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Brane matter, hidden or mirror matter, their various avatars and mixings: many faces of the same physics

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Numerous papers deal with the phenomenology related to photon-hidden photon kinetic mixing and with the effects of a mass mixing on particle-hidden particle oscillations. In addition, recent papers underline the existence of a geometrical mixing between branes which would allow a matter swapping between branes. These approaches and their phenomenologies are reminiscent of each other but rely on different physical concepts. In the present paper, we suggest there is no rivalry between these models, which are probably many faces of the same physics. We discuss some phenomenological consequences of a global framework.

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I. INTRODUCTION

Since a long time, there is a growing interest for physical models related to hidden matter or hidden worlds. Numerous theoretical and experimental works result from this line of thought [1–24]. In those approaches, the standard model of particles splits into a visible sector (to give the usual "baryonic" matter) and one (or many) hidden sector(s) (to give hidden "baryonic" matter). Two approaches are then considered: the hidden matter could be dissimulated in other braneworlds located somewhere in a higher dimensional bulk [3–7] or could be sterile particles in our four-dimensional spacetime [8, 9, 11, 12]. In both cases, it is expected that many puzzling phenomena could be explained in such a framework.

For instance, in the mirror matter concept, the parity violation in the weak interaction could be explained by adding a mirror sector to the particle content. This idea was first evoked by Lee and Yang in 1956 [1]. Thus, for each left-handed neutrino, a right-handed mirror neutrino restores parity. Commonly, in the mirror-world scenario, the standard model splits in two sectors with an opposite parity symmetry breaking [8, 9, 11, 12]. Usually, it is assumed that particles and mirror particles do not interact except through gravitation. Then, mirror matter may exist with similar properties as usual matter but would be undetectable to us through electromagnetic radiations. Mirror particles are then considered as plausible candidates for dark matter^a by some authors [9, 10, 13, 14]. In recent years, extensions of the original idea were suggested which allow couplings between matter and hidden matter at the quantum level [8–12].

These couplings can include photon-mirror photon kinetic mixing [11, 12] or neutrino-mirror neutrino [8] and neutron-mirror neutron [9, 10] mass mixing. It must be noted that this work is all reminiscent of the concept of a shadow universe first considered in 1965 by Nishijima and Saffouri [2]. The idea has also been extended to many coexisting hidden sectors without necessarily mirror symmetry [4].

In the other hand, the braneworld idea is considered as a relevant approach to unify physics [17–19]. The idea is that our visible Universe is a three-dimensional space sheet embedded in a hyperspace (called the bulk). The particles of the standard model are then trapped along such a sheet, which is called a braneworld. In some work, several braneworlds, invisible to each other, live in the bulk each one with its own copy of the standard model. Several issues are concerned by this approach: the hierarchy between the electroweak and the Planck scales [18], the cosmic acceleration or the dark matter origin [3] for instance. Dark matter could be explained as a hidden baryonic matter localized on another braneworld, provided that gravitation can spread enough into the bulk. In addition to gravitational interaction, photon-hidden photon kinetic mixing are expected to allow matter coupling between branes [5–7, 11, 12]. In recent theoretical work it has also been shown that a geometrical mixing must occur between the matter fields of two braneworlds [20–22]. As a consequence, usual matter (related to the standard model) could leap from our braneworld toward a hidden one, and vice versa [23, 24].

Obviously, all these approaches – related to braneworlds or mirror symmetry for instance – share a similar phenomenology where a hidden matter state can oscillate with the visible matter state. If such a kind of effect would exist, it would then be legitimate to assume that there should exist a single unified model which could adequately describe this phenomenon, avoiding the multiplicity of exotic mathematical and physical solutions. In the present work, we show that such a model exists indeed and that there is therefore no competition between the previously mentioned approaches. The geometrical

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^a Mirror matter is able to self-interact. Though it is usually assumed that dark matter does not interact with itself, contrary to visible matter, some authors underlined that dark matter could be self-interacting after all [15, 16].

mixing and the matter swapping between branes, the mass mixing and the particle-mirror(-hidden) particle oscillations, the photon-mirror(-hidden) photon kinetic mixing are different phenomena which probably share a common underlying physics.

In section II, we will recall the theoretical description of a two-brane universe and we consider quantum dynamics of a spin-1/2 particle in this setup. Next, in section III we underline the different phenomenologies related to the two-brane universe and show that there exists a unified approach supporting the idea that matter may exist in two states, each state having a different location in a higher dimensional bulk. The existence of a swapping mechanism between these two states (which refer here to the duality matter/hidden matter or matter/mirror matter) follows then trivially. Finally, in section IV, we discuss possible experimental conditions in the lab-scale for demonstrating with cold neutrons the existence of these two states.

II. FERMION DYNAMICS AND ELECTRODYNAMICS IN A TWO-BRANE WORLD

Any Universe with two braneworlds is equivalent to a noncommutative two-sheeted spacetime $M_4 \times Z_2$ when one follows the dynamics of particles at low energies [20, 21]. Let us consider a two-brane Universe made of two domain walls (which are two kink-like solitons of a scalar field Φ) on a continuous $M_4 \times R_1$ manifold, the relevant Lagrangian is:

$$\mathcal{L}_{M_4 \times R_1} = -\frac{1}{4G^2} \mathcal{F}_{AB} \mathcal{F}^{AB} + \frac{1}{2} (\partial_A \Phi) (\partial^B \Phi) - V(\Phi) + \bar{\Psi} (i\Gamma^A (\partial_A + i\mathcal{A}_A) - \lambda\Phi) \Psi \quad (1)$$

where $\mathcal{F}_{AB} = \partial_A \mathcal{A}_B - \partial_B \mathcal{A}_A$. \mathcal{A}_A is the $U(1)$ bulk gauge field with the coupling constant G . Φ is the scalar field. The potential $V(\Phi)$ is assumed to allow the existence of kink-like solutions, i.e. of domain walls by following the Rubakov-Shaposhnikov concept [17]. Ψ is the massless fermionic matter field. Ψ is coupled to the scalar field Φ through a Yukawa coupling term $\lambda \bar{\Psi} \Phi \Psi$ with λ the coupling constant. An effective phenomenological discrete two-point space Z_2 can then replace the continuous real extra dimension R_1 . This result is obtained from an approach inspired by the construction of molecular orbitals in quantum chemistry, here extended to fermionic bound states on branes [20]. At each point along the discrete extra dimension Z_2 there is then a four-dimensional spacetime M_4 with its own metric. Both branes can then be considered as separated by a phenomenological distance $\delta = 1/g$. g is proportional to an overlap integral of the fermionic wave functions of each 3-brane over the extra dimension R_1 (see Ref. [20]). In the following, our brane (respectively the hidden brane) will be conveniently la-

beled (+) (respectively (-)). An effective $M_4 \times Z_2$ effective Lagrangian can then be defined [20]:

$$\mathcal{L}_{M_4 \times Z_2} = -\frac{1}{4e^2} \mathcal{F}_{+\mu\nu} \mathcal{F}_+^{\mu\nu} - \frac{1}{4e^2} \mathcal{F}_{-\mu\nu} \mathcal{F}_-^{\mu\nu} - \varepsilon \mathcal{F}_{+\mu\nu} \mathcal{F}_-^{\mu\nu} + \bar{\Psi} (i\mathcal{D}_A - M) \Psi \quad (2)$$

from which one gets the two-brane Dirac equation $(i\mathcal{D}_A - M) \Psi = 0$, such that [20]:

$$\begin{pmatrix} i\gamma^\mu (\partial_\mu + iqA_\mu^+) - m & ig\gamma^5 - im_r + i\gamma^5 \Upsilon \\ ig\gamma^5 + im_r + i\gamma^5 \bar{\Upsilon} & i\gamma^\mu (\partial_\mu + iqA_\mu^-) - m \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \quad (3)$$

ψ_\pm are the wave functions in the branes (\pm). A_μ^\pm (respectively $\mathcal{F}_{\pm\mu\nu}$) are the electromagnetic four-potentials (respectively the electromagnetic tensors) in each brane (\pm). e and ε are effective coupling constants. m is the mass of the bound fermion on a brane. The off-diagonal mass term m_r results from the two-domain-wall Universe [20]. The derivative operator is then: $D_\mu = \mathbf{1}_{8 \times 8} \partial_\mu$ ($\mu = 0, 1, 2, 3$) and $D_5 = ig\sigma_2 \otimes \mathbf{1}_{4 \times 4}$, and the Dirac operator is defined as $\mathcal{D} = \Gamma^N D_N = \Gamma^\mu D_\mu + \Gamma^5 D_5$ where: $\Gamma^\mu = \mathbf{1}_{2 \times 2} \otimes \gamma^\mu$ and $\Gamma^5 = \sigma_3 \otimes \gamma^5$. γ^μ and $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$ are the usual Dirac matrices and σ_k ($k = 1, 2, 3$) the Pauli matrices. Note that Eq. (3) is typical from noncommutative $M_4 \times Z_2$ two-sheeted spacetimes [20].

Concerning the electromagnetic field, it has been proved [20] that the five-dimensional $U(1)$ bulk gauge field must be substituted by an effective $U(1)_+ \otimes U(1)_-$ gauge field in the $M_4 \times Z_2$ spacetime. $U(1)_+$ is the gauge group of the photon field localized on our brane, while $U(1)_-$ is that of the photon field located on the hidden brane. Here, the Dvali-Gabadadze-Shifman mechanism [19] leads to the gauge field localization on the branes [20]. As two branes are considered, the bulk gauge field \mathcal{A}_A splits into A_μ^\pm . The electromagnetic field [20]

$$\mathcal{A} = \begin{pmatrix} iq\gamma^\mu A_\mu^+ & \gamma^5 \Upsilon \\ \gamma^5 \bar{\Upsilon} & iq\gamma^\mu A_\mu^- \end{pmatrix} \quad (4)$$

is introduced in the Dirac equation through $\mathcal{D}_A \rightarrow \mathcal{D} + \mathcal{A}$, according to the $U(1)_+ \otimes U(1)_-$ gauge group. We get [20]

$$\begin{cases} \Upsilon = \varphi + \gamma^5 \phi \\ \bar{\Upsilon} = \varphi^* - \gamma^5 \phi^* \end{cases} \quad (5)$$

where φ and ϕ are the scalar components of the field Υ and $\bar{\Upsilon} = \gamma^0 \Upsilon^\dagger \gamma^0$.

One underlines that the equivalence between two-brane models and the present noncommutative two-sheeted spacetime approach is rather general and relies neither on the domain-wall features nor on the bulk dimensionality [20].

III. INTERPRETATION AND PHENOMENOLOGY

From the above two-brane description it is easy to show that the whole phenomenology of the interactions between particles and hidden particles (or between matter and mirror matter) can be recovered.

A. Photon-hidden (or mirror) photon kinetic mixing

If one assumes a Universe made of two branes, it is logical to deal with a $U(1)_+ \otimes U(1)_-$ gauge field theory as explained above. Though each photon fields live in their own brane, they must undergo a kinetic mixing given by the Lagrangian term (see Eq.(2)):

$$\mathcal{L}_k = -\varepsilon \mathcal{F}_{+\mu\nu} \mathcal{F}_{-}^{\mu\nu} \quad (6)$$

with ε the coupling strength. In fact, the hidden photon can be any kind of photon, such as mirror photon [12] or pseudo-photon [5, 6] for instance. The relation between such a coupling and a brane description was still shown in a stringy context: a $U(1)$ gauge field on the hidden brane is coupled to the $U(1)$ photon field of our brane thanks to a one-loop process [5, 6].

We are not discussing here the phenomenology of such a coupling which has been widely studied by other authors [5, 6, 11, 12]. We just note that a specific mixing between matter and hidden matter can occur through the photon-hidden photon kinetic mixing. This can be illustrated in a naive way in the case of the positronium-hidden positronium oscillations for instance [11]. Indeed, positronium can decay into photons. Since such photons are coupled to hidden photons, which can be materialized into hidden positronium, this allows a coupling between positronium and hidden positronium [11]. Such a second order coupling must occur for neutral particles only. In the following, we are focusing rather on more original matter-hidden matter coupling.

B. Mass and geometrical mixing

Let us focus on the matter-hidden matter swapping. The off-diagonal terms in the two-brane Dirac equation (3) are related to three terms: $ig\gamma^5$, $-im_r$ and $i\gamma^5\Upsilon$.

i. The first term $ig\gamma^5$ is a geometrical mixing. It is specific to the braneworld formalism [20, 21]. Indeed, as explained above g is proportional to the overlap integral of the extra-dimensional fermionic wave functions of each 3-brane over the fifth dimension R_1 . The dramatic influence of this term is emphasized in the next section III C.

ii. The second term $-im_r$ is a mass mixing term. In the following, we will demonstrate that its phenomenology is somewhat different from that arising from geometrical mixing. The difference results from the γ^5 matrix which is not present in the mass mixing term. Mass mixing is often considered for neutron-mirror neutron and neutrino-mirror neutrino couplings [8, 9] or for a coupling with hidden sectors [4]. The present brane approach is fully compatible with all this work. This is because no restrictive hypothesis was done concerning the exact nature of the domain walls. If a wall is a kink-soliton which supports left-handed neutrino, then the second wall can be an antikink-soliton with right-handed neutrino [20]. But we can also imagine a domain-wall pair where particles share the same parity. Note that in the references cited above, neutral particles are mainly considered. The present model is much more general and is fully applicable to any spin-1/2 standard model particle be it neutral or charged.

Another point which deserves further attention concerns neutrinos. In the present model, neutrino-hidden neutrino coupling is only possible through mass mixing. Indeed, no geometrical mixing is possible for particles without any magnetic moment (see section III C).

iii. The third term $i\gamma^5\Upsilon$ can be considered as an electromagnetic coupling resulting from the $U(1)_+ \otimes U(1)_-$ gauge field. Nevertheless, it must not be confused with the coupling resulting from the photon kinetic mixing. In addition, its existence is closely related to $ig\gamma^5$ and $-im_r$. Indeed, if $g = 0$ and $m_r \neq 0$ then $i\gamma^5\Upsilon$ reduces to $i\phi$, whereas if $g \neq 0$ and $m_r = 0$ it reduces to $i\gamma^5\varphi$. If $g = 0$ and $m_r = 0$, the extra term Υ is not required. In addition, it can be shown that $|\varphi|$ (respectively $|\phi|$) should present an amplitude comparable to that of g (respectively m_r) [20]. As a consequence, the two-brane Dirac equation (3) can be rewritten in the more relevant form:

$$\begin{pmatrix} i\gamma^\mu(\partial_\mu + iqA_\mu^+) - m & i\tilde{g}\gamma^5 - i\tilde{m}_r \\ i\tilde{g}^*\gamma^5 + i\tilde{m}_r^* & i\gamma^\mu(\partial_\mu + iqA_\mu^-) - m \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \quad (7)$$

with

$$\begin{cases} \tilde{g} = g + \varphi \\ \tilde{m}_r = m_r - \phi \end{cases} \quad (8)$$

where we have just replaced the field Υ and the coupling constants g and m_r by the effective coupling parameters \tilde{g} and \tilde{m}_r . The gauge field term Υ acts as a correction to the geometrical and mass mixing terms. In addition, without loss of generality, we will consider now that $\tilde{g} \approx g$ and $\tilde{m}_r \approx m_r$ since $|\varphi|$ (respectively $|\phi|$) should not exceed g (respectively m_r) as explained before. This choice allows to further simplify the model.

C. Mass and geometrical mixing at non-relativistic energies

As shown in previous work [20–22], it is relevant to derive the non-relativistic limit of the two-brane Dirac equation. Indeed, the non-relativistic limit leads to a nice and simple Pauli equation where the two-state structure and its phenomenological consequences are more easily tackled. This two-state Pauli equation is: $i\hbar\partial_t\Psi = \{\mathbf{H}_0 + \mathbf{H}_{cm} + \mathbf{H}_c\}\Psi$, with $\mathbf{H}_0 = \text{diag}(\mathbf{H}_+, \mathbf{H}_-)$ and $\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$. Here, ψ_{\pm} are Pauli spinors. Note that \mathbf{H}_{\pm} are the usual four-dimensional Pauli Hamiltonian expressed in each branes. Moreover, new fundamental coupling terms appear (in natural units) [20]:

$$\mathbf{H}_c = \begin{pmatrix} 0 & im_r c^2 \\ -im_r c^2 & 0 \end{pmatrix} \quad (9)$$

which is simply the mass mixing term, and

$$\mathbf{H}_{cm} = ig\mu \begin{pmatrix} 0 & -\sigma \cdot \{\mathbf{A}_+ - \mathbf{A}_-\} \\ \sigma \cdot \{\mathbf{A}_+ - \mathbf{A}_-\} & 0 \end{pmatrix} \quad (10)$$

where \mathbf{A}_{\pm} are the magnetic vector potentials in the branes (\pm) and μ is the magnetic moment of the particle. Clearly, \mathbf{H}_{cm} relates to a mixed geometrical/electromagnetical coupling. The coupling strength becomes clearly dependent from the magnetic potential in each branes. The effect of these new terms is discussed hereafter.

IV. MASS MIXING VERSUS GEOMETRICAL MIXING

In the following, we will mainly study the effect of the mass and geometrical couplings. The second order coupling related to the kinetic mixing is not explicitly studied since it can be described through a correction to the mass mixing term.

A. Spontaneous oscillations between visible and hidden sectors

Let us first consider a situation where V_{\pm} are the gravitational fields felt by the particle in each brane and where there is a magnetic field $\mathbf{B}_+ = \mathbf{B}$ in our brane. We assume that $\mathbf{B}_- = \mathbf{0}$ in the second brane: the existence of hidden magnetic fields will be discussed in section IV E. We set $\mathbf{A} = \mathbf{A}_+ - \mathbf{A}_-$, the ambient magnetic vector potential such that $\nabla \times \mathbf{A} \approx \mathbf{0}$ (see section IV C). In addition, we assume that $\mathbf{A} \gg \mathbf{A}_0$ where $\nabla \times \mathbf{A}_0 = \mathbf{B}$. For the sake of

simplicity, we will consider separately the role of \mathbf{H}_c and \mathbf{H}_{cm} . Therefore, we have to deal with the Hamiltonians:

$$\mathbf{H}_{gm} = \begin{pmatrix} V_+ + \mu\sigma \cdot \mathbf{B} & -ig\mu\sigma \cdot \mathbf{A} \\ ig\mu\sigma \cdot \mathbf{A} & V_- \end{pmatrix} \quad (11)$$

for the geometrical/electromagnetic mixing, and

$$\mathbf{H}_{mm} = \begin{pmatrix} V_+ + \mu\sigma \cdot \mathbf{B} & im_r c^2 \\ -im_r c^2 & V_- \end{pmatrix} \quad (12)$$

for the mass mixing.

Let us set $\eta = (V_+ - V_-)/\hbar$, $b = \mu B/\hbar$ and consider a particle located initially ($t = 0$) on our brane with a polarization state $\mathcal{P} = (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow})$ (with $N_{\uparrow} + N_{\downarrow} = 1$). N_{\uparrow} (respectively N_{\downarrow}) is the probability to find the particle in a spin-up state (respectively in a spin-down state) at $t = 0$ in our brane. At time t , the probability $P_{+,\uparrow}$ (respectively $P_{+,\downarrow}$) to detect the particle in the up state in our brane (respectively in the down state in our brane) is given by:

$$P_{+,\uparrow} = N_{\uparrow} \left(1 - \frac{4\Omega^2}{4\Omega^2 + (\eta - b)^2} \times \sin^2 \left((1/2)\sqrt{4\Omega^2 + (\eta - b)^2}t \right) \right) \quad (13)$$

and

$$P_{+,\downarrow} = N_{\downarrow} \left(1 - \frac{4\Omega^2}{4\Omega^2 + (\eta + b)^2} \times \sin^2 \left((1/2)\sqrt{4\Omega^2 + (\eta + b)^2}t \right) \right) \quad (14)$$

where $\Omega = |m_c|c^2/\hbar$ if we consider the mass mixing or $\Omega = g\mu A/\hbar$ if we consider the geometrical mixing. Eqs. (13) and (14) show that the particle undergoes Rabi-like oscillations between its two states, i.e. between the two branes. It is important to note that the oscillations are strongly suppressed when $|\eta|$ becomes greater than Ω , i.e. when the particle is strongly interacting with its environment. The most striking point with this example, is that there is no difference between mass mixing and geometrical mixing for spontaneous oscillations. The differences will occur in the interpretation of the experimental results. In the following, we discuss the values of the unknown parameters of the problem: g , m_r , \mathbf{A} , η , and \mathbf{B}_- .

B. Magnitude of mass mixing versus geometrical coupling

In a previous work [20], it was shown that g and m_r are related to different overlap integrals of the extra-dimensional fermionic wave functions over the extra dimension (see Eqs. (44) and Appendix B in Ref. [20]).

Though the physical interpretation of each integral is different from each other [20], it is possible to specify some constraints. For a brane and a mirror brane, g and m_r have a similar magnitude [20]:

$$g \approx m_r \propto (1/\xi) \exp(-d/\xi) \quad (15)$$

where d is the distance between each brane and ξ is the brane thickness (with $\hbar = c = 1$). In this case, it is trivial to check from Eqs. (9) and (10) that when $A > A_c = 4mc/eg_s$ (where m is the mass of the particle, and g_s its Landé factor), then the effect of the geometrical mixing becomes larger than that of the mass mixing. This critical value is $A_c = 3.4 \cdot 10^{-3}$ T·m for the electron, while it is $A_c = 3.3$ T·m for the neutron.

These values constitute an important indication for the model. Indeed, values of the order 10^9 T·m are expected for astrophysical magnetic potentials (see section IV C). In that case, the geometrical/electromagnetic coupling \mathbf{H}_{cm} would be larger than the mass mixing \mathbf{H}_c by nine orders of magnitude. Of course, we cannot fully reject the possibility of a domain-wall pair such that $g = 0$ and $m_r \neq 0$, but there is absolutely no evidence for this to date.

C. Magnitude of ambient magnetic vector potentials

Following our previous remark, let us now consider the exact influence that an astrophysical magnetic potential may have on the particle dynamics. It must be stressed that the level of magnitude of such potentials has recently been discussed in the literature [25, 26] in order to give a constraint to the photon mass (which is still assumed massless in the present work). Let \mathbf{A} be the sum of the potentials of the astrophysical objects (galaxy, star, planet) surrounding us. Since each astrophysical object is endowed with a magnetic moment \mathbf{m} , it produces a potential $\mathbf{A}(\mathbf{r}) = (\mu_0/4\pi)(\mathbf{m} \times \mathbf{r})/r^3$ corresponding to a magnetic field $\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r})$.

Following Eq. (10), we note that it is the difference $\mathbf{A} = \mathbf{A}_+ - \mathbf{A}_-$ between the magnetic potentials of each world which is important. As \mathbf{A}_- depends on hidden sources in the other braneworld, we cannot set its value. We should then consider \mathbf{A} as an unknown parameter. Nevertheless, unless that \mathbf{A}_+ and \mathbf{A}_- are fortuitously anti-collinear and almost equal, which is a very unlikely situation, \mathbf{A}_- cannot significantly change the magnitude of \mathbf{A}_+ . As a consequence, an estimation of \mathbf{A}_+ should be a good approximation of the magnitude of \mathbf{A} .

It can be noticed that in the neighborhood of Earth, and at large distances from sources, \mathbf{A} is almost constant (i.e. $\nabla \times \mathbf{A} \approx \mathbf{0}$) and cannot be turned off with magnetic shields. Since $A \sim RB$ (R is the distance from the astrophysical source) the magnitude of the contributions to \mathbf{A} can be deduced. Considering the galactic magnetic field ($B \approx 1 \mu\text{G}$) in relation to the Milky Way core

($R \approx 1.9 \times 10^{19}$ m) then $A \approx 2 \times 10^9$ T·m [26, 27]. When one considers the Coma galactic cluster, values around $A \approx 10^{12}$ T·m are also suggested [25, 27]. Nevertheless, some authors consider this last value as irrelevant as a consequence of the uncertainties on the magnitude of the magnetic fields inhomogeneities at extragalactic scales [26]. The order of magnitude $A = 10^9$ T·m is then usually considered as reliable. The Earth and Sun contributions (200 T·m and 10 T·m respectively) can be neglected [24].

D. Magnitude of gravitational fields in each brane

In the present context, a crucial issue concerns the values of the gravitational fields V_{\pm} felt by the particle in each brane and more specifically the value of $|\eta| = |V_+ - V_-|/\hbar$.

Although gravitation is expected to spread into the bulk, its stretching along the extra dimensions is probably limited [19, 28]. Hence, it can be assumed that the gravitational effects exerted by a mass of a given brane on masses located in the other brane depend on the distance between both branes. Typically, any mass M in the hidden brane acts approximately as a mass M' in our brane such that [28]:

$$M' \sim M \exp(-kd) \quad (16)$$

where d is the distance between branes, and $k = 2\sigma/3\mathcal{M}^3$ with σ the brane tension, and \mathcal{M} the bulk Planck mass [28]. Then, the mass M does not act on our visible world as in its own brane: the gravitation felt by the particle is different in each brane and one gets $\eta \neq 0$ unless that $d = 0$. As a consequence, in previous work [20–24], it was fairly considered that the gravitational fields of each brane are independent and so $\eta \neq 0$. Contrarily, if both branes are close enough such that they share almost the same gravitational field, then $\eta \approx 0$. But since g and m_r also depend on the brane distance in an exponential way (see Eq.15), for very close branes in the limit $d \rightarrow 0$, g and m_r should then take unrealistic high values. As a consequence, it is hard to believe that there is no gravitational role (i.e. that $\eta = 0$) in the amplitude of the matter-hidden matter oscillations.

Of course, the value of $\eta\hbar$ is difficult to determine as the gravitational contribution from the hidden brane (V_-) is unknown. However, rough estimates for gravitational potential energy of neutron give V_+ of the order of 500 eV for the Milky Way core, while the Sun, the Earth, and the Moon have contributions of about 9 eV, 0.65 eV, and 0.1 meV [22–24]. As a consequence, it can be assumed that the value of $|\eta|\hbar$ should range between a few meV up to a few keV. At last, it is clear that $\eta\hbar$ is probably time-dependent. This mainly comes from the Earth motion around the Sun ($\Delta\eta\hbar \sim 0.31$ eV for a neutron on one year) [22–24]. Indeed, it seems unlikely that

Earth can be "close" enough to a stellar-like mass distribution (hidden in the other brane) able to induce a time dependence on timescales of a year or less.

E. Magnitude of hidden magnetic fields

Let us now consider the case of hidden magnetic fields. Magnetic fields becomes significant if $b \approx \eta$ (see Eqs. (13) and (14)), i.e. if the gravitational potential energy contributions are similar to or smaller than the particle energy in a magnetic field. For instance, a magnetic field about 10^5 G leads to $b \approx 600$ neV. The possibility of a gravitational potential energy less or equal to this value is doubtful according to section IV D. However, let us first imagine that the gravitational potentials are similar in both branes (i.e. $\eta = 0$). In that case, the influence of magnetic fields becomes non-negligible and the hidden magnetic fields created by the invisible masses (those located on the other brane) also contribute to the particle dynamics. Obviously, the hidden magnetic fields can be extrapolated from the dark matter distribution (which is here implicitly considered as being the matter located on the other brane). In recent years, dark matter maps have been obtained from its gravitational effects, on the basis of methods relying on gravitational lensing or calculations related to the dynamics of visible objects (stars, nebulae, ...) in interstellar or intergalactic medium [29–35].

Recent observations have confirmed that dark matter constitutes a halo around the visible Milky Way. In addition, accurate simulations show that existence of local dense clouds of dark matter can be fairly excluded (the probability that the solar system is in a dense subhalo is 10^{-4} [32, 33]). Therefore, the density of local dark matter cannot exceed that of the halo which is $\rho_{DM} = 4 \cdot 10^{-25}$ g·cm $^{-3}$ [34]. If one assumes the existence in our solar system of a hidden molecular cloud made of hidden H $_2$ (mirror dihydrogen for instance), it should then exhibit a particle density lower than 0.1 cm $^{-3}$. Such a density is two orders lower than the lower densities of known molecular clouds in the visible Universe [36]. From observations, the strongest magnetic fields in dense parts of clouds (with densities ρ about 10^4 up to 10^7 cm $^{-3}$) varies between 0.1 and 30 mG [36]. For the most common parts in clouds (with densities ρ about 10 up to 10^3 cm $^{-3}$), the magnetic fields are about 5 up to 30 μ G. On the whole, the magnetic field in a molecular cloud varies as ρ^α where $0.47 \leq \alpha \leq 2/3$ according to the kind of molecular cloud [36]. As a consequence, the magnetic field induced by dark matter cannot exceed 0.5 μ G and could be even lower by many orders of magnitude. This value must be compared with the lowest magnetic fields in current ultracold neutrons experiments which are around 10 μ G [37, 38]. Then, when the magnetic field is switched off in these experiments, the hidden magnetic fields still remain negligible in comparison with the residual magnetic

fields in the neutron vessel. Finally, if the gravitational potentials are different in both branes ($\eta \neq 0$), it is expected that $\eta \gg b$. As a consequence, in all cases, the influence of hidden magnetic fields can be neglected.

F. Consequences

The fact that particle and hidden particle could undergo strictly the same gravitational influence ($\eta = 0$), is the less convincing hypothesis due to the reasons mentioned in section IV D. In the following, we admit that $\eta \neq 0$ and $\Omega \ll \eta$, such that the interactions between the particle and its environment ensure its confinement in the brane. Then, the following situations can be considered:

i. The case $b \approx \eta$ where b is related to "weak" magnetic fields (10^{-1} G $< B < 10^4$ G). For this level of magnitude, the neutron magnetic energy is extremely weak if compared with usual gravitational potentials (a magnetic field of about 10^4 G corresponds to an energy of about 60 neV for a neutron). Clearly, $b \approx \eta$ would require some kind of fine-tuning of the distance between the branes and their mass content. Even if this seems rather unlikely, this situation cannot be completely rejected. Though it is not relevant to consider hidden magnetic fields (see section IV E), it is nevertheless interesting to assess the neutron disappearance rate against the magnetic field intensity in our visible world. In the following, it must be kept in mind that neutrons can undergo elastic or inelastic collisions which will inhibit any coherent oscillation behavior. From the point of view of a single particle, each collision resets the oscillatory behavior in a quantum Zeno like effect. This freezing of the oscillations is expected to increase with temperature and neutron density. Let us set $b = b_0 + \delta b$ such that $b_0 \equiv \eta$. From Eqs. (13) and (14), it is clear that a resonance occurs whenever $b = \eta$, i.e. when $\delta b = 0$ (we consider that $|\delta b| \leq 2\Omega$). If $\langle t \rangle$ is the typical time between two consecutive collisions on the vessel walls, we get:

$$P_{+, \uparrow} \sim N_{\uparrow} \left(1 - \Omega^2 \langle t \rangle^2 \right) \quad (17)$$

and

$$P_{+, \downarrow} \sim N_{\downarrow} \left(1 - \frac{\Omega^2}{2b_0^2} \left(1 - \frac{\delta b}{b_0} \right) \right) \quad (18)$$

assuming that $b \gg \langle t \rangle^{-1}$, a condition which is easily achieved in the present context. For instance, for the lowest magnetic field here considered ($B = 10^{-1}$ G) we get $b \approx 920$ s $^{-1}$ while typically $\langle t \rangle^{-1} \approx 10$ s $^{-1}$ [24]. Here it is interesting to note that there is an asymmetry in the swapping rate for different polarization states of the neutron. The swapping should then lead to a shift of the polarization of a neutron gas. Nearly similar situations were studied in detail by Berezghiani *et al.* for the

neutron-mirror neutron mass mixing (see Refs. [9, 10] for instance). But in these cases, no gravitational effects were considered, and the role of η was played by the neutron magnetic energy due to hypothetical hidden magnetic fields [10]. One notes that for a usual statistical thermodynamical set of neutrons at temperature T , one gets: $N_{\uparrow} = (1/2)N_0 \text{sech}(\hbar b/kT) \exp(\mp \hbar b/kT)$ with N_0 the number of neutrons. Then, any measurement should consider the natural polarization shift to avoid a false positive signal. For instance, in Ref. [10] such a thermodynamical constraint was not explicitly considered and could be a source of fallacy. In return, the results of Berezhiani and Nesti [10] could be also a clue of a geometrical mixing instead a mass mixing. This can be justified by the magnitude of \mathbf{H}_{cm} likely higher than that of \mathbf{H}_c as explained in section IV B. As the hidden magnetic fields are negligible (see section IV E), the resonance according to the magnetic field can then be due to the gravitational potentials as shown by Eqs. (13), (14), (17) and (18).

ii. The situation $b \ll \eta$ where b is related to "weak" magnetic fields ($1 \text{ mG} < B < 10^4 \text{ G}$), is the most probable situation. In this case, the role of the magnetic field is negligible and there is no polarization dependence. Due to the expected weakness of Ω related to g or m_r , we get $4\Omega \ll \eta$. Then the particle would exhibit oscillations of high frequency and low amplitude between the two worlds. The probability to observe the particle in the hidden brane can be time-averaged to:

$$p \sim \frac{2\Omega^2}{\eta^2} \quad (19)$$

Considering ultracold neutrons stored in a vessel for instance, they have a probability p to leak from our world toward the hidden one at each wall collision. This topic has been discussed in a recent paper [24] where an upper limit on p has been assessed from experimental values. Constraints on the parameters of the two-brane world were also specified.

G. Resonant oscillations between visible and hidden sectors

The two-brane Pauli equation also supports resonant solutions [20, 22, 23] thanks to the geometrical mixing term \mathbf{H}_{cm} (Eq. (10)). We may consider for instance, a neutron subject to a rotating magnetic vector potential $\mathbf{A}_p = A_p (\cos \omega t \sin \omega t 0)$ (in our brane with an angular frequency ω). We neglect magnetic fields and still consider the gravitational interaction. The probability to find the neutron in the hidden brane is then:

$$P(t) = \frac{4\Omega_p^2}{(\eta - \omega)^2 + 4\Omega_p^2} \sin^2 \left((1/2) \sqrt{(\eta - \omega)^2 + 4\Omega_p^2} t \right) \quad (20)$$

where $\Omega_p = g\mu A_p/\hbar$ and $\eta = (V_+ - V_-)/\hbar$. When $\omega = \eta$, the particle then resonantly oscillates between the branes. Such a resonant matter exchange has been investigated and discussed in recent papers [20, 22–24]. It was suggested that a device involving a frequency comb laser source could be a very efficient way to force the matter swapping between the visible and the hidden sector. In that case, the intensity of the laser source dictates the efficiency of the matter swapping rate, which is then potentially unlimited [23, 24].

V. CONCLUSION

We have shown that the geometrical mixing and the matter swapping between branes, the mass mixing and the particle-hidden particle oscillations, the photon-hidden photon kinetic mixing are the many faces of the same physics involving quantum dynamics of particles in a bulk containing several braneworlds (at least two). Since all these phenomena are deeply interconnected, any positive result coming from an experiment devoted to one of them would be a strong signal for the reality of the others. A rich phenomenology emerges then if one considers hidden matter as matter localized on a hidden brane. The effects of the geometrical mixing are probably the most important ones due to the magnitude of the coupling and because they open the door to an artificial matter exchange between neighboring branes.

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