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Published in:
Physica A: Statistical Mechanics and its Applications

DOI:
10.1016/j.physa.2006.08.032

Publication date:
2007

Document Version
Publisher's PDF, also known as Version of record

Available online at www.sciencedirect.com
ScienceDirect
Physica A 375 (2007) 227–232
PHYSICA A
www.elsevier.com/locate/physa

Energy and number of collision fluctuations in inelastic gases

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Received 24 October 2005; received in revised form 20 July 2006
Available online 7 September 2006

Abstract

The two-dimensional Inelastic Maxwell Model (IMM) is studied by numerical simulations. It is shown how the inelasticity of collisions together with the fluctuations of the number of collisions undergone by a particle lead to energy fluctuations. These fluctuations are associated to a shrinking of the available phase space. We find the asymptotic scaling of these energy fluctuations and show how they affect the tail of the velocity distribution during long time intervals. We stress that these fluctuations relax like power laws on much slower time scales than the usual exponential relaxations taking place in kinetic theory.

Keywords: Granular models of complex systems; Random walks and Lévy flights; Kinetic theory

1. Introduction

Inelastic gases are low-density systems composed of macroscopic particles themselves performing *inelastic* collisions [1,2]. Due to their inelasticity, these systems dissipate kinetic energy, so that their total energy asymptotically vanishes if they are not supplied by an external energy source. Nonetheless, it has been shown [3–7] that their velocity distribution, when homogeneous, usually reaches a self-similar solution,

$$f(v;t) = \frac{1}{\sqrt{T(t)}} f_S\left(\frac{v}{\sqrt{T(t)}}\right), \tag{1}$$

i.e. a form preserving solution whose time dependence occurs through one parameter $T(t)$, the *granular temperature*, defined kinetically by

$$T(t) = \left\langle \frac{1}{d}\frac{mV^2}{2} \right\rangle, \tag{2}$$

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.08.032

where $\mathbf{V} = \mathbf{v} - \mathbf{u}$ is the random velocity, $\mathbf{u}$ is the local mean velocity and d is the dimension of the system. The average is performed with the one particle velocity distribution $f(\mathbf{v}; t)$. Such scaling velocity distributions have been observed in a large variety of kinetic models [5] and have been shown to generically highlight overpopulated high energy tail, whose details depend on the model [8–10].

In statistical physics, interactions usually play a mixing role whose effects are to bring the system into its equilibrium state [11]. This justifies, for instance, in kinetic theory, to use the average number of collisions in order to measure relaxation to equilibrium. In the case of inelastic gases, on the other hand, it is well known that the dissipative collisions prevent the system to reach its equilibrium state, i.e., additionally to a mixing effect that randomises the particle velocities, inelastic collisions act as an energy sink that makes their kinetic energy decrease forever.

In this paper, we focus on the effect of this additional effect on simplified kinetic models, the so-called Inelastic Maxwell Model (IMM) [8–10,12–15] which is a mean field approximation of the Boltzmann equation for inelastic hard discs. After introducing the model in Section 2.1, we perform numerical simulations, thereby showing that the average energy of particles depends on their collision history, i.e., on the number of collisions they have undergone in the course of time. This relation that also appears in simpler one-dimensional models [16] is related to the fluctuating character of the number of inelastic collisions. We show that the resulting energy fluctuations relax like power laws toward the asymptotic state.

2. Collision number statistics

2.1. Inelastic Maxwell Model

Usually, inelastic gases are defined as assemblies of smooth inelastic hard spheres (IHS), i.e., particles whose interactions do not transfer angular momentum, and are instantaneous and dissipative. In the absence of external force, the grains move freely between successive collisions and undergo a collision when particles i and j are in contact, with the collision rule:

$$\mathbf{v}'_i = \mathbf{v}_i - \frac{(1+\alpha)}{2\alpha}\boldsymbol{\varepsilon}(\boldsymbol{\varepsilon}.\mathbf{v}_{ij}),$$

$$\mathbf{v}'_j = \mathbf{v}_j + \frac{(1+\alpha)}{2\alpha}\boldsymbol{\varepsilon}(\boldsymbol{\varepsilon}.\mathbf{v}_{ij}), \tag{3}$$

where $\mathbf{v}_{ij} \equiv \mathbf{v}_i - \mathbf{v}_j$ and $\boldsymbol{\varepsilon}$ is the unitary vector along the axis joining the centres of the two colliding spheres, $\boldsymbol{\varepsilon} \equiv \mathbf{r}_{ij}/|\mathbf{r}_{ij}|$. The primed velocities are the velocities before the collision, the unprimed ones are the post-collisional velocities. Energy dissipation is accounted through the so-called normal restitution coefficient, $\alpha \in]0:1]$. $\alpha = 1$ corresponds to the elastic limit. Let us also stress that the restitution coefficient α is taken as a constant, i.e., we neglect the dependence of α on the relative velocity of the particles [18,19].

In the low-density limit, by assuming that pre-collisional correlations may be neglected [17], the system is described by the inelastic Boltzmann equation. In the following, we assume that the system is and remains homogeneous. Moreover, we use standard mean field methods in order to simplify the mathematical structure of the collision operator. To do so, the collision frequency between particles i and j, which is proportional to v_{ij} in the case of hard spheres, is approximated by the mean field quantity $\sqrt{T(t)}$. This leads to the following kinetic equation:

$$\frac{\partial f(\mathbf{v}_i; t)}{\partial t} = \frac{1}{2\pi}\int \mathrm{d}\theta \int \mathrm{d}\mathbf{v}_2 \left[\frac{1}{\alpha} f(\mathbf{v}'_i; t) f(\mathbf{v}'_j; t) - f(\mathbf{v}_i; t) f(\mathbf{v}_j; t)\right], \tag{4}$$

where $f_i(t) \equiv f(\mathbf{v}_i; t)$ and $f'_i(t) \equiv f(\mathbf{v}'_1; t)$ and θ is defined by the relation $\cos\theta \equiv \boldsymbol{\varepsilon}.\mathbf{v}_{12}/v_{12}$. Angular integrals and the factor $\sqrt{T(t)}$ have been absorbed into the time scale. This model is usually called IMM, and several variations of it have been considered in the literature [8–10,12–15]. It is well known that it leads to an exactly solvable sets of equations for the velocity moments $m_n(t) = \langle (v^2)^n \rangle$, and that its asymptotic scaling solutions are characterised by a power-law tail $v^{-\mu(\alpha)}$, where $\mu(\alpha)$ goes to infinity in the elastic limit $\alpha \to 1$ [13].

2.2. Power-law relaxation and scaling

In order to explore the energy statistics of the particles, we have performed direct simulation Monte-Carlo (DSMC) simulations of the IMM [20]. They are based on the stochastic interpretation of the kinetic equation (4) and consist in picking randomly pairs of colliding particles at each step. It is straightforward to show that the number of inelastic collisions N undergone by a particle at a given time t fluctuates. This quantity is distributed according to the Poisson law $t^N e^{-t}/N!$, where, by construction, t is the average number of collisions suffered by one particle $\langle N \rangle_t = t$. Let us remind that the Poisson distribution satisfies

$$\sigma \equiv \sqrt{\frac{\langle N^2 \rangle_t - \langle N \rangle_t^2}{\langle N \rangle_t^2}} = t^{-1/2}. \tag{5}$$

In order to evaluate the effect of these N fluctuations on the particle energies, we have started simulations from a Maxwell–Boltzmann initial condition, and let the simulation run during t collisions per particle. At that time, the simulation is paused, the particle velocities are rescaled so that the average energy $\langle E \rangle = 1$ and we measure the average energy $\langle E \rangle_\Delta$ of particles having performed $N = t + \Delta$ collisions. The results (Fig. 1) clearly show that the average energy of particles is a decreasing function of their number of collisions, thereby showing that ensembles of particles, discriminated by their number of collisions, are characterised by different quantities of energy on average. This feature comes from the fact that particles having undergone less collisions have also dissipated less energy due to inelastic collisions. Simulations also show that this relation obviously does not take place when the collisions are elastic, as expected. Finally, let us stress that the relation depicted in Fig. 1 does not depend on the initial conditions. This has been verified by starting the simulations with various initial conditions, including the asymptotic scaling solution of the dynamics.

When $\alpha < 1$, the following scaling expression is found for long times $t > 10$ and small values of $\delta \equiv \Delta/t$ (Fig. 2)

$$e(\Delta) = -r(\alpha)\frac{\Delta}{t} + \cdots, \tag{6}$$

where $e(\Delta)$ is the deviation to the average defined by $e \equiv \langle E \rangle_\Delta - 1$ and $r(\alpha)$ is a positive parameter that vanishes in the limit $\alpha \to 1$. This parameter has been measured for different values of the restitution parameter α, and is well approximated by the empirical expression $r(\alpha) \sim 1 - \alpha$ for α close to 1 (Fig. 3). For small times, in contrast, we observe in the simulations a breaking of relation (6), and a non-vanishing value of $e(\Delta)$ at $\Delta = 0$,

$$e(\Delta) = e(0) - e'(0)\Delta + \cdots. \tag{7}$$

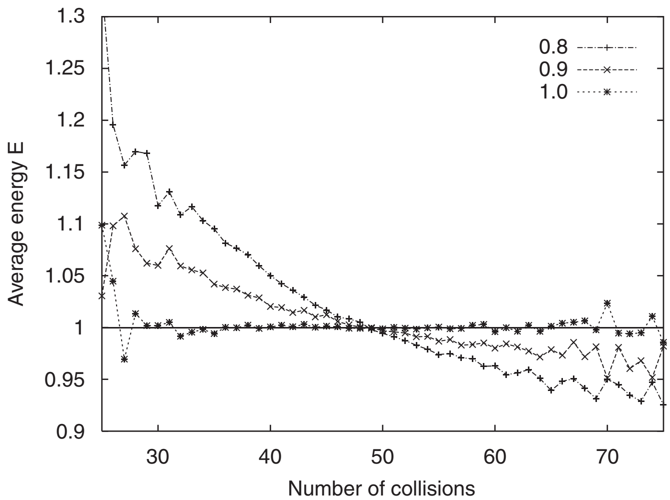

Fig. 1. Average energy of particles as a function of their number of collisions, after 50 collisions per particle. The solid line is the constant value $\langle E \rangle = 1$. The system is composed of 10 000 000 particles with $\alpha = 0.8$, $\alpha = 0.9$ and $\alpha = 1.0$.

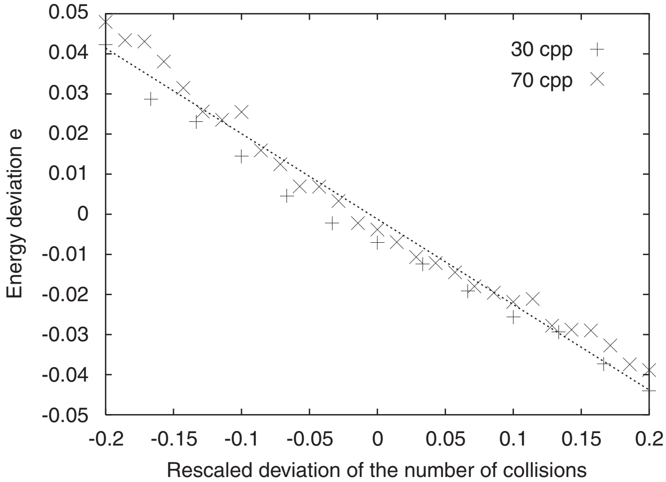

Fig. 2. Energy deviation $e = \langle E \rangle_\Delta - 1$ as a function of the rescaled deviation of the number of collisions $\delta = \Delta/t$, after 30 and 70 collisions per particles (cpp). The gas is composed of 10 000 000 particles with $\alpha = 0.8$. The dashed line is the linear fit Eq. (6) of this relation.

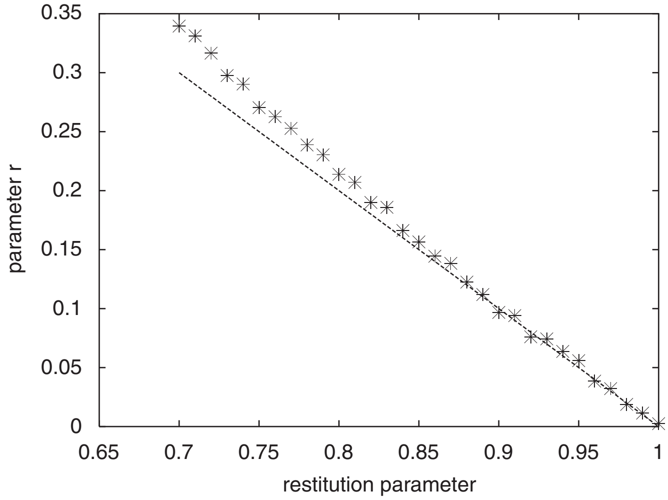

Fig. 3. Parameter $r(\alpha)$ as a function of the restitution parameter. The parameter is measured by performing simulations of a gas composed of 10 000 000 particles and by evaluating the best linear fit of Eq. (6) after 50 collisions per particle. The dashed line $1 - \alpha$ is a fit of $r(\alpha)$ for $\alpha \to 1$.

Let us note that $e(0)$ increases with t for small times, reaches a maximum around $t = 5$ and decreases very fast later on.

Relations (5) and (6) imply that the fluctuations of $\langle E \rangle_\delta$ decrease asymptotically like power laws,

$$\sigma_e = \sqrt{\langle e^2 \rangle} \sim r(\alpha) t^{-1/2}. \tag{8}$$

We observe this behaviour by DSMC (Fig. 4), namely $\sqrt{\langle e^2 \rangle}$ decreases like the power law $t^{-1/2}$ after an increase that takes place during a few mean collision times. This initial increase followed by a crossover is due to the breaking of relation (6) for small times, and to the non-vanishing value of $e(0)$. In the case of elastic collisions $\alpha = 1$, $\sqrt{\langle e^2 \rangle}$ is strictly zero, i.e., the dynamics are restrained on the initial constant energy surface. This property ceases to be true when the collisions dissipate energy, i.e., when $\alpha < 1$, as confirmed by

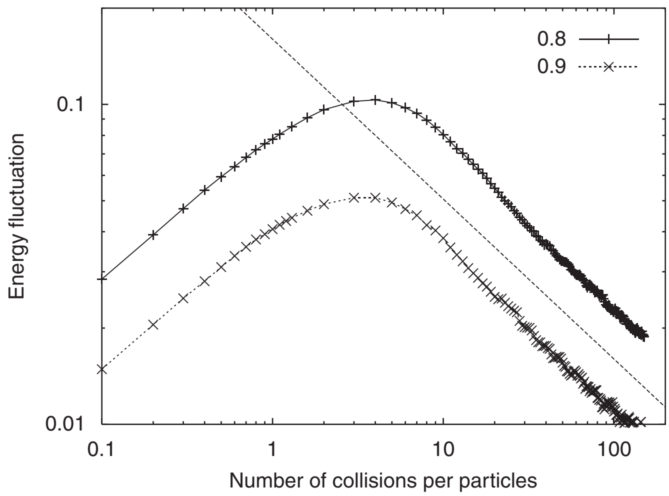

Fig. 4. Energy fluctuation σ_e as a function of the number of collisions per particle. The gas is composed of 10 000 000 particles with $\alpha = 0.8$ and $\alpha = 0.9$. The dashed line is a guide for the eye and corresponds to a power law $\sim t^{-1/2}$.

the non-vanishing values of $\langle e^2 \rangle$ in the course of time. Let us stress that result (8) suggests that the energy fluctuations behave asymptotically in the same way as the number of collision fluctuations.

Amongst others, the energy fluctuations contribute to the rapid emergence of fat tails in the velocity distribution. Indeed, an averaging of distributions with different mean energies is well known to overpopulate the tail of the distribution [21,22]. In the present case, the existence of these very energetic particles (as compared to the average) arises due to the fluctuations in the number of collisions. We have checked this effect by DSMC. To do so, we have focused on particles belonging to the tail of the distribution, i.e., the N_t particles whose energy is larger than k times the average, say $k = 5$. Among these N_t particles, we consider the proportion p_t of particles having performed less collisions than the average t, $p_t = N_t^- / N_t$. The time evolution of p_t shows a power-law, i.e., slow, relaxation that confirms the important role played by the observed fluctuations in the high-energy tail for very long times. Let us note that energy fluctuations may also lead to non-Gaussian velocity distributions in the case of systems with fluctuating temperature [21] or mass [23].

3. Conclusion

In this paper, we describe the phase space dynamics of an inelastic gas. To do so, we study a two-dimensional Inelastic Maxwell Model (IMM), which is a mean field approximation of the Boltzmann equation for inelastic hard discs. By performing DSMC simulations of the model, we show a statistical relation between the number of *inelastic* collisions undergone by a particle and its average energy. The resulting non-equilibrium fluctuations of energy lead to an anomalous exploration of phase space and to a discrimination of particles depending on their collisional history. One should note that the observed fluctuations relax on much slower time scales than the usual exponential relaxations taking place in kinetic theory. Moreover, this mechanism is specific to inelastic gases, where energy is dissipated at each collision, and has no counterpart in elastic gases. Finally, let us stress that this work opens perspectives for a phase space description of inelastic gases, e.g., the use of Liouville equations for IHS [24,25].

Acknowledgements

This work has been done thanks to a financial support of FRIA, ARC and COST P10 (Physics of Risk).

References

[1] N.V. Brilliantov, T. Pöschel, Kinetic Theory of Granular Gases, Oxford University Press, Oxford, 2004.
[2] M. Ausloos, R. Lambiotte, K. Trojan, Z. Koza, M. Pękala, Physica A 357 (2005) 337.
[3] T.P.C. van Noije, M.H. Ernst, Granular Matter 1 (1998) 57.
[4] J.J. Brey, M.J. Ruiz-Montero, D. Cubero, Phys. Rev. E 54 (1996) 3664.
[5] M.H. Ernst, R. Brito, in: T. Poeschel, N.V. Brilliantov (Eds.), Granular Gas Dynamics, Lecture Notes in Physics, vol. 624, Springer, Berlin, 2003, p. 3.
[6] M.H. Ernst, R. Brito, J. Stat. Phys. 109 (2002) 407.
[7] A. Barrat, T. Biben, Z. Rácz, E. Trizac, F. van Wijland, J. Phys. A 35 (2002) 463.
[8] E. Ben-Naim, P.L. Krapivsky, Phys. Rev. E 61 (2000) R5.
[9] A. Baldassarri, U.M.B. Marconi, A. Puglisi, Europhys. Lett. 58 (2002) 14.
[10] R. Lambiotte, L. Brenig, Phys. Lett. A 345 (2005) 309 cond-mat/0507573.
[11] P. Résibois, M. De Leener, Classical Kinetic Theory of Fluids, Wiley, New York, 1977.
[12] M.H. Ernst, R. Brito, Europhys. Lett. 58 (2002) 182.
[13] M.H. Ernst, R. Brito, Phys. Rev. E 65 (2002) 040301(R).
[14] E. Ben-Naim, P.L. Krapivsky, Lecture Notes in Physics, vol. 624, 2003, p. 65.
[15] R. Lambiotte, L. Brenig, Phys. Rev. E 72 (2005) 042301 cond-mat/0508768.
[16] R. Lambiotte, L. Brenig, J.M. Salazar, Physica A 366 (2006) 250.
[17] R. Soto, M. Mareschal, Phys. Rev. E 63 (2001) 041303.
[18] R. Ramirez, et al., Phys. Rev. E 60 (1999) 4465.
[19] N.V. Brilliantov, T. Pöschel, Phys. Rev. E 61 (2000) 1716.
[20] G.A. Bird, Molecular Gas Dynamics, Clarendon Press, Oxford, 1976.
[21] C. Beck, Phys. Rev. Lett. 87 (2001) 180601.
[22] R. Lambiotte, M. Ausloos, Physica A 362 (2006) 513.
[23] M. Ausloos, R. Lambiotte, Phys. Rev. E 73 (2006) 011105.
[24] J.M. Salazar, L. Brenig, Phys. Rev. E 59 (1999) 2093.
[25] J.W. Dufty, A. Baskaran, Ann. NY Acad. Sci. 1045 (2005) 93.