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Optimization by a genetic algorithm of the light-extraction efficiency of a GaN light-emitting diode

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Abstract. Light extraction from light-emitting materials is fundamentally limited by internal reflections due to the high dielectric-constant contrast between the material that produces the light and the emergent medium. These internal reflections can however be reduced significantly by a well-designed texturation of the surface of the emitting material. We used a genetic algorithm to determine optimal geometrical and material parameters for this texturation, the objective being to maximize the extraction of light of a GaN light-emitting diode (LED). This study, which was restricted to two-dimensional texturations, shows that symmetric triangles actually correspond to the optimal shape. The dielectric constant of the material used for this texturation should ideally have the same dielectric constant as the GaN. The optimal texturation determined in this work leads to a light-extraction efficiency of 11.1%, which improves significantly the value of 3.7% obtained with a flat surface and the value of 5.7% achieved in previous work.

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1. Introduction

The efficiency of light-emitting diodes (LED) has increased significantly over the past few years, but the overall efficiency is still limited by total internal reflections due to the high dielectric-constant contrast between the material that produces the light and the emergent medium. Various approaches are considered in order to increase the extraction of light from high dielectric-constant materials. The bulk of the semiconductor may contain microstructures, which act as tiny lenses, mirrors or other optical devices for guiding light out of the active material.[1, 2, 3] Many efforts actually focus on finding an efficient structuration of the surface in order to enhance the extraction of light.[4, 5, 6, 7, 8, 9, 10] Diffusion by surface roughness for example will open new optical channels for the extraction of light.[11] Some structurations specifically aim at achieving a graded refractive index at the surface in order to reduce total internal reflections.[12, 13, 14, 15] Surface plasmonics[16] is also used to facilitate the extraction of light.

The bioluminescent organ of fireflies gave incentive for the study of nature-inspired structures able to enhance the extraction of light.[17, 18, 19] This study showed that the surface of the cuticle of fireflies presents jagged scales, which turn out to increase light extraction significantly in comparison to a planar surface. In order to achieve a similar effect with light-emitting diodes, the structure needs to be adapted. A theoretical investigation is required in order to determine optimal parameters. The time required for this optimization grows however exponentially with the number of parameters. Previous analysis was therefore restricted to only two adjustable parameters. Extending this study to a wider variety of shapes involves the consideration of trillions of parameter combinations, which is intractable by methods based on a systematic scan on parameters. Genetic algorithms[20, 21, 22, 23, 24] provide however a more efficient way to explore these parameters.

We present in this work a genetic algorithm (GA) we developed for the optimization of the light-extraction efficiency of a GaN light-emitting diode. The details of this algorithm are presented in Sec. II. Sec. III then presents the results obtained when optimizing the light-extraction efficiency of the GaN light-emitting diode. We discuss technical aspects of this approach in Sec. IV. Sec V finally concludes this work.

2. Description of the genetic algorithm

Let $f = f(\vec{x})$ be an objective function of n physical parameters x_i, where $x_i \in [x_i^{\min}, x_i^{\max}]$ with a specified granularity of Δx_i in the representation of each parameter. We want to find, amongst this whole set of possibilities for the parameters x_i, the values that maximize globally the objective function f.

Each parameter x_i is represented by a string of n_i bits (0 or 1), also called a "gene". The length n_i of each gene is chosen so that $(x_i^{\max} - x_i^{\min})/(2^{n_i} - 1) \leq \Delta x_i$. The value of the physical parameter x_i is then given by $x_i = x_i^{\min} + \langle \text{gene } i \rangle \times \Delta x_i$, where

$\langle \text{gene } i \rangle \in [0, 2^{n_i} - 1]$ stands for the value coded by the gene i in Gray binary coding.[23] The genetic algorithm must reject gene values that lead to $x_i > x_i^{\max}$. A given set of physical parameters $\{x_i\}_{i=1}^n$ is finally represented by the juxtaposition of the n genes used for the representation of each parameter. These strings of n genes are also called "DNA".

We work with a population of n_{pop}=100 individuals. Each individual has its own DNA. It is therefore representative of a given set of physical parameters $\{x_i\}_{i=1}^n$. The initial population consists of random individuals. These individuals must be evaluated in order to determine their "fitness". The fitness is taken in this context as the value of the objective function f. The individuals are then sorted according to their fitness. This classification determines the part of the population that will be replaced by random individuals in the next generation (n_{rand}) and the part of the population that will be replaced by individuals for which the fitness has already been calculated (n_{rec}). The remaining part of the population ($N = n_{\text{pop}} - n_{\text{rand}} - n_{\text{rec}}$ individuals) will be determined by a classical game of selection, crossover and mutations applied to the top N individuals of the current population.

We select for this purpose N individuals ("the parents") amongst the top N individuals of the population by a rank-based *Roulette Wheel Selection*.[23, 24] This is a random selection procedure in which the probability for an individual to be selected is proportional to its weight on a "wheel". The best individual in terms of fitness receives a weight of N, the second-best receives a weight of $N - 1$, etc. The last individual in this selection receives a weight of 1. Individuals with a higher fitness have thus more chance to be selected. A given individual can be selected several times. This enables the best individuals to progressively dominate the population.

For any pair of parents, we determine two "children" for the next generation. These children are obtained either (i) by a one-point crossover of the parents' DNA (probability of 70%), or (ii) by a simple replication of the parents' DNA (probability of 30%). The position in the chain of bits at which the two parts of the parents' DNA is exchanged is chosen randomly.[23, 24] The rate of crossover (70%) controls the balance between the exploration of new solutions (individuals obtained by crossing the parents' DNA) and the conservation of good solutions (transmission of unchanged individuals to the next generation). With smaller crossover rates, the fittest individual will rapidly dominate the population and convergence may be too fast. With higher crossover rates, there is more exploration before the genetic algorithm converges to a given optimum of the objective function. We finally introduce random mutations in the DNA of the children obtained when crossing the parents' DNA. Each bit of the children's DNA has a probability of 1% to be reversed. These mutations constitute another driving force for the exploration of parameters. When the rate of mutation is too small, the genetic algorithm may converge too rapidly, without finding the global optimum. When the rate of mutation is too high, the exploration of parameters tends to be essentially random and therefore inefficient. We use a value of 1% based on experience with previous problems.

When repeated from generation to generation, this evolutionary strategy will generally converge to the global maximum of the objective function f. One can monitor the progress achieved by the GA by representing generation after generation the fitness of the best individual (f_{best}), the average fitness in the population (f_{mean}) and the genetic similarity (s). The genetic similarity represents the fraction of the bits in the population whose value is the same as for the best individual.[23] The genetic similarity s typically takes an initial value of 0.5 (random population) and raises up to 1 (complete dominance by the best individual). One can finally define a progress indicator $p = |s - 0.5|/0.5$, which will range between 0 (start of the GA) and 1 (end of the GA) along the optimization. The number n_{rand} of random individuals and the number n_{rec} of recycled individuals introduced at each generation are then given by $n_{\mathrm{rand}} = 0.1 \times n_{\mathrm{pop}} \times (1 - p)$ and $n_{\mathrm{rec}} = 0.1 \times n_{\mathrm{pop}} \times (1 - p)$. Both n_{rand} and n_{rec} actually decrease when the algorithm enters a phase of refinement of the solution ($s \to 1$). Elitism is implemented in order to make sure that the best individual is not lost when going from one generation to the next.

3. Application to the optimization of the light-extraction efficiency of a GaN light-emitting diode

The light-extraction efficiency of light-emitting materials is fundamentally limited by internal reflections due to the high dielectric-constant contrast between the material that produces the light and the emergent medium. A previous study by Bay *et al.* on the bioluminescent organs of fireflies showed that the cuticle of these fireflies presents jagged scales, which turn out to increase the extraction of light.[17, 18, 19] This observation was the starting point for the study of nature-inspired structures able to increase light extraction from a GaN light-emitting diode.

Parameter optimization in this previous study was achieved by a systematic scan on parameters. As the evaluation of each structure required important computational resources, the study was restricted to shapes that involve only two structural parameters (basically, the period P and the height H). The structures considered were (i) two-dimensional symmetric triangles, (ii) two-dimensional right-centered triangles (similar to a "factory-roof"), (iii) three-dimensional pyramids, and (iv) cones. Parameter optimization for each structure was achieved by considering values between 1 and 15 μm for P and H. A discretization step of 1 μm was used in order to reduce the number of evaluations. This study concluded that the light-extraction efficiency of the GaN light-emitting diode is actually higher with the "factory-roof" geometry. This result was consistent with the observation of jagged scales on the cuticle of fireflies. The optimal parameters determined for this structure were a period P of 5 μm and a height H of 6 μm. These parameters were associated with a light-extraction efficiency of 5.7%, which improved the value of 3.7% obtained with a flat surface. Since the step used for this parameter exploration was rather large (1 μm), it is likely that a better structure can actually be found. We seek in this work not only at refining the determination of optimal parameters, but also at exploring a wider range of possible shapes for the surface texturation of the GaN. The genetic algorithm actually makes it possible for two reasons: (i) all individuals of a given generation can be evaluated in parallel (thanks to a multi-agent implementation of the GA), (ii) the number of fitness evaluations required by the GA is way smaller than the actual number of possibilities (typically, a thousand of evaluations instead of trillions).

The light-extraction efficiency of a light-emitting diode is defined by $\eta = I_{\text{trans}}/I_{\text{inc}}$, where I_{inc} refers to the intensity of the light emitted in the active material and I_{trans} refers to the intensity of the light extracted into free space. It is actually calculated by

$$\eta = \int_0^{2\pi} \int_0^{\pi/2} T(\theta, \phi, \lambda) \sin\theta d\theta d\phi, \tag{1}$$

where $T(\theta, \phi, \lambda)$ is the transmittance for a radiation of wavelength λ that encounters from inside the surface of the LED. The main wavelength λ for the GaN LED is 425 nm. θ and ϕ refer to the directional angles of the incident radiation. The transmittance $T(\theta, \phi, \lambda)$ is actually calculated by a Rigorous Coupled-Waves Analysis (RCWA).[25, 26] This method accounts for three-dimensional aspects of the problem and for the specificities of the LED. The dielectric constant of GaN at the wavelength considered is 6.4.[27] A current-spreading layer (nickel and gold alloy) is added on top of the light-emitting material for functioning purposes of the LED. Its dielectric constant was calculated by considering ϵ_{Ni}=-3.7+i 8.1 and ϵ_{Au}=-1.6+i 6.3.[28] The material considered by Bay *et al.*[18, 19] for the surface texturation of the GaN was a photoresist, whose dielectric constant ϵ is 2.763 (manufacturer's value for photoresist AZ 9245®).[29]

The structures considered in this work for the surface texturation of the GaN include as particular cases the two-dimensional structures considered by Bay *et al.*,[18, 19] i.e. symmetric and right-centered triangles. This work also accounts for all intermediate situations regarding the apex position of these triangles. For the left and right sides of these structures, we consider either concave, straight or convex edges. The height $h(x)$ that describes the two-dimensional structures considered in this work is actually given by

$$h(x) = \begin{cases} H \times \left[1 - \frac{(c.P-x)^{\alpha_{\text{left}}}}{(c.P)^{\alpha_{\text{left}}}}\right] & \text{when } 0 \leq x \leq c.P \\ H \times \left[1 - \frac{(x-c.P)^{\alpha_{\text{right}}}}{(P-c.P)^{\alpha_{\text{right}}}}\right] & \text{when } c.P \leq x \leq P \end{cases} \tag{2}$$

where P and H refer as previously to the period and the height of the periodic structures considered for the surface texturation of the GaN. The parameter c determines the apex position. For c=0, 0.5 and 1, the apex is respectively on the left, in the middle and on the right of the period P. The coefficients α_{left} and α_{right} determine the concavity of the left and right edges. Straight edges are achieved for $\alpha_{\text{left}}=\alpha_{\text{right}}=1$. Values of α_{left} or α_{right} higher than 1 determine convex edges that extend beyond the reference triangular shape achieved when $\alpha_{\text{left}}=\alpha_{\text{right}}=1$. Values of α_{left} or α_{right} smaller than 1 determine concave edges that keep within the reference triangular shape.

The geometrical parameters to consider for the optimization of the light-extraction efficiency of the LED are hence P, H, c, α_{left} and α_{right}. The dielectric constant ϵ of the material used for the surface texturation of the GaN can also be adjusted. We consider in this work discretization steps of 0.1 μm for P and H (instead of 1 μm in previous work). This value corresponds to $\lambda/4$, which is the shortest length scale expected to influence light extraction from the LED. The calculation of $T(\theta, \phi, \lambda)$ in Eq. 1 also relied on a discretization of the physical system by steps of $\lambda/4 \simeq 0.1$ μm. The objective of this refined exploration is to achieve higher light-extraction efficiencies. The values considered for the period P and the height H actually range between 1 and 10 μm (by steps of 0.1 μm). For the apex position c, we consider values between 0.5 and 1 (step of 0.01). The coefficients α_{left} and α_{right} that determine the concavity of the two edges take values between 0.2 and 5 (by steps of 0.01). The dielectric constant ϵ of the material used for the surface texturation of the GaN takes values between 1.2 and 6.35 (by steps of 0.01). These parameter specifications leave us with 50,418,836,005,356 possibilities to explore.

The results achieved by the genetic algorithm are summarized in Table 1. The different lines correspond to optimizations on different sets of parameters. There are two main groups of results. They correspond to optimizations in which ϵ was either (i) fixed to the photoresist's value (first group of results), or (ii) included in the optimization (second group of results). For each group of results, the first two lines correspond to optimizations in which only two geometrical parameters were considered (P and H). The other parameters were fixed at c=0.5 or 1, α_{left}=1 and α_{right}=1. These optimizations hence correspond to simple symmetric or right-centered triangles. The third line corresponds to optimizations in which the full set of geometrical parameters was considered (P, H, c, α_{left} and α_{right}).

Table 1. Optimal parameters for the surface texturation of the GaN with corresponding values for the light-extraction efficiency. The six lines correspond to different optimizations by the GA. The parameters that were included in each optimization are underlined.

P (μm)	H (μm)	c	α_{left}	α_{right}	ϵ	η
$\underline{6.8}$	$\underline{4.8}$	0.5	1	1	2.763	7.0%
$\underline{5.9}$	$\underline{6.1}$	1	1	1	2.763	6.1%
$\underline{6.5}$	$\underline{4.6}$	$\underline{0.55}$	$\underline{1.12}$	$\underline{1.07}$	2.763	7.1%
$\underline{3.5}$	$\underline{2.8}$	0.5	1	1	$\underline{6.34}$	11.0%
$\underline{2.7}$	$\underline{2.2}$	1	1	1	$\underline{6.34}$	7.5%
$\underline{3.3}$	$\underline{2.8}$	$\underline{0.57}$	$\underline{1.07}$	$\underline{1.09}$	$\underline{6.34}$	11.1%

In the work of Bay *et al.*,[18, 19] the highest light-extraction efficiency was achieved with the factory-roof geometry (right-centered triangle). We consider therefore this simple case first and include only the period P and the height H in the optimization. The remaining geometrical parameters are fixed to $c = 1$ (right-centered apex) and α_{left}=α_{right}=1 (straight edges). The dielectric constant of the material used for the surface texturation of the LED is fixed to ϵ=2.763 (manufacturer's value for the photoresist). As there are only two parameters to determine (P and H), we can reduce the population size to $n_{\text{pop}} = 50$ individuals. The parameters found by the GA are given in the second line of Table 1. They show that a light-extraction efficiency η of 6.1% can actually be obtained with a simple factory-roof geometry, by taking a period P of 5.9 μm and a height H of 6.1 μm. This result improves the value of η=5.7% achieved by Bay *et al.* for a period P of 5 μm and a height H of 6 μm. It proves that working with finer discretization steps (0.1 μm) leads indeed to solutions with higher efficiencies. Only 270 evaluations of the fitness were required by the genetic algorithm for this first case, out of 8,281 possible parameter combinations for P and H.

We can now generalize the shape of the surface texturation and include the whole set of geometrical parameters (P, H, c, α_{left} and α_{right}) in the optimization. We keep for the moment $\epsilon = 2.763$. As there are five parameters to determine, we work this time with a population size of $n_{\text{pop}} = 100$ individuals. The parameters found by the GA are given in the third line of Table 1. The GA required in this case 2058 evaluations of the fitness, out of 97,710,922,491 possible parameter combinations for P, H, c, α_{left} and α_{right}. This study shows that a light-extraction efficiency η of 7.1% can actually be obtained by generalizing the shape of the surface texturation. This improves the result of η=6.1% achieved with a simple factory-roof geometry and the value of η=3.7% achieved with a flat surface. The optimal structure determined by the GA is represented in Fig. 1. The solution found by the GA turns out to be essentially symmetric ($c \sim 0.5$), with straight edges ($\alpha_{\text{left}}, \alpha_{\text{right}} \sim 1$). Amongst the whole range of possible shapes considered this time by the genetic algorithm, it appears therefore that symmetric triangles actually maximize the extraction of light. This result is confirmed by an additional optimization, in which only P and H were included, the remaining geometrical parameters being fixed to c=0.5 (centered apex) and α_{left}=α_{right}=1 (straight edges). A value of η=7.0% is actually achieved for a period P and a height H, whose values are indeed very close to those obtained when considering the whole set of geometrical parameters (see first line of Table 1). Simple symmetric triangles appear therefore to be the optimal shape.

Since the fundamental reason that limits light extraction from the GaN is the high dielectric-constant contrast between the GaN and the emergent medium (air), we should also consider variations of the dielectric constant ϵ of the material used for the surface texturation. This study should suggest materials to use for this texturation in addition to the optimal shape. The results achieved when ϵ is included in the optimization are given in the last three lines of Table 1. We can distinguish again the results achieved when only P, H and ϵ are included in the optimization (fourth and fifth line) from those achieved when the full set of parameters is included (sixth line). If we include the whole set of parameters (P, H, c, α_{left}, α_{right} and ϵ) in the optimization, we finally obtain a light-extraction efficiency η of 11.1% (sixth line of Table 1). The GA required in this case 2235 evaluations of the fitness, out of 50,418,836,005,356 possible parameter combinations. This value of η=11.1% improves the value of 7.1% achieved previously when considering $\epsilon = 2.763$ (photoresist's value). Compared to the value of η=3.7%

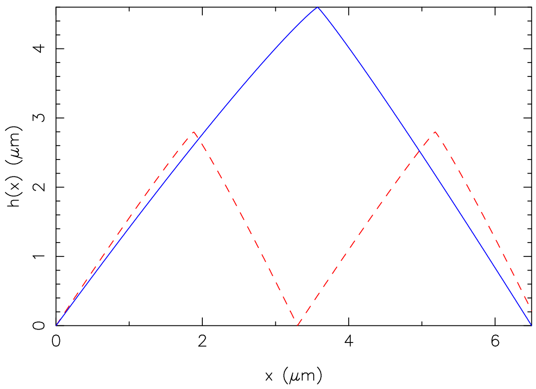

Figure 1. (Color online) Optimal shapes for the surface texturation of the GaN when optimizing on (i) P, H, c, α_{left} and α_{right} with ϵ=2.763 (solid), and (ii) P, H, c, α_{left}, α_{right} and ϵ (dashed). The parameters associated with these structures are given respectively in the third and sixth line of Table 1.

obtained with a flat surface, the light-extraction efficiency has been multiplied by three ! The optimal structure determined by the GA is represented in Fig. 1. The solution found by the GA turns out again to be essentially symmetric ($c \sim 0.5$), with straight edges ($\alpha_{\text{left}}, \alpha_{\text{right}} \sim 1$). Symmetric triangles appear again to be the optimal shape. The solution found by the GA also suggests that the material used for the surface texturation should have the same dielectric constant as the GaN. This conclusion is consistent with the fact that the internal reflections that limit the extraction of light are indeed related to contrasts in the dielectric constant. These conclusions are supported by additional optimizations in which only P, H and ϵ were included, the remaining geometrical parameters being fixed to c=0.5 (centered apex) or 1 (right-centered apex) and α_{left}=α_{right}=1 (straight edges). With symmetric triangles (fourth line of Table 1), we obtain a light-extraction efficiency of 11.0% with parameters that are comparable with those achieved when including the whole set of parameters in the optimization. With right-centered triangles (fifth line of Table 1), we obtain a light-extraction efficiency of 7.5%. It confirms our previous conclusion that symmetric triangles are better than right-centered triangles regarding the extraction of light. These additional optimizations also confirm our conclusion that the material used for the surface texturation should ideally have the same dielectric constant as the GaN.

4. Discussion

Initial training of the genetic algorithm was done on simple $f(x_1, \ldots, x_n) = \prod_{i=1}^{n} \cos^2 x_i \exp[-x_i^2/10]$ functions, where each $x_i \in [-5, 5]$ with a step Δx_i=0.01. The number n of parameters considered ranged between 2 and 6 as in this article. For an implementation of the GA in which parents are not transferred automatically to the next generation, a rate of crossover of 70% turns out to provide an ideal compromise between the conservation of good solutions and the exploration of new ones. We also appreciate the fact that this value implies a typical lifetime of three generations for a given individual (from a heuristic argument based on the fact that $0.3^4 < 0.01$). This initial training also revealed that a mutation rate of 1% provides a good compromise between achieving a sufficient exploration of the parameter space and getting the algorithm finally converge. For problems in which the optimum is more difficult to localize, a mutation rate of 2% may be used instead in order to boost the exploration. The data collected by the algorithm can finally be used in regular attempts to guess the optimal solution. This will help the GA converge still more rapidly.

When using $x_i = x_i^{\min} + \langle \text{gene } i \rangle \times \Delta x_i$ to achieve a strict representation of the parameter specifications, we must reject gene values that lead to $x_i > x_i^{\max}$. For the problem considered in this article, this implies an intrinsic rejection rate of 78% when all six parameters are included in the optimization. We can actually easily deal with this issue. When generating random individuals, we only accept individuals with gene values such that $x_i \leq x_i^{\max}\ \forall i$. When crossing the DNA of two parents, we make $\sum_{1=1}^{n} n_i - 1$ random attempts to get children with acceptable gene values. We keep the parents unchanged if these attempts do not succeed. When introducing random mutations in a DNA, we make repetitive attempts until these mutations lead to an acceptable individual (only mutations introduced in this successful attempt are then considered for the modified DNA). Unexpected cases can finally be assigned a fitness of zero. The time required for this management was actually negligible compared to that required by the fitness calculations. Compared to a GA implementation in which $x_i = x_i^{\min} + \langle \text{gene } i \rangle \times \frac{x_i^{\max} - x_i^{\min}}{2^{n_i} - 1}$ is used for the representation of parameters (with all gene values accepted), working with a strict representation of parameters turns out to reduce the number of parameter combinations to consider and therefore the time required to converge to the optimal solution.

Fig. 2 shows the evolution of the best fitness (f_{best}) achieved at each generation for the different cases presented in Table 1. The figure shows that the genetic algorithm converges indeed rapidly to optimal solutions, which improve significantly the light-extraction efficiency obtained with a flat surface ($\eta = 3.7\%$). The best solutions are obtained with symmetric triangles. Optimizations that include the dielectric constant of the material used for the surface texturation of the LED provide still better solutions. Fig. 3 shows the corresponding evolution of the genetic similarity (s). It shows that the genetic algorithm had a regular dynamics, with s growing progressively from an initial value of 0.5 (random population) to 1 (dominance of the population by the best individual). The best solution achieved in this article provides a light-extraction efficiency of 11.1%, which improves significantly the value of 5.7% achieved in previous work.[18, 19]

The application considered in this work required extensive computational resources because of the time required by each fitness calculation (up to 35 hours of cpu time on a Tier-1 supercalculator). With a total of 50,418,836,005,356 possible parameter combinations to consider, optimization methods based on a systematic scan on parameters would have been untractable. Genetic algorithms provide however an efficient approach to global optimization problems. They involve indeed a collective

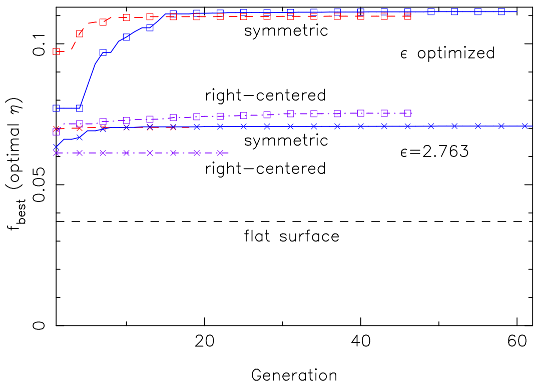

Figure 2. (Color online) Best fitness in the population for each generation. Solid lines correspond to optimizations in which P, H, c, α_{left} and α_{right} are included, dashed lines to optimizations restricted to P and H with c=0.5 and α_{left}=α_{right}=1, and dot-dashed lines to optimizations restricted to P and H with c=1 and α_{left}=α_{right}=1. Crosses indicate optimizations for which ϵ=2.763. Squares indicate optimizations in which ϵ was included. The horizontal line indicates the light-extraction efficiency achieved with a flat surface ($\eta = 3.7\%$).

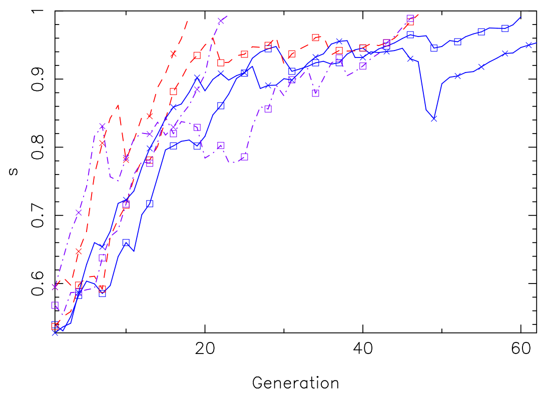

Figure 3. (Color online) Genetic similarity for each generation. Solid lines correspond to optimizations in which P, H, c, α_{left} and α_{right} are included, dashed lines to optimizations restricted to P and H with c=0.5 and α_{left}=α_{right}=1, and dot-dashed lines to optimizations restricted to P and H with c=1 and α_{left}=α_{right}=1. Crosses indicate optimizations for which ϵ=2.763. Squares indicate optimizations in which ϵ was included.

exploration of the parameter space and all fitness computations can be done in parallel. When including for example all six parameters in the optimization (sixth line of Table 1), 2235 fitness evaluations were required along 60 generations. With an average cpu time of 20 hours per evaluation, this required a total cpu time of the order of 45,000 hours. As these fitness calculations could however be done in parallel, the wallclock time for this optimization was reduced to 2,100 hours (3 months).

5. Conclusion

This work aimed at optimizing the light-extraction efficiency of a GaN light-emitting diode by considering two-dimensional periodic texturations of the surface of the LED. A genetic algorithm was used for the optimization of parameters. Starting from a light-extraction efficiency η of 3.7% for a flat surface, the solution finally found by the GA enables a light-extraction efficiency of 11.1%. The results show that symmetric triangles actually correspond to the optimal shape for the surface texturation. They also indicate that the material used for this surface texturation should ideally have the same dielectric constant as the GaN. The consideration of three-dimensional structures for the surface texturation of the LED may lead to still higher efficiencies. This will be explored in future work.

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