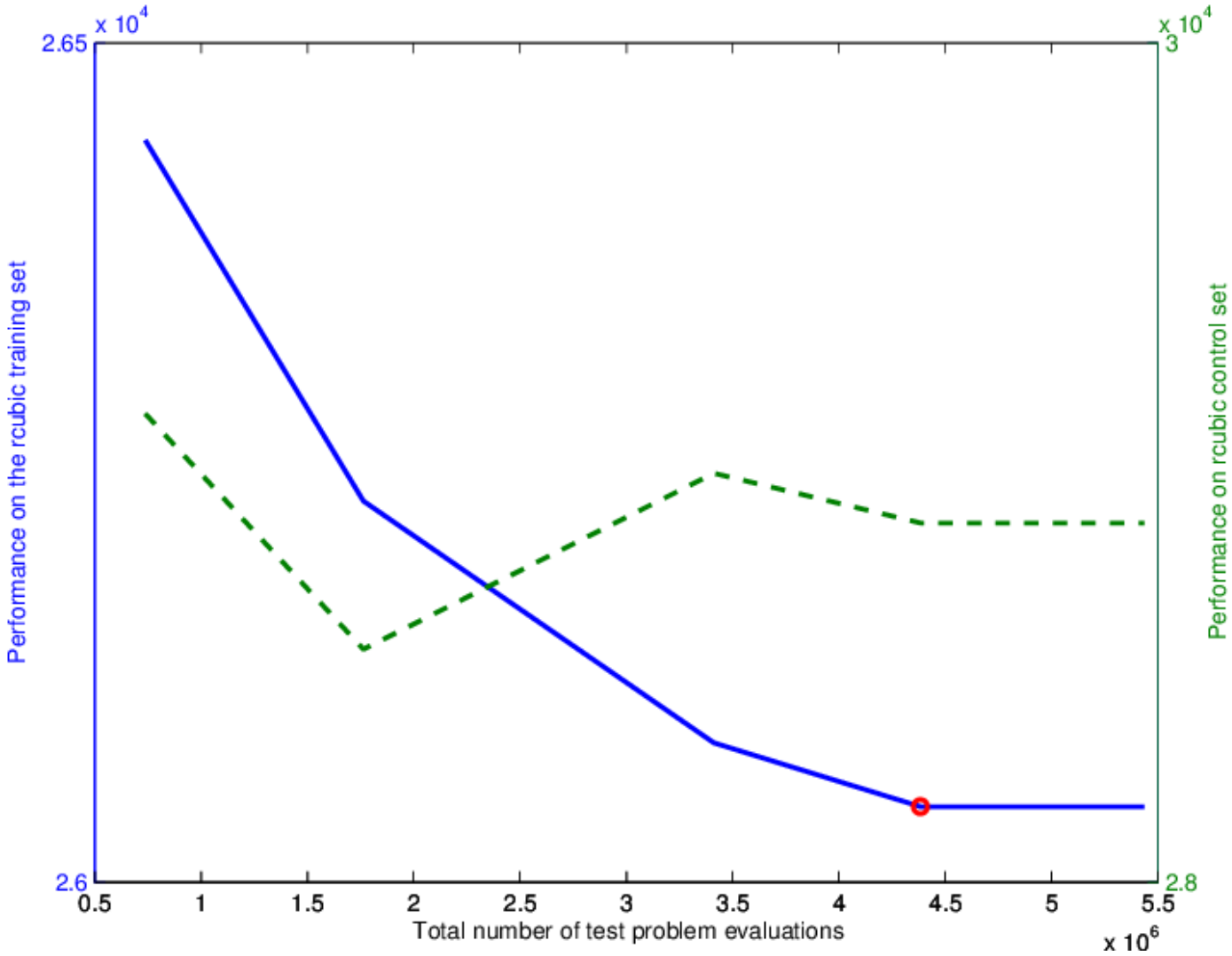

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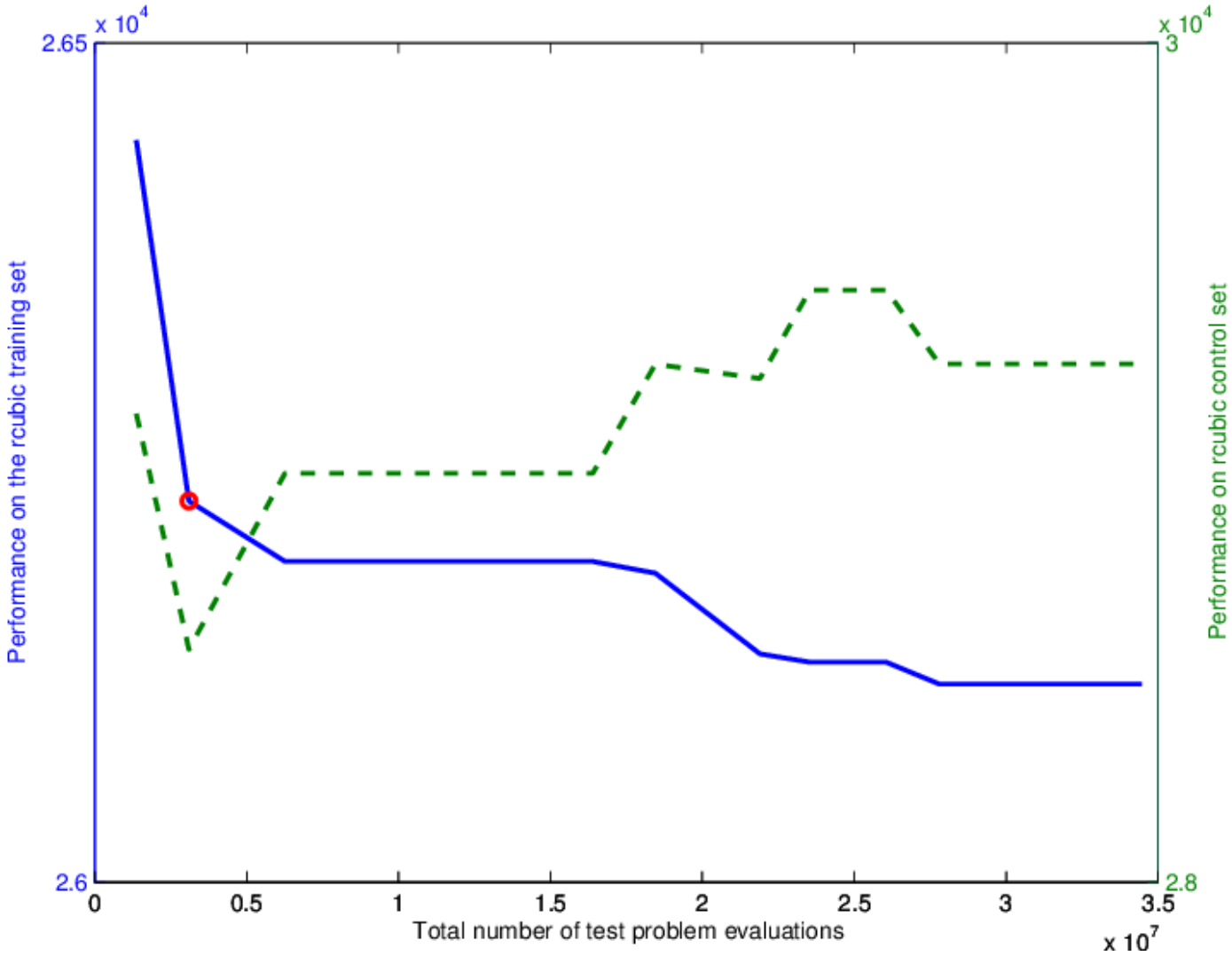

Examples of calls

- `[ x, fx ] = bfo( @banana, [ -1.2, 1 ] )`
- `[ x, fx ] = bfo( @banana, [ -1.2, 1 ], 'xtype', 'ic' )`
- `[ x, fx ] = bfo( @banana, [ -1.2, 1 ], 'xlower', 0, 'epsilon',0.01)`
- `[ x, fx ] = bfo( @banana, [ -1.2, 1 ] , ...`
  `'save-freq',10,'restart-file','bfo.rst')`
- `[ x, fx ] = bfo( @banana, [ -1.2, 1 ] , ...`
  `'training-mode', 'train', ...`
  `'training-parameters', 'fruity', ...`
  `'training-problems', {@banana,@apple},...`
  `'training-problems-data', {@fruit_data} )`
- `[ x, fx ] = bfo( @robust_training, [ 0, -1, 0, 1 ] , ...`
  `'xlevel', [ 1 1 2 2 ], ...`
  `'max-or-min', [ 'min', 'max'] )`

And now...

Some conclusions

*** Use BFO! ***

*** Use BFO to tune your algorithm! ***

(you can even tune BFO to tune your own algorithms)

More user-tunable codes?

Perspectives for the immediate future

- more complicated constraints
- use the original interpolation DFO algorithm in the search step!
- ...

Many thanks for your attention!

Further reading...

M. Porcelli and Ph. L. Toint,
"BFO, a trainable derivative-free Brute Force Optimizer for nonlinear bound-constrained optimization and equilibrium computations with continuous and discrete variables",

naXys Tech Report naXys-06-2015,
Namur Center for Complex Systems, University of Namur (Belgium), 2015.

available from http://perso.unamur.be/~phtoint/toint.html