University of Namur, Belgium

Social Physics

The dynamics of human behaviour and economy

Tuesday the 18th December 2012

SVTN “J.D. van der Waals”
Eindhoven University of Technology

Workshop: Modeling in social sciences

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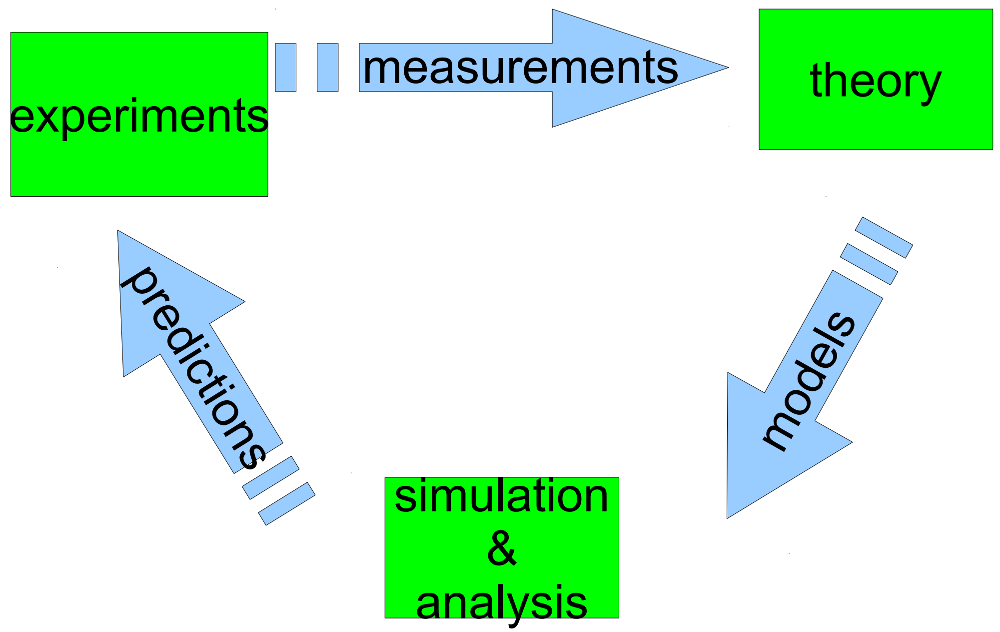
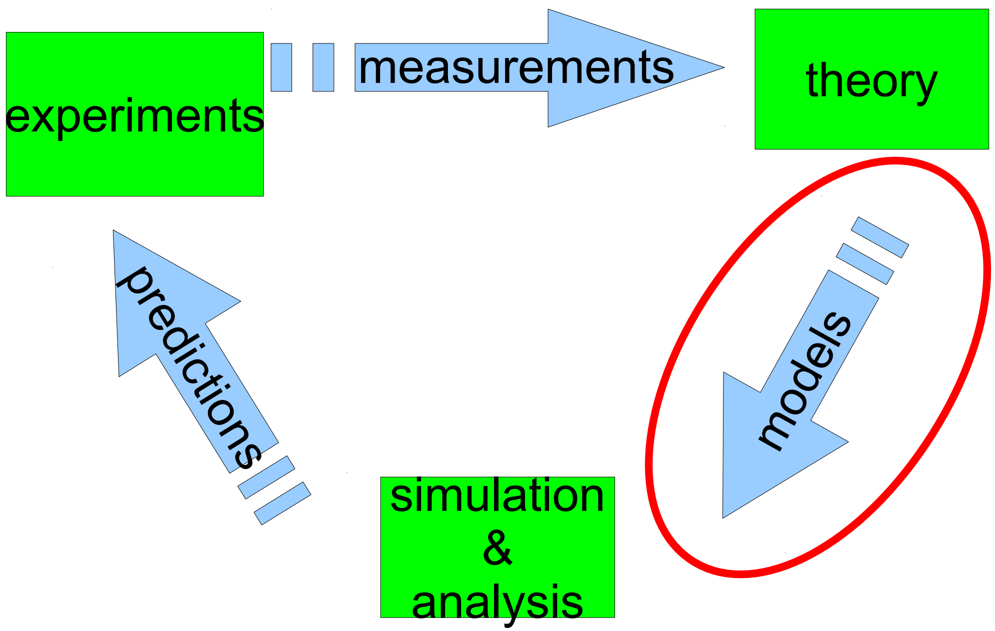
The modeling cycle



The modeling cycle

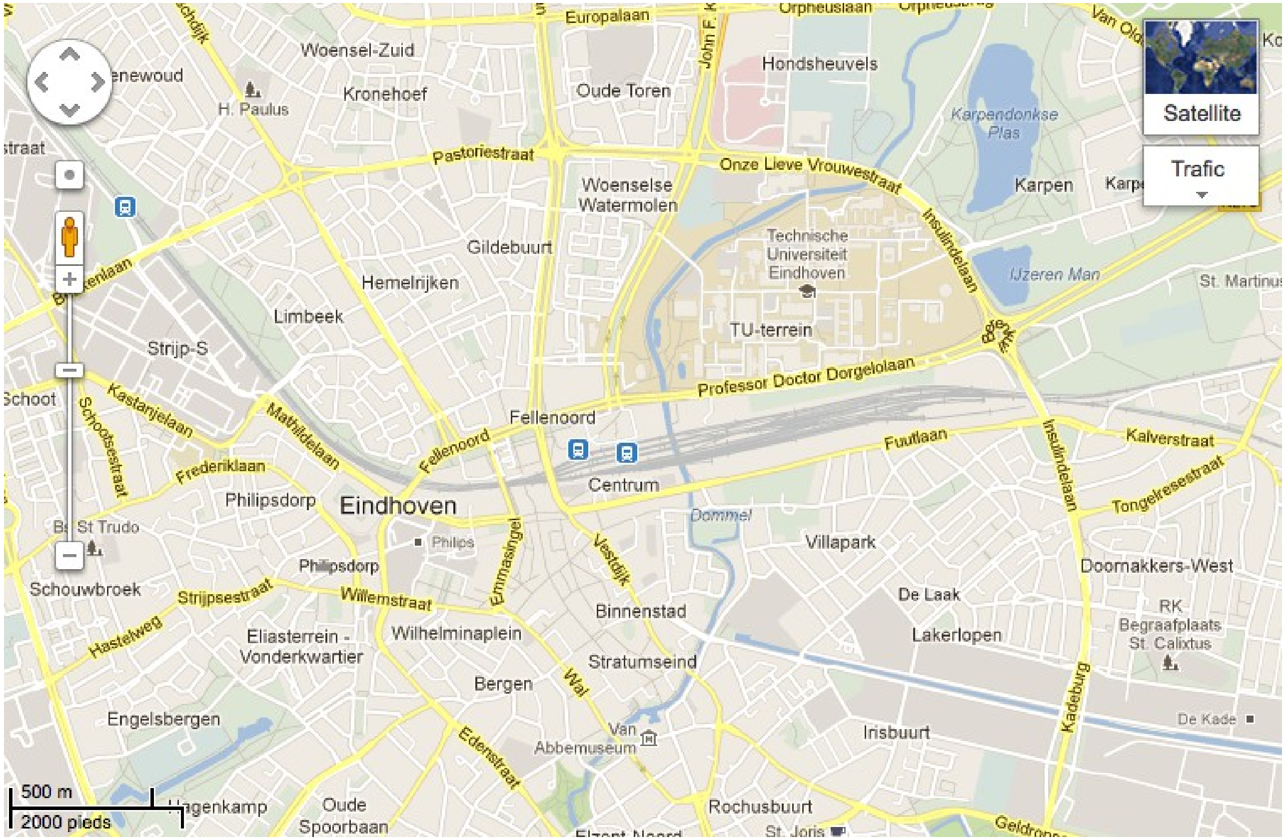


A simple example: a map

You can browse google maps to find a location

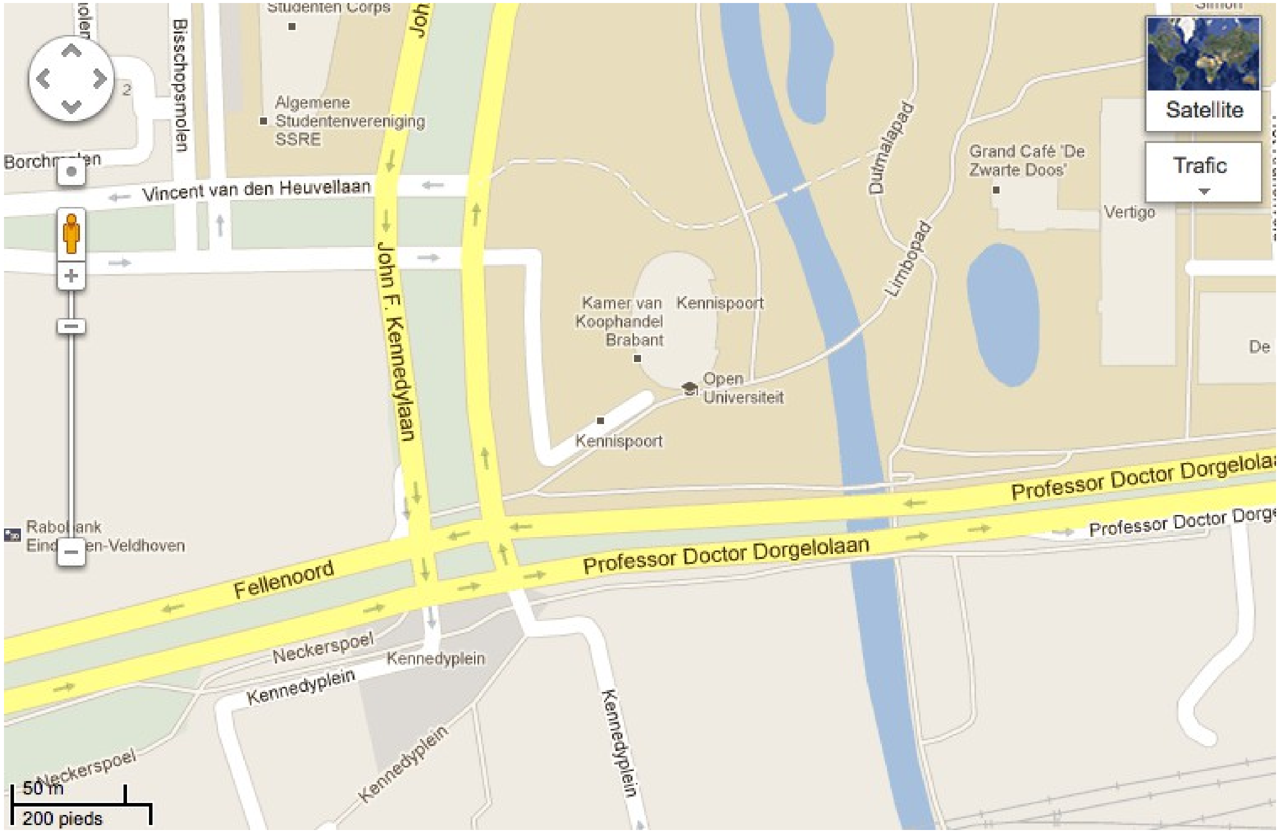
A simple example: a map

You can zoom over to have more details



A simple example: a map

And even more



But you still don't have a complete information

A simple example: a map

So you can add a layer more

Can you conclude that the best map is the one with plenty of details?

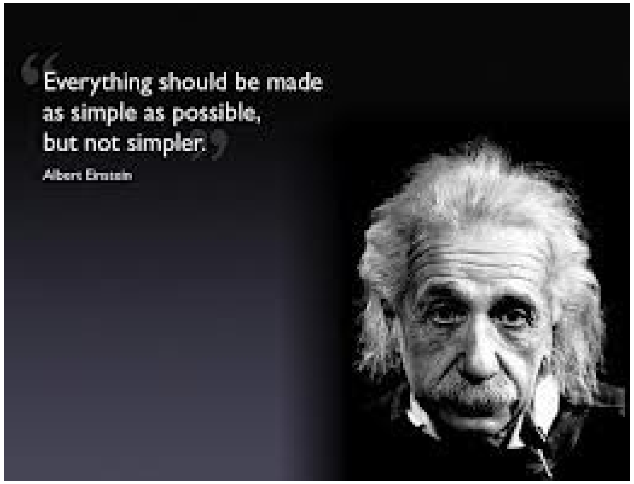
A simple example: a map

No, because such a map would be useless.
It would be as large as the real world!

A simple example: a map

No, because such a map would be useless.
It would be as large as the real world!

Modeling is the same: you should add as much details as you need to answer to your question, but no more.



Once you got the answer you can improve the model by adding more details. But take care to be able to find again your previous results as particular cases.

Opinion dynamics in social networks

The goal of this workshop is to present and study some **opinion dynamics models** with particular attention to the final state, **consensus** vs **fragmentation**, and to the **underlying social network**.

I will briefly present two main classes of models: the "**Agent Based Models**" (microscopic models) and the "**Mean Field Models**" (macroscopic models).

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I will briefly present two main classes of models: the "**Agent Based Models**" (microscopic models) and the "**Mean Field Models**" (macroscopic models).

In the former, we start by modeling the **individual behavior** (local interaction rules) to deduce collective behavior. In the latter, we directly work at the **average individual** or collectivity.

We will analyze the limits of each model and the impact of the network topology. You will be asked to write, run and analyze **numerical codes**, to get the goal.

Remark: these notes are part of a course I give to 5^{th} year math students at the University of Namur.

Opinion dynamics in social networks

Individuals in a society (or group) interact by exchanging information, ideas, opinions, etc. Actually this is the very first definition of the group, because share such features allows us to recognize the group identity.

Robert Axelrod, Journal of Conflict Resolution, 41, 2, 1997, pp. 203.

If people tend to become more alike in their beliefs, attitudes, and behavior when they interact, why do not all these differences eventually disappear ? ... Despite the existence of so many mechanisms for the maintenance of differences, none of them takes into account the fundamental principle of human communication that « the transfer of ideas occurs most frequently between individuals ... who are similar in certain attributes such as beliefs, education, social status, and the like ».

Opinion dynamics in social networks

Questions :

- **How to model opinions ?**
- How to model a group of individuals exchanging opinions
- How to model social interactions?

Opinion dynamics in social networks

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One opinion can be defined through a question. In some cases the answer is simply : **yes** or **not**

Do you like soccer ?

Answer to a referendum

Are you republican or democrat (in US)?

We are thus dealing with **binary opinions** 0 & 1 (for maths) or up & down (for physicists) left & right (for politicians).

This can be generalized to a finite (small) set of possible choices (political elections)

Opinion dynamics in social networks

However it is **not always possible** to reduce the answer to yes or not. Actually this depends on the **amount of information** you want to put into your answer.

Examples.
“Do you believe in God?” The short answer is yes or not, but we could create a graduation of believes: how often do you go to the church? How many times do you pray? Etc ...

“Are you republican or democrat?” The short answer is yes or not, but once again we can add nuances via subquestions: what do you think of Afghanistan war? What do you think of public health? Taxes? Economics? ... We could thus obtain a fine grid from completely agree with Rep to completely agree with Dem.

In this case we would speak of **continuous opinions**.

Opinion dynamics in social networks

Questions :

- How to model opinions ? OK
- **How to model a group of individuals exchanging opinions**
- How to model social interactions?

Modeling means to represent the group in mathematical language to be able to study (analytically and/or numerically) its temporal behavior, the role of parameters and the assumptions used.

This corresponds to define the level at which we want to stufy the group: **individual level** or the **whole group**.

Opinion dynamics in social networks

At individual level we are dealing with Agent Based Model. We should define each individual by some internal variables (e.g. opinion, sex, spatial localization, ...) et and then define the interaction laws. If needed we could finally aggregate the data to have informations at the group level.

Example :
If agent 1 with internal state A meets agent 2 with internal state B, then the former will change his internal state according to $A'=f_1(A,B)$ and the latter to $B'=f_2(A,B)$.

Opinion dynamics in social networks

At group level, we are dealing with Mean Field Model (i.e. averages). We completely loose the notion (and any information) of individual and we work on ratio/number of agents with a given set of features. Assuming a complete mix of the agents, the interaction laws depend on such ratio/number of.

Example :
Let $N_a(t)$ (respectively by $N_b(t)$) the number of agents in the group at time t with the feature a (respectively b). Then at some time $t'>t$ we will get $N_a(t')=F(N_a(t),N_b(t))$ et $N_b(t')=F(N_a(t),N_b(t))$.

Opinion dynamics in social networks

Questions :

- How to model opinions ? OK
- How to model a group of individuals exchanging opinions? OK
- **How to model social interactions?**

In the real life, people exchange opinion (informations, etc …) through interpersonal meetings, that can be generically divided into: direct ones (groups meeting, ...) or indirect ones (mails, gsm, ...)

The formers arise more often once people share close geographical positions, while the latter can be done even at very large distances.

Both cases can be modeled using networks. Nodes are individuals and links can represent telephonic wire, a geographical distance, mail exchange …), upon which opinions diffuse.

G. Weisbuch, G. Deffuant, F. Amblard, J-P Nadal, *Meet, Discuss, and Segregate!*, Complexity, **7**, (3) , 2002, pp. 55.

Let us consider a group of N individuals, each one with an opinion $O_i(t)$ at time t, we assume the opinion to be continuous and normalized (at $t=0$) into $[0,1]$.

Each agent can interact with all the other ones (complete network).

At each time step, two agents are randomly drawn from the population and each one tells his own opinion to the other.

Each agent has an internal parameter $\sigma_i \in [0,1]$ (openness of mind) used to select the incoming opinion : people do accept only opinions close enough to his own.

G. Weisbuch, G. Deffuant, F. Amblard, J-P Nadal, *Meet, Discuss, and Segregate!*, Complexity, **7**, (3) , 2002, pp. 55.

If agents i and j are selected, then each one compare his opinion distance form the other $\Delta_{ij}(t) = O_i(t) - O_j(t)$ to his openness of mind σ_i

If $|\Delta_{ij}(t)| \leq \sigma_i$ then agent i is ready to accept the opinion of j; Otherwise i will not change his opinion.

Let us observe that the process is not symmetric because in general $\sigma_i \neq \sigma_j$

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The simplest way to model the opinion exchange is to move toward the other opinion:

$$O_i(t') = O_i(t) - \mu\Delta_{ij}(t)$$

where $\mu \in (0, 1/2]$

$\mu = 1/2$ corresponds to compute the average.

G. Weisbuch, G. Deffuant, F. Amblard, J-P Nadal, *Meet, Discuss, and Segregate!*, Complexity, **7**, (3) , 2002, pp. 55.

Problem 1.

If at time *t=0* the opinions are uniformly distributed into *[0,1]*, which will be the time evolution of the system? Under which conditions the agents will converge to consensus?

Assuming $\sigma_i = \sigma \quad \forall i$ and $\mu = 1/2$ then the model has only two parameters: N and σ

Remark.

Even if the model is quite simple, the number of analytical results is very limited. There is a conjecture that in the limit of large groups, the number of clusters of agents sharing the same opinion is given by $\lfloor 1/(2\sigma) \rfloor$

Consider the case where agents cannot interact with all the others but only with a subset.

S. Galam, *Local dynamics vs. Social mechanisms : A unifying frame*, Europhys. Lett., **70**, (6) , 2005, pp. 705.

Let us consider a group composed by N individuals, each one with a binary opinion + or – at time t.

Agents change their opinions because of small groups meetings (say 3 persons). During the meeting, they discuss and when they left each one will have the **majority opinion**.

Let p(t) the ration of individuals with opinion + at time t, hence 1-p(t) will denote the ratio of agents with opinion -. This is the aggregated variable we are interested in.

At each time step, the whole population is split into groups of size 3 randomly generated. Can be deduce p(t+1) by the knowledge of p(t)?

S. Galam, *Local dynamics vs. Social mechanisms : A unifying frame*, Europhys. Lett., **70**, (6) , 2005, pp. 705.

Inside each group (local dynamics) we can have 4 cases:

$(+++) \rightarrow (+++)$ Three agents with opinion +, they keep the same opinion +.

$(++-) \rightarrow (+++)$ Two agents with opinion +, one with opinion -, the latter will move to +.

$(+--) \rightarrow (---)$ Two agents with opinion -, one with opinion +, the latter will move to -.

$(---) \rightarrow (---)$ Three agents with opinion -, they keep the same opinion -.

Inside each group (local dynamics) we can have 4 cases:

$(+++) \quad \rightarrow \quad (+++)$ Three agents with opinion +, they keep the same opinion +.

this can happen with probability $P_{+++} = \frac{n}{N}\frac{n-1}{N-1}\frac{n-2}{N-2} \sim \frac{n^3}{N^3} = [p(t)]^3$
being *n(t)=p(t)N* the number of agents with opinion +

$(++-) \quad \rightarrow \quad (+++)$ Two agents with opinion +, one with opinion -, the latter will move to +.

this can happen with probability

$$P_{++-} = 3\frac{n}{N}\frac{n-1}{N-1}\frac{N-n}{N-2} \sim 3\frac{n^2}{N^2}\left(1-\frac{n}{N}\right) = 3[p(t)]^2(1-p(t))$$

$(+--) \quad \rightarrow \quad (---)$ Two agents with opinion -, one with opinion +, the latter will move to -.

$(---) \quad \rightarrow \quad (---)$ Three agents with opinion -, they keep the same opinion -.

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In conclusion:

$$p(t+1) = [p(t)]^3 + 3[p(t)]^2(1-p(t)) = -2[p(t)]^3 + 3[p(t)]^2$$

Let $f(x) = -2x^3 + 3x^2$ then the time evolution of p(t) is given by the dynamical system (discrete time map)

$$p(t+1) = f(p(t))$$

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Let $f(x) = -2x^3 + 3x^2$ then the time evolution of p(t) is given by the dynamical system (discrete time map)

$$p(t+1) = f(p(t))$$

One can study the fixed points: $f(x) = x$

$$p_1 = 0 \qquad p_2 = 1 \qquad p_3 = 1/2$$

And their (in)stability: $|f'(x)| < 1$

$$f'(0) = 0 \qquad f'(1) = 0 \qquad f'(1/2) = 3/2$$

Modification of the local interaction law

In the former model we assumed a strict majority rule: 2 individuals with the same opinion can always push the third one with a different opinion on their side.

We can now hypothesize that this is not always possible and thus to define a **persuasion coefficient** $m_{3,i} \in [0,1]$ corresponding to the strength that i agents with opinion + can be use to force the k-i agents with opinion – to change their minds.

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Of course: $m_{3,0} = 0 \qquad m_{3,3} = 1$

The we can compute once again the time evolution of p(t)

$$p(t+1) = [p(t)]^3 + 3m_{3,2}[p(t)]^2(1-p(t)) + 3m_{3,1}p(t)[1-p(t)]^2$$

That is

$$p(t+1) = g(p(t))$$

where

$$g(x) = x^3 + 3m_{3,2}x^2(1-x) + 3m_{3,1}x(1-x)^2$$

One can study the fixed points: $g(x) = x$

$$p_1 = 0 \qquad p_2 = 1 \qquad p_3 = \frac{3m_{3,1}-1}{1-3m_{3,2}+3m_{3,1}}$$

And their (in)stability: $|g'(x)| < 1$

$$g'(0) = 3m_{3,1} \qquad g'(1) = 3(1-m_{3,2}) \qquad g'(p_3) = ...$$

Using a sort of symmetry argument one could assume $m_{3,j} = 1 - m_{3,3-j}$ and thus

$$g'(0) = 3m_{3,1} \qquad g'(1) = 3m_{3,1} \qquad g'(p_3) = ...$$

S. Galam, *Local dynamics vs. Social mechanisms : A unifying frame*, Europhys. Lett., **70**, (6) , 2005, pp. 705.

Modification of group size

We can assume that in general the population is split into groups of size k and after some computations we end up with

$$p(t+1) = \sum_{j=0}^{k} m_{k,j} \binom{k}{j} p^j (1-p)^{k-j}$$

$m_{k,j} \in [0,1]$ is the strength that j agents with opinion + are able to do to convince k-j agents with opinion – to change their minds.

Remark.

Because $m_{k,0} = 0$ $p_1 = 0$ is always a fixed point and its multiplier is: $km_{k,1}$

Because $m_{k,k} = 1$ $p_2 = 1$ is always a fixed point and its multiplier is: $k(1 - m_{k,k-1})$ under the assumption of symmetry we get $km_{k,1}$

Problem 2.

Study the first model (i.e. k=3) using both the equation for p(t) and an Agent Based Model.

Compare both results and study what happen in the case of larger and larger group sizes

Problem 3.

As problem 2 but using an arbitrary k using both the equation for p(t) and an Agent Based Model.

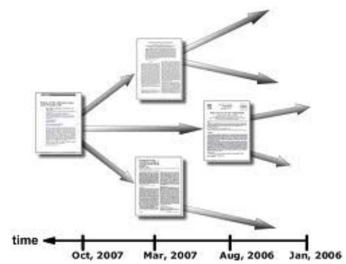
Compare both results and study what happen in the case of larger and larger group sizes

Networks

- A graph is a representation of a set of objects, nodes, where some pairs of the objects are connected by links because of the existence of some relationship

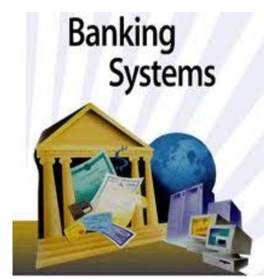
- A (complex) network is a graph with non-trivial features

nodes = persons
links =friendship



nodes = scientists
links = co-authorship

nodes = persons
links =mails exchanges



nodes = banks
links = money fluxes

nodes = web pages
links = hyperlinks

Networks

- A undirected graph is an ordered pair G = (V, E), formed by a set V of vertices (nodes) together with a set E of edges (links), which are unordered pairs of vertices.

- A two standard way to represent a graph are:

Adjacency list: for each node write down a list of all nodes linked to it

Adjacency matrix: for each i and j define

$A_{i,j} = 1$ If there exists a link between i and j

$A_{i,j} = 0$ otherwise

Adjacency list ~ (Number nodes) x (Numer links)

Adjacency matrix ~ (Number nodes)^2

Networks

- The most studied quantity related to a network is the node degree, that is the number of links a node has

$$k_i = \sum_j A_{i,j} = \|\text{adj list}\|$$

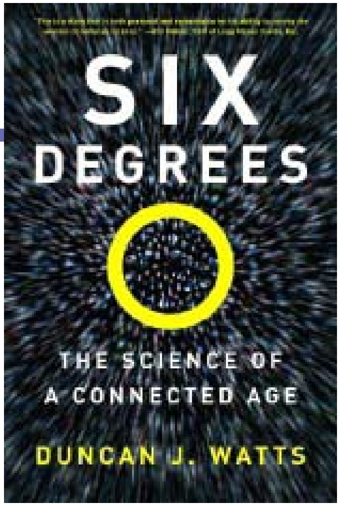
- and the degree distributions, P(k), that is the probability a generic node has degree k

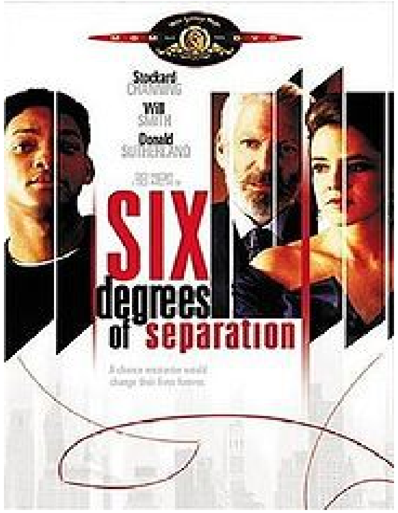
- In a random (or Erdős-Reny) network P(k) is a Poisson distribution (or a Gaussian distribution if N is large)

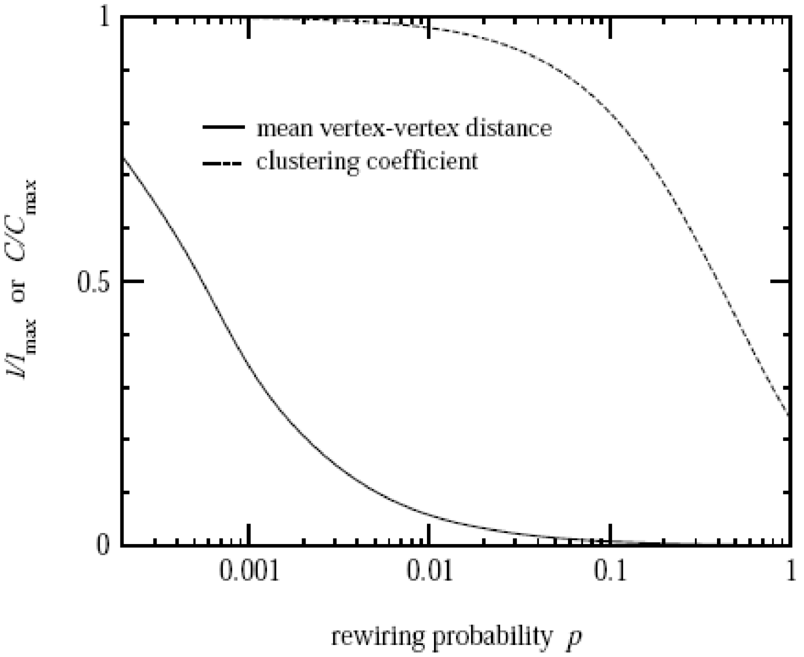
- In a scale free network P(k) is a power law

Small world

- A network has the small world property if most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of steps







Milgram 1967 experiment,
Kavin Bacon Number, Erdős number …

Watts, D. J. & Strogatz, S. H. (1998), Nature 393 (6684), 440–442

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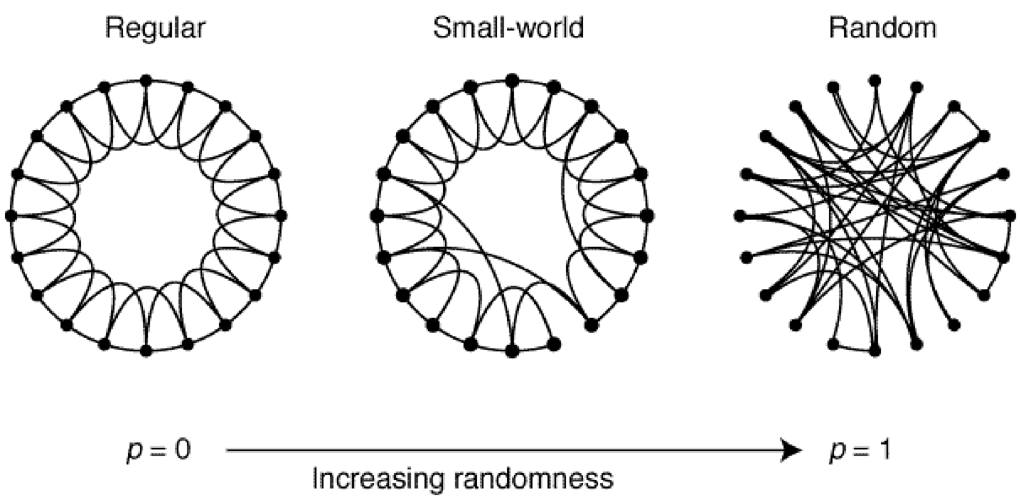
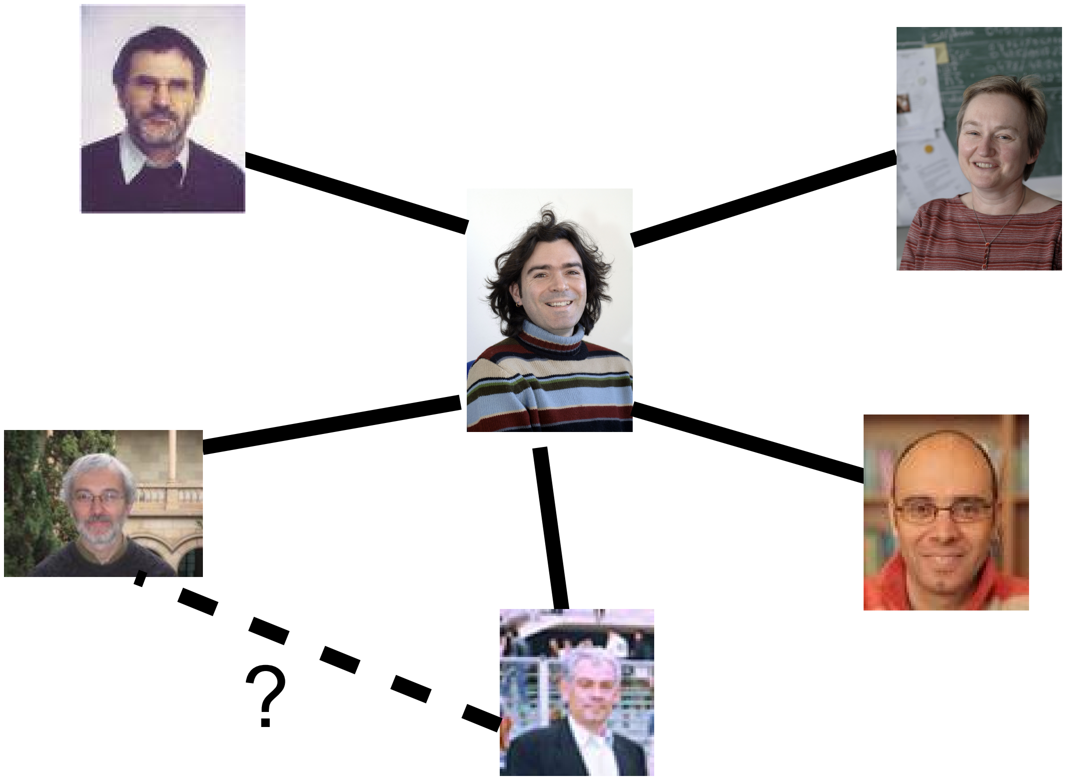
Start with a regular network (a lattice), think of n points on a circle and connect each one with few neighbors, say **2m**, then at any time step **rewire randomly** one edge, i.e. change its extremities, with a given probability **p**.

For **p = 0** the original graph is not modified, while if **p = 1** all edges are randomly rewired; thus **small values** of p imply **regularit**y while **large values randomness**, but there are then ranges of p (close to 1 but not too much) for which $< \ell >$ is very small but C is still large.



Clustering, ...

- How many co-authors of mine, are also relatively co-authors ?



$$C_i = \frac{\text{existing triangles}}{N(N-1)/2}$$

Normalizing factor : N co-authors can form at most N(N-1)/2 triangles

- Social network have large clustering coefficient, my friends are often friends with each other