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Theoretical analysis of the energy exchange and cooling in field emission from the n-type semiconductor

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Field emission has been theoretically found to contribute to the cooling only for the semiconductor cathodes. Using the formal theory developed recently by authors, we have calculated the energy exchange $\Delta\varepsilon$ as a function of temperature T and field F. It is found that the obtained $\Delta\varepsilon$ is positive for all T and large enough for the considerable cooling at the room temperature. Even though the Joule heat is considered, field emission yields the net cooling effect. It is also found that the cooling is more effective for the n-GaN cathode than for the n-Si.

I. INTRODUCTION

Field emission is the quantum mechanical tunneling of electrons from a cathode material to the vacuum under an applied electric field. Compensating electrons are injected into the cathode from a back contact, which is called the replacement (see Fig. 1). During the emission-replacement process, the outgoing energy is not equal to the incoming energy whereas charge is conserved within the cathode. An energy transfer takes place between the cathode and the external in the circuit. Thus the cathode can be heated or cooled by the so-called the Nottingham effect.[1] It is known that field emission from a metallic cathode contributes to heating at low temperatures and to cooling at high temperatures.[2-4] It seems that a metallic cathode cannot produce the cooling effect accessible at room temperature. In fact, it may not result in a success to make the direct refrigeration by electron emission from carbon nanotubes at room temperature.[5]

The Nottingham effect plays an important role for stabilizing temperature in field emission. However, the mechanism of the replacement process in metallic emitters had

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been in question for a half century since the controversy between Nottingham[1] and Fleming and Henderson.[2] Each predicted different values of the average energy, $<\varepsilon_r>$, of replacement electrons. Engle and Cutler[6] and Miskovsky et al.[7] treated $<\varepsilon_r>$ as an adjustable parameter to obtain the inversion temperature in improved agreement with experiment. Later, Chung et al.[8] did a systematic theoretical treatment of the replacement process to obtain the inversion temperature T_i in agreement with experimental observation of Swanson et al.[9] and Drechsler.[10] Very recently, we have developed a formal theory for replacement in field emission from semiconductors.[11,12] The theory leads to a surprising result that field emission from semiconductors yields the cooling only to the cathode. In the current paper, we use the same formalism to calculate the energy exchange in field emission from n-type semiconductors of Si and GaN. Further we make analysis of cooling effect due to field emission.

II. ENERGY EXCHANGE IN FIELD EMISSION

Consider the steady state of field emission. Many electrons are emitted from the cathode and the same number of electrons are simultaneously injected into it. The energy exchange per electron between the two types of electrons is defined as

$$\Delta\varepsilon = <\varepsilon_e> - <\varepsilon_r>, \tag{1}$$

where $<\varepsilon_e>$ and $<\varepsilon_r>$ are the average energies of emission and replacement electrons, respectively. This definition denotes the energy loss of the cathode through field emission of one electron, implying that the cathode is cooled by the amount of $\Delta\varepsilon$ per electron.

As usual, the average energy $<\varepsilon>$ is given by

$$<\varepsilon> = \frac{\int \varepsilon j(\varepsilon) d\varepsilon}{\int j(\varepsilon) d\varepsilon}, \tag{2}$$

where $j(\varepsilon)$ is the energy distribution of carrier. The average energies $<\varepsilon_e>$ and $<\varepsilon_r>$ are obtained using the emission or replacement electron energy distributions, $j_e(\varepsilon)$ and $j_r(\varepsilon)$, respectively. For field emission from the n-type semiconductor under the field applied along the x-axis, they are given by[11,13]

$$j_e(\varepsilon) = (4\pi m e / h^3)\int_{\varepsilon_c}^{\varepsilon} f(\varepsilon) D(\varepsilon_x) d\varepsilon_x \,, \tag{3}$$

$$j_r(\varepsilon) = (4\pi m e / h^3)\int_{\varepsilon_c}^{\varepsilon} f(\varepsilon)[1 - f(\varepsilon) + f(\varepsilon) D(\varepsilon_x)] d\varepsilon_x \,, \tag{4}$$

where ε_c is the conduction band minimum, $f(\varepsilon)$ the Fermi distribution function, and $D(\varepsilon_x)$ the transmission coefficient with normal energy ε_x. In Eq. (4), the terms 1-$f(\varepsilon)$ and $f(\varepsilon)D(\varepsilon_x)$ represent the probabilities for the empty state to be produced by thermal excitation and tunneling, respectively. At given temperature T and field F, evaluations of Eqs. (3) and (4) can be easily made with material parameters of the cathode. Substitutions of the obtained $j_e(\varepsilon)$ and $j_r(\varepsilon)$ into Eq. (2) yields the average energies $<\varepsilon_e>$ and $<\varepsilon_r>$, respectively. Then we use Eq. (1) to find $\Delta\varepsilon$ as a function of T and F. It is clear that exact calculations should be made numerically. It begins with the exact calculation of $D(\varepsilon)$. Such a calculation has been formulated in the several ways.[14,15] Without much difficulty, $D(\varepsilon)$ was exactly calculated and used in the calculation for the cooling effect.[16,17]

Analytic calculations of $\Delta\varepsilon$ are possible if we use the WKB(Wentzel-Kramers-Brillouin) transmission coefficient[18]

$$D(\varepsilon_x) = \exp[-c + (\varepsilon_x - \varepsilon_c)/d] \,, \tag{5}$$

where c and d are the field-dependent number and energy, respectively. Substituting Eq. (5) into Eqs. (3) and (4), we obtain the analytic forms

$$j_e(\varepsilon) = \frac{4\pi m e}{h^3} d\, e^{-c} \frac{\exp[(\varepsilon - \varepsilon_c)/d] - 1}{1 + \exp[(\varepsilon - \mu)/kT]} \,, \tag{6}$$

$$j_r(\varepsilon) = \frac{4\pi m e}{h^3} \frac{(\varepsilon - \varepsilon_c)\exp[(\varepsilon - \mu)/kT] + d\, e^{-c}\,(\exp[(\varepsilon - \varepsilon_c)/d] - 1)}{(1 + \exp[(\varepsilon - \mu)/kT])^2} \,. \tag{7}$$

Since $j_e(\varepsilon)$ and $j_r(\varepsilon)$ given by Eqs. (6) and (7) play as the weighting factor, the ε-independent factors may be dropped out in the calculation of $<\varepsilon_e>$ and $<\varepsilon_r>$. Thus we normalize $j_e(\varepsilon)$ and $j_r(\varepsilon)$, and plot them as a function of T and F in Figs 2-5 for both the n-type Si and GaN semiconductors. Using these graphs, we can investigate the features of $<\varepsilon_e>$ and $<\varepsilon_r>$.

It is seen that all $j_e(\varepsilon)$ are spread further over the higher energy range with respect to the associated $j_r(\varepsilon)$. This implies that $<\varepsilon_e>$ is always higher than $<\varepsilon_r>$ at given T and F. That is, $\Delta\varepsilon$ is positive for all T. In Figs. 2 and 3, it is seen that the shift of $j_e(\varepsilon)$ toward the higher energy becomes larger at higher T, indicating that $\Delta\varepsilon$ increases with increasing T. It is also seen in Figs. 4 and 5 that the shift becomes larger at weak F, indicating that $\Delta\varepsilon$ decreases with increasing F. These features are confirmed by quantitative analysis of $\Delta\varepsilon$ later.

To investigate the F-dependence behaviors of $j_e(\varepsilon)$ and $j_r(\varepsilon)$, we look at Eqs. (6) and (7). What matters is the roles of field-dependent number c, more specifically, e^{-c}. According to Good,[18] e^{-c} is almost zero only except for strong F. By the way, the term is cancelled out in normalized $j_e(\varepsilon)$. Thus $j_e(\varepsilon)$ has only $d\,(\propto F/\phi^{1/2})$ as the F-dependent quantity. Since it is in the denominator of the exponent, $j_e(\varepsilon)$ decreases with increasing F as shown in Figs. 4 and 5. This reflects that the deeper (i.e., lower) states in the conduction band can contribute increasingly to field emission at stronger F.

This becomes a different story in the normalized $j_r(\varepsilon)$. Since e^{-c} appears in one term of the two in Eq. (7), the term is negligible in the normalized $j_r(\varepsilon)$. Thus the normalized $j_r(\varepsilon)$ deos not change with F except for strong F. This is why $j_r(\varepsilon)$ remains almost the same even for considerable change of F as shown in Figs. 4 and 5. Such dependences can be understood in the following reason. For the semiconductor emitter, field emission can not make a considerable change in the density of empty states because there are already a lot of empty states in the conduction band. As a result, $j_r(\varepsilon)$ is rarely changed even by the meaningful change of F.

Figs. 2 and 3 show the T-dependence of the two distributions at given F for the n-type Si and GaN semiconductors, respectively. The value of F is chosen to be V/nm for n-Si and 1.2 V/nm for n-GaN so that field emission produces the current density $j \sim$ mA/cm^2. Figs. 4 and 5 exhibits the F-dependence of the two distributions at room T for the n-type Si and GaN semiconductors, respectively. Comparisons between Figs. 2 and 3 and between Figs. 4 and 5 exhibit the difference. This is due to the material parameters such as the electron affinity (i.e., ϕ), the Fermi level E_F, and dielectric constant κ. It is easily seen that the effect of ϕ is considered in the quantity d ($\propto F/\phi^{1/2}$). The effects of E_F and κ are included implicitly and are not considered to large.

III. DISUSSIONS

The energy exchange $\Delta\varepsilon$ is calculated as a function of T and F. The current formalism can be used to calculate over the very wide range of T and F except the case of high T and weak F. Using both $j_e(\varepsilon)$ and $j_r(\varepsilon)$ given by Eqs. (6) and (7), we make numerical calculations of Eq. (2) to obtain the average energy, $<\varepsilon_e>$ and $<\varepsilon_r>$, as a function of T and F. In the previous work, we use Eqs. (6) and (7) to obtain the analytic forms of $<\varepsilon_e>$ and $<\varepsilon_r>$.[11,12] However, they are expressed in terms of a little sophisticated mathematical functions. In addition, the results are often obtained with rough approximation. Even though worthwhile to be derived , the analytic forms may sometimes yield rough values of $\Delta\varepsilon$. Thus we make numerical integration of Eq.(2). The current work must yield more exact $\Delta\varepsilon$ in comparison with the previous work even if some approximation is made in obtaining the analytic form of $j_e(\varepsilon)$ and $j_r(\varepsilon)$. Of course, the entire numerical work is possible and can be made without difficulty. However, we would like use the analytic forms of $j_e(\varepsilon)$ and $j_r(\varepsilon)$ because they show the T- and F-dependence more in more explicit way.

The obtained $<\varepsilon_e>$, $<\varepsilon_r>$, and $\Delta\varepsilon$ are plotted as a function of T in Fig. 6 for n-Si and in Fig. 7 for n-GaN. The two values of F are chosen so that the field emission current density $j \sim 10^{-3}$ and 10^{-6} A/cm^2, respectively (see Table 1). We choose the doping concentration n=10^{19}/cm^3. With large n, E_F changes. It is lower than ε_c for Si but is higher than ε_c for GaN. Since all energies are represented with respect to E_F, the two energies, $<\varepsilon_e>$ and $<\varepsilon_r>$, changes with n. By cancellation, however, $\Delta\varepsilon$ doens change with E_F. Thus both $<\varepsilon_e>$ and $<\varepsilon_r>$ are located higher for n-Si or lower in n-GaN with respect to $\Delta\varepsilon$ at low T (see Figs. 6 and 7).

As shown in Figs. 6 and 7, $\Delta\varepsilon$ increases as T increases. If T increases from 300K to 600K, then $\Delta\varepsilon$ increases from 8 meV to 61 meV for Si. For GaN, $\Delta\varepsilon$ increases from 16 meV to 204 meV for the same increase of T. It is noted that at high T, $\Delta\varepsilon$ increases rapidly with increasing T. This change of $\Delta\varepsilon$ sometimes depend strongly on F. The weak F enhance the T-dependence of $\Delta\varepsilon$. This is due the property of $<\varepsilon_e>$. As mentioned above, $<\varepsilon_r>$ does not change with F. Thus the two graphs of $<\varepsilon_r>$ for the two F overlap as seen in Figs. 6 and 7. As seen in Table 1 and in Fig. 6 for Si, $\Delta\varepsilon$ increases from

8 meV to 18 meV as F decreases from 2.0 V/nm to 1.3 V/nm at T=300K. It changes from 34 meV to 102 meV for the same decrease at T=500K. The F-dependence of $\Delta\varepsilon$ becomes larger for high T. In all, the cooling effect is very efficient at high T and weak F. It seems that the obtained value of $\Delta\varepsilon$ is enormous amount of energy because it is obtained by emission of one electron.

Now consider the cooling effect of the cathode. The cooling power (or rate) is defines as the total energy loss of the cathode per unit time. It is $\Delta\varepsilon(I/e)$ where I is the current. The $\Delta\varepsilon$ is the energy loss per electron and I/e is the total number of electrons passing through the cathode per unit time. Instead, the current I also produces the Joule heat in the cathode. The heating power is $I^2 R_c$, where R_c is the resistance of the cathode. Thus the net cooling power Γ is the sum of the two:

$$\Gamma = \Delta\varepsilon I/e - I^2 R_c . \qquad (8)$$

We change the form of Eq. (8): $\Gamma = -R_c(I - \Delta\varepsilon/2eR_c)^2 + (\Delta\varepsilon)^2/4e^2R_c$. There are the two important features. First, field emission contributes to cooling of the cathode for all T in the range of I between 0 and $\Delta\varepsilon/eR$. Second, the maximum net cooling power is $\Gamma = \Gamma_m = (\Delta\varepsilon)^2/4e^2R_c$ at $I = I_m = \Delta\varepsilon/2eR$. It is interesting that at $\Gamma = \Gamma_m$, Γ is a half the cooling power of field emission (i.e., $\Delta\varepsilon(I/e)$) and equal to the Joule heating power ($I^2 R_c$) in magnitude. These features are shown in Fig. 8. It is found that since I_m is large enough, field emission from semiconductor contributes to cooling in almost every situation.

Now we introduce the cooling efficiency η of the field emission cooling device.[19] It is assumed that the cathode is in thermal contact with the sample to be cooled.[5] Then η is defined as the rate of heat removed from the sample to the power input:

$$\eta = \frac{\Gamma}{IV}, \qquad (9)$$

Here, V is the input bias. Using the relation $V = IR_t$, where R_t is the total resistance of the entire circuit, we obtain the simple relation $\eta = R_c / R_t$ at $I = I_m$ (i.e., when the cooling power is maximum). This is not the maximum. As seen in Fig. 8, the maximum η should be obtained for $I < I_m$.

V. CONCLUSIONS

We calculated the energy exchange $\Delta\varepsilon$ in field emission from n-type Si and GaN semiconductors. The energy exchange $\Delta\varepsilon$ is positive for all T, indicating that field emission contributes to cooling for all T. When the Joule heat is considered, the net cooling effect is obtained in the certain range of the current I. The maximum cooling power of I^2R_c is obtained at $I = \Delta\varepsilon / 2eR_c$, where R_c is the resistance of the cathode. This implies that the use of field emission leads to the fabrication of a new generation of cooler.

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Table 1. Energy exchange $\Delta\varepsilon$ at T=300K. The values of F are chosen so that the field emission current density $j \sim 10^{-9}$, 10^{-6}, and 10^{-3} A/cm^2 for n-type Si and GaN semiconductor, respectively. For both Si and GaN, we choose the doping density n=10^{19}/cm^3.

	n-Si			n-GaN		
F (V/nm)	1.3	1.6	2.0	0.8	1.0	1.2
j(A/cm^2)	1.1x10^{-9}	2.3x10^{-6}	1.7x10^{-3}	7.2x10^{-9}	2.8x10^{-5}	7.1x10^{-3}
$\Delta\varepsilon$ (meV)	18	12	8	37	23	16

Figure Captions

Fig. 1. Schematic of field emission from the n-type semiconductor. The electron injected in the cathode occupies the empty state evacuated by thermal excitation or tunneling.

Fig. 2. Energy distributions, $j_e(\varepsilon)$ and $j_r(\varepsilon)$, of field and replacement electrons for n-Si at T=300 and 800 K. At higher T, $j_e(\varepsilon)$ is spread further over the higher energy range with respect to $j_r(\varepsilon)$.

Fig. 3. Energy distributions, $j_e(\varepsilon)$ and $j_r(\varepsilon)$, of field and replacement electrons for n-GaN at T=300 and 800 K. As T becomes high, $j_e(\varepsilon)$ is spread further over the higher energy range with respect to $j_r(\varepsilon)$.

Fig. 4. Energy distributions, $j_e(\varepsilon)$ and $j_r(\varepsilon)$, of field and replacement electrons for n-Si at F=1.3 and 2.0 V/nm. For the increase of F, $j_e(\varepsilon)$ decreases while $j_r(\varepsilon)$ remains unchanged.

Fig. 5. Energy distribution of field and replacement electrons for n-GaN at F=0.8 and 1.2 V/nm. For the increase of F, $j_e(\varepsilon)$ decreases while $j_r(\varepsilon)$ remains unchanged.

Fig. 6. $\Delta\varepsilon$ vs. T for n-Si. The energy exchange $\Delta\varepsilon$(solid lines) is the difference between $<\varepsilon_e>$(dash lines) and $<\varepsilon_r>$(dotted lines). The $\Delta\varepsilon$ increases with increasing T and decreasing F

Fig. 7. $\Delta\varepsilon$ vs. T for n-GaN. The energy exchange $\Delta\varepsilon$(solid lines) is the difference between $<\varepsilon_e>$(dash lines) and $<\varepsilon_r>$(dotted lines). The $\Delta\varepsilon$ increases with increasing T and decreasing F

Fig. 8. Cooling power of the semiconductor field emission device. The net cooling power(solid line) is given as the cooling power (dash line) minus the heating power (dotted line). The net cooling effect is accessible in the region of current I between 0 and $\Delta\varepsilon / eR$.

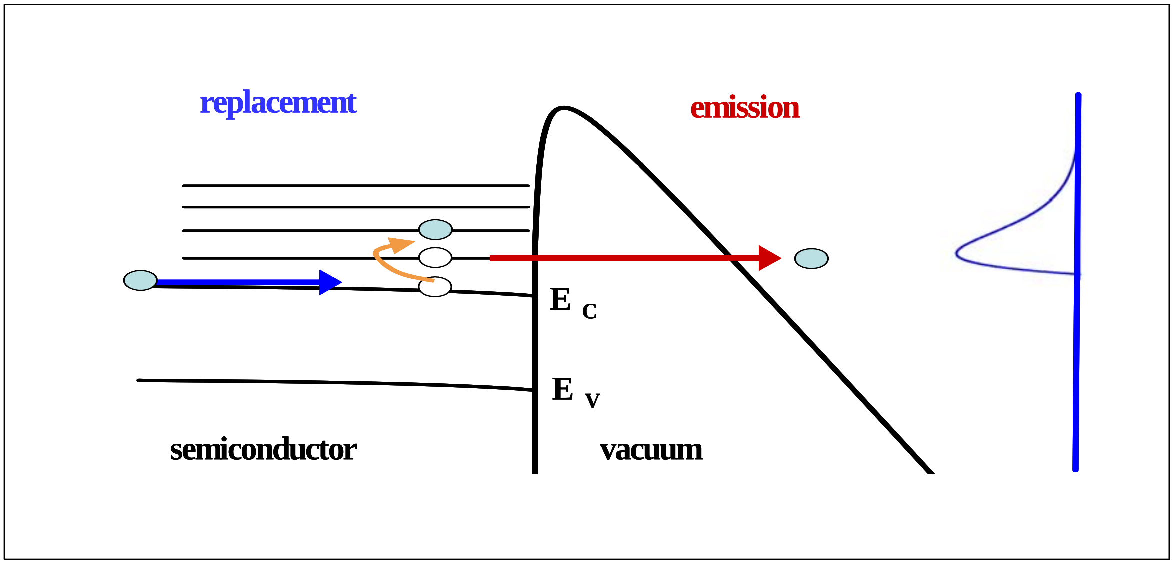


Fig. 1

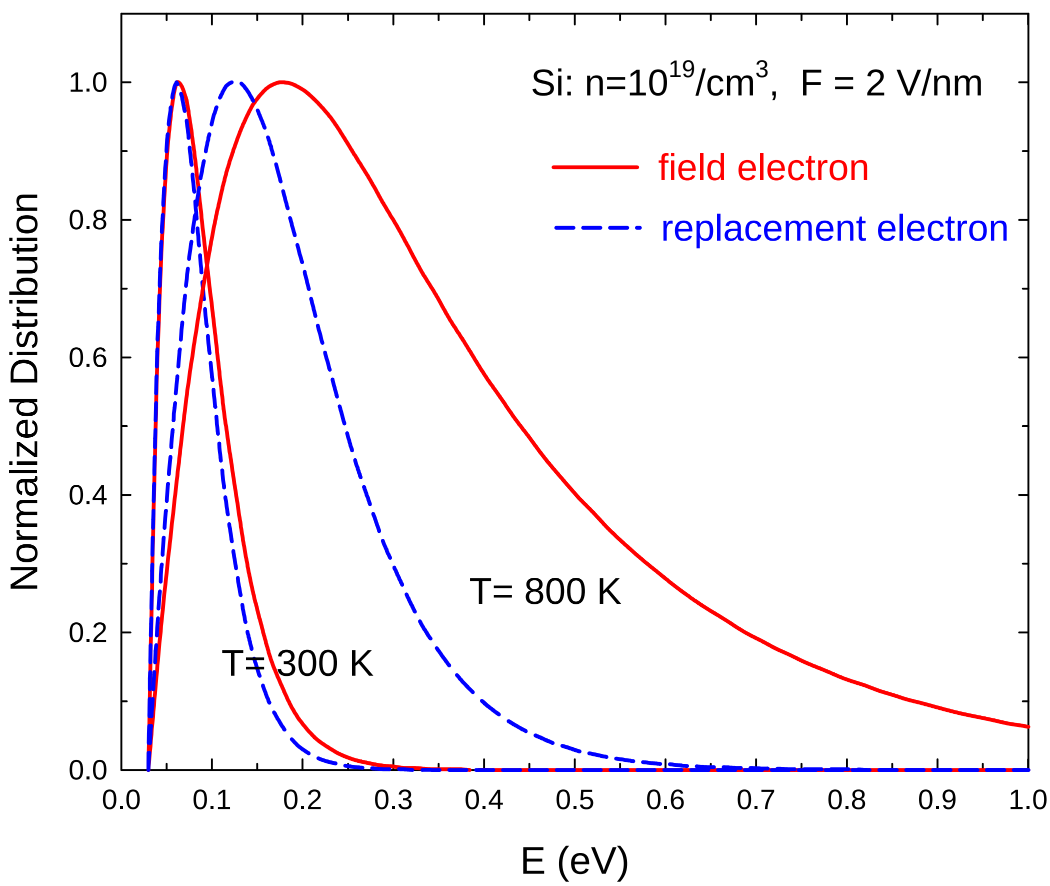
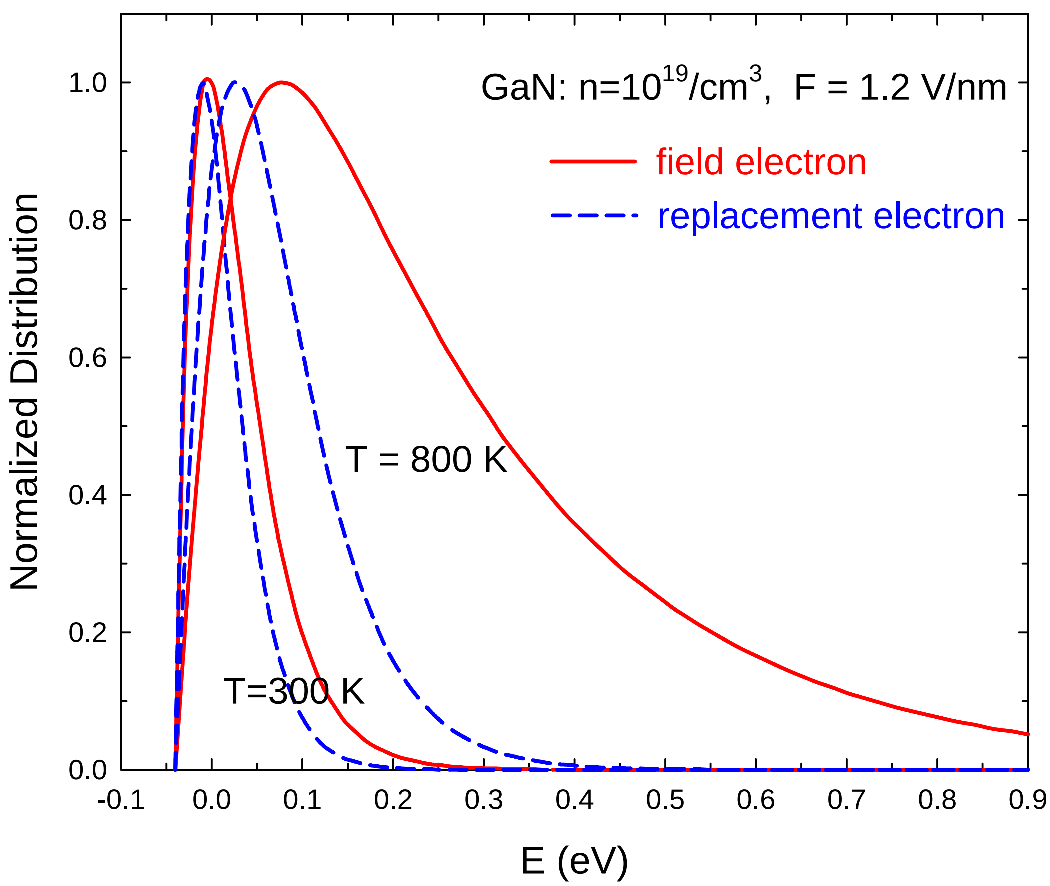

Si: $n=10^{19}/cm^3$, F = 2 V/nm
field electron
replacement electron
T= 800 K
T= 300 K
Normalized Distribution
E (eV)
0.0
0.2
0.4
0.6
0.8
1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Fig. 2



GaN: n=10^19/cm^3, F = 1.2 V/nm
field electron
replacement electron
T = 800 K
T=300 K
Normalized Distribution
E (eV)
0.0
0.2
0.4
0.6
0.8
1.0
-0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

Fig. 3

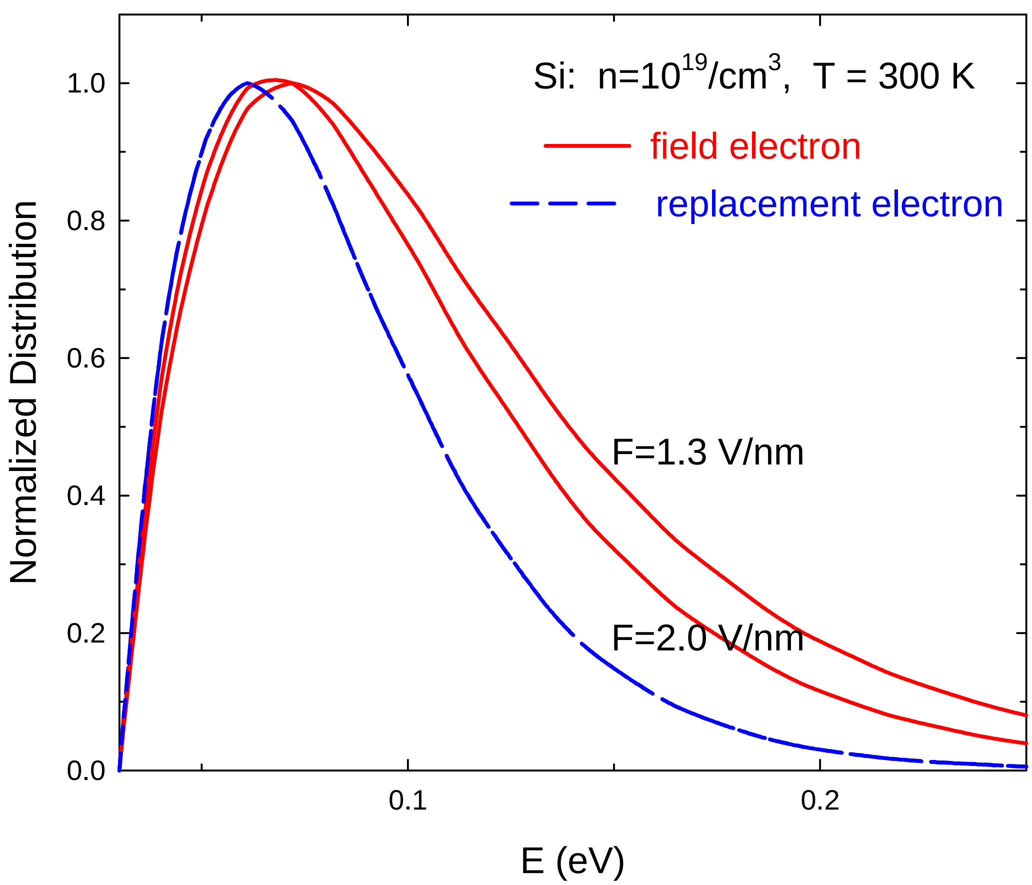
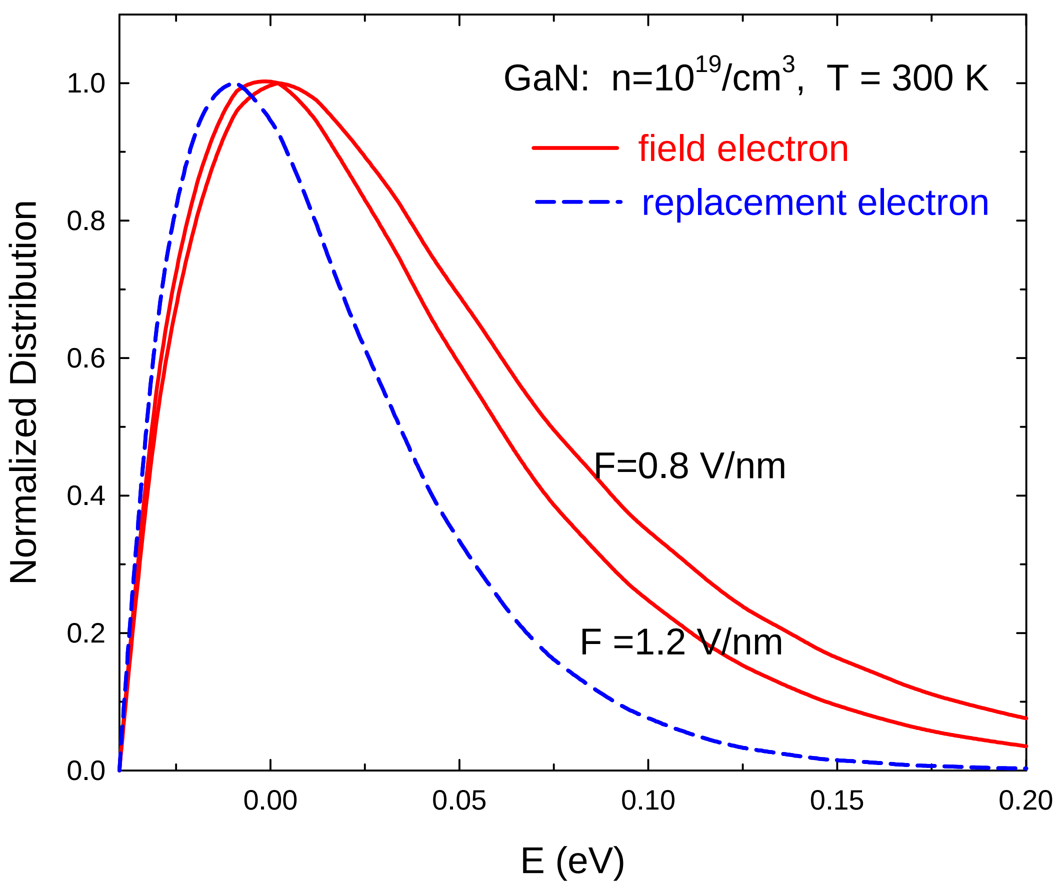

Si: n=10^19/cm^3, T = 300 K
field electron
replacement electron
F=1.3 V/nm
F=2.0 V/nm
Normalized Distribution
E (eV)
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.2

Fig. 4



GaN: n=10^19/cm^3, T = 300 K
field electron
replacement electron
F=0.8 V/nm
F =1.2 V/nm
Normalized Distribution
E (eV)
0.0
0.2
0.4
0.6
0.8
1.0
0.00
0.05
0.10
0.15
0.20

Fig. 5

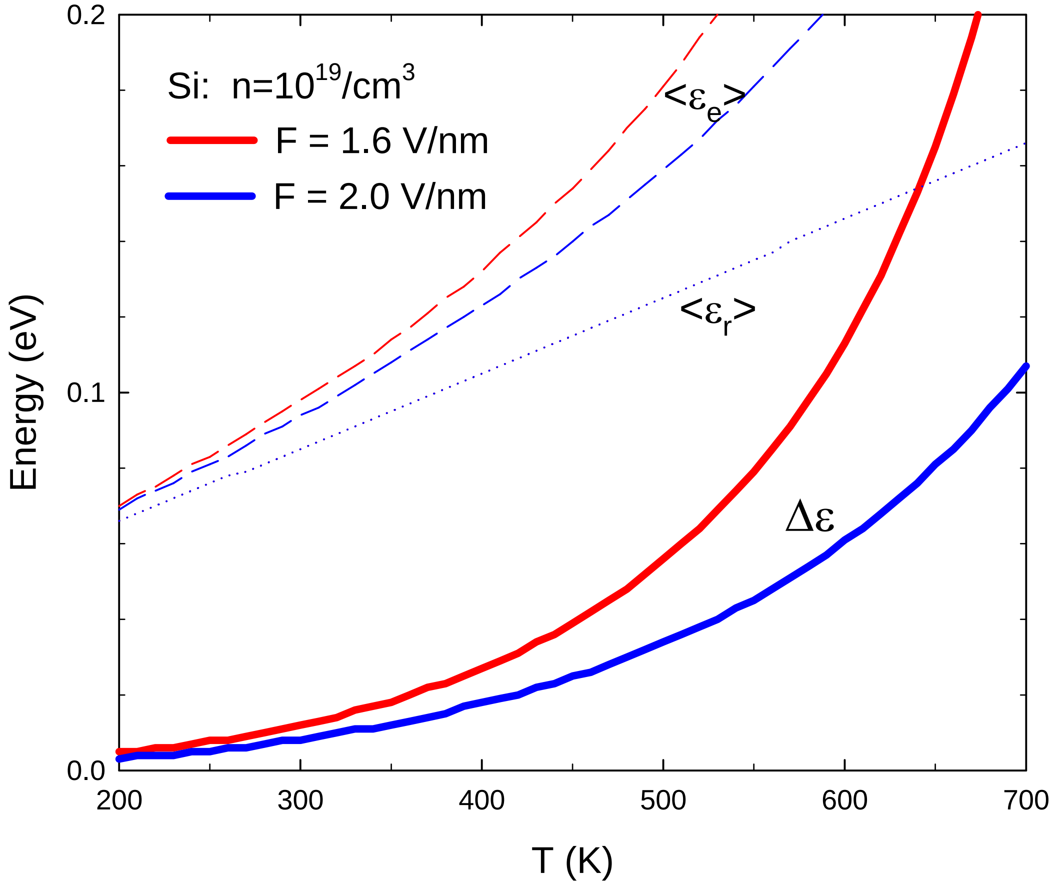
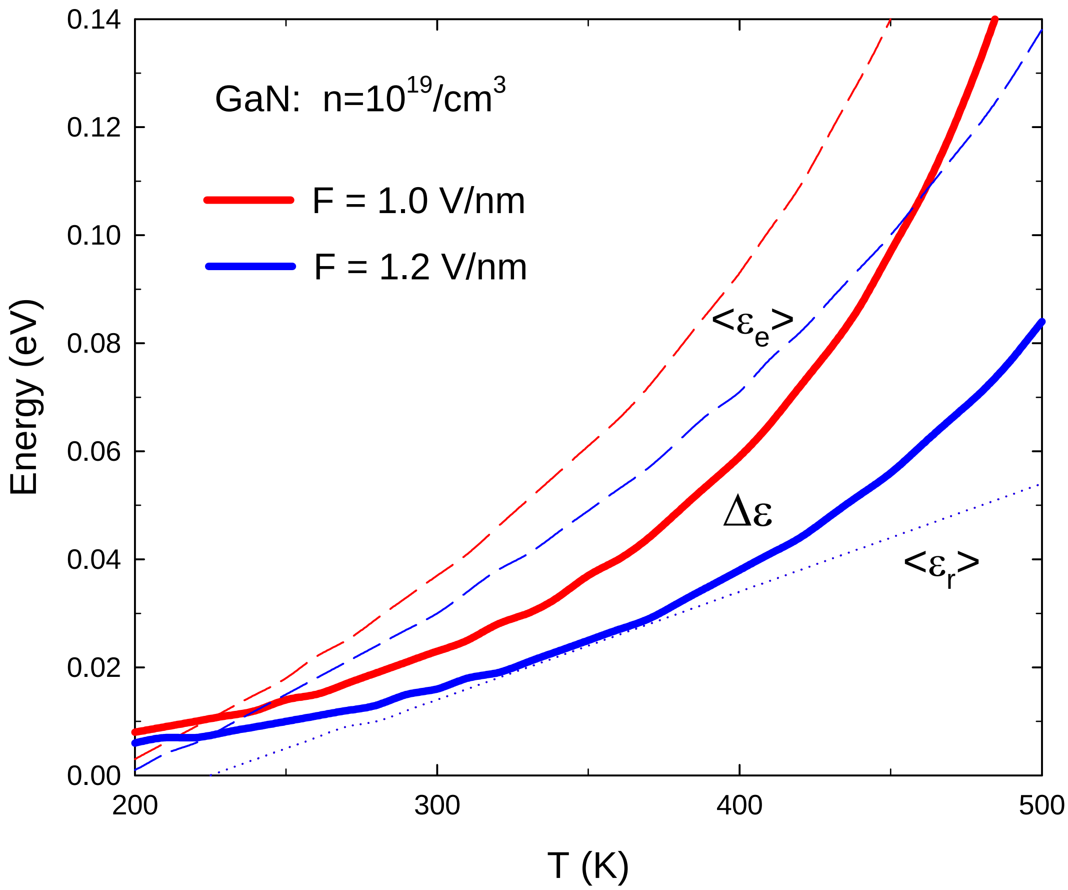

Si: n=10^19/cm^3
F = 1.6 V/nm
F = 2.0 V/nm
<εe>
<εr>
Δε
Energy (eV)
T (K)
0.0
0.1
0.2
200
300
400
500
600
700

Fig. 6



GaN: n=10^19/cm^3
F = 1.0 V/nm
F = 1.2 V/nm
<ε_e>
Δε
<ε_r>
Energy (eV)
T (K)
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
200
300
400
500

Fig. 7

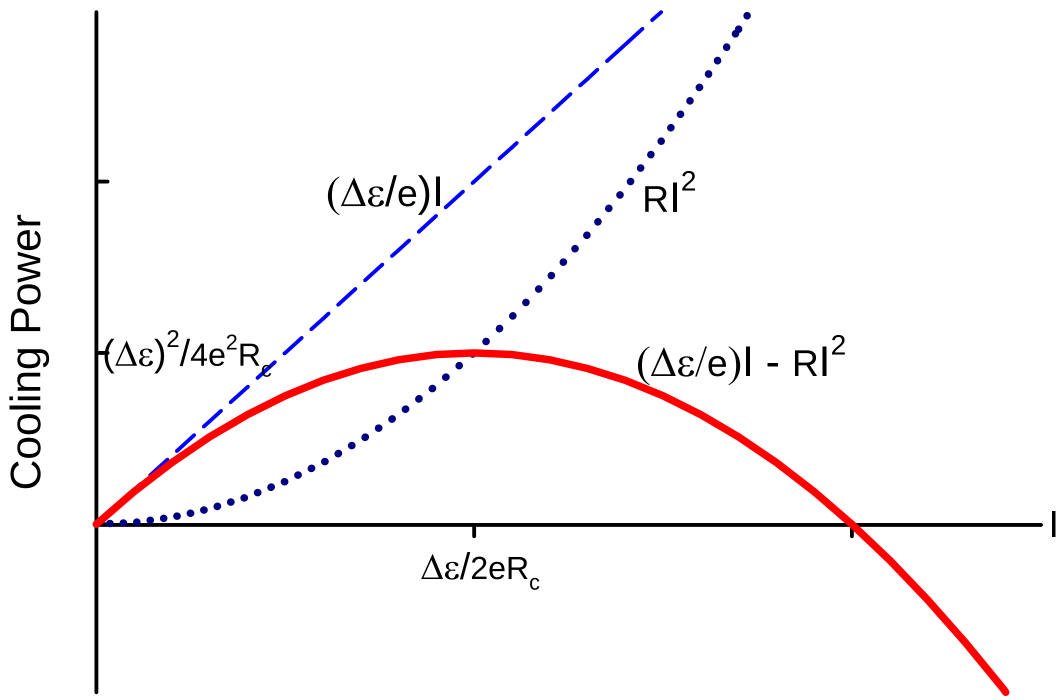

Cooling Power
$(\Delta\varepsilon/e)I$
RI^2
$(\Delta\varepsilon)^2/4e^2R_c$
$(\Delta\varepsilon/e)I - RI^2$
$\Delta\varepsilon/2eR_c$
I

Fig. 8